open up. He expressed his wish to leave the Institute for Advanced Study and to move to the West Coast with facilities better suited to his plans than those available at the Institute. Had the opportunity been granted him, he would probably have overshadowed his earlier achievements. Unless this testimony about von Neumann's last and possibly brightest scientific goal is placed on record, no balanced view of him as scientist can be formed and no fair measure of his career or his motives established.

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Fixed point theory: *An introduction*, by Vasile I. Istratescu, Mathematics and Its Applications, Vol. 7, D. Reidel Publishing Company, Dordrecht, Holland, 1981, xv + 466 pp.

The fixed point theory started almost immediately after the classical analysis began its rapid development. The further growth was motivated mainly by the need to prove existence theorems for differential and integral equations. Thus the fixed point theory started as purely analytical theory. In 1920 S. Banach formulated and proved the general contraction principle in complete metric spaces, which became soon a powerful tool in both classical and modern analysis. Due to its simplicity and generality, the contraction principle has drawn attention of a very large number of mathematicians. After the period of enormous development of linear functional analysis the time was ripe to focus on nonlinear problems. Then the role of the analytical fixed point theory became even more important. On the other hand, the topological fixed point theory started with the famous Brouwer fixed point theorem (1910) and his notion of topological degree of a map. But the big leap for the topological fixed point theory came not until after Schauder (1930) had proved his fixed point theorem in Banach spaces which he successfully applied to existence theorems in partial differential equations.

A new era for the topological fixed point theory began with the Leray-Schauder topological degree of mappings (1934) and its applications to partial differential equations. The development of the fixed point theory is still strong and rapidly growing in both directions, and there is no sign that the fascination of many mathematicians with this subject has stopped. In 1974, D. R. Smart **[10]** published his rather brief (92 pages) introduction to *Fixed point theorems* which contains a number of central theorems and applications motivated by the author's interest. The present book is, in contrast, considerably larger (466 pages) and constitutes a rather successful attempt to provide an informative introduction to the fixed point theory. Although the book presents both theories, the main focus is on the analytical fixed point theory. To make the monograph accessible to a broader audience of nonmathematicians, two preliminary chapters in functional analysis are included. Fortunately, J. Dugundji and A. Granas **[5]** have written their monograph on *Fixed point theory*, volume I, which appeared in 1982, emphasizing the topological aspect of the subject, especially the recent topological development related to the Leray-Schauder theory. Having mentioned this lucky coincidence let us return to the main subject. The book under review contains a host of very interesting applications of the fixed point theory. One of them is Lomonosov's invariant subspace theorem via Schauder's fixed point theorem. This was a real surprise. However, not denying the ingenuity of Lomonosov's proof as an even greater surprise came *Hilden's simple proof of Lomonosov's invariant subspace theorem* **[8]**. The proof is amazingly simple, short, and elementary, indeed, and deserves a footnote in the book.

Since the book is intended for a broader audience, Zorn's lemma could be eliminated from the proof of Caristi's fixed point theorem and replaced, e.g., by an argument from the interesting article of Ekeland **[6]** dealing with the now classical principle on ordered sets which has been discovered by Brézis and Browder **[4]** and which has already made an impact on the fixed point theory. Another new trend in the analytical fixed point theory not mentioned in the book, results from the theory of contractors and contractor directions **[1]** (see also **[2]**). A great many fixed point theorems can be unified and generalized from this angle, and some new results have also been obtained in this way in the random fixed point theory, e.g. **[3, 7, 9]**. In conclusion let us mention that a contraction mapping F in a Banach space has the following interesting property: the sequence

$$x_{n+1} = (1 - \varepsilon_n)x_n + \varepsilon_n F x_n, \qquad (n = 0, 1, \ldots),$$

converges to the fixed point if $0 < \varepsilon_n \leqslant 1$ and $\Sigma_n \varepsilon_n = \infty$. This follows from the theory of contractor directions without using the Banach theorem. This result reminds one of a similar one for nonexpansive mappings F in a uniformly convex Banach space. The first related result was proved by Krasnoselskii for F compact and nonexpansive with $\varepsilon_n = \frac{1}{2}$.

The monograph is well written. Despite some omissions, the book, which is intended for a broader audience, can also be an excellent reference book for a mathematican interested in nonlinear functional analysis. The bibliography of 39 pages is very impressive.

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Groups, trees and projective modules, by Warren Dicks, Lecture Notes in Math., vol. 790, Springer-Verlag, Berlin and New York, 1980, 126 pp., $9.80.

Trees, by Jean-Pierre Serre, Springer-Verlag, Berlin and New York, 1980, ix + 142 pp., $29.80.

Classically, one studies a discrete subgroup Γ of a Lie group G by its action on the homogeneous space $X = G/K$ where K is a maximal compact subgroup of G; for torsion-free Γ, the form $\Gamma\backslash X$ is a $K(\Gamma, 1)$ space. When $G = SL_2(\mathbf{R})$ and $\Gamma = SL_2(\mathbf{Z})$ one has the well-studied reduction theory for Γ and its subgroups acting on the upper half-plane $\mathcal{H} = SL_2(\mathbf{R})/SO_2$ by linear fractional transformations. A program initiated by Bruhat and Tits makes available certain simplicial complexes called buildings which play the role of the symmetric space for p-adic groups [**20, 22, 28**]. For $G = SL_n(\mathbf{Q}_p)$ the building is a contractible $n - 1$ dimensional complex, $T_n(\mathbf{Q}_p)$, in which the vertices are the elements of $SL_n(\mathbf{Q}_p)/SL_n(\mathbf{Z}_p)$ and the simplices come from flags of $\mathbf{Z}_p$-submodules of $\mathbf{Q}_p^n$ which "cover" flags of subspaces of $(\mathbf{Z}/p\mathbf{Z})^n$. When $n = 2$ this Bruhat-Tits tree provides the background fiber to the first part of