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Bases in Banach spaces. II, by Ivan Singer, Springer-Verlag, Berlin and New York, 1981, viii + 880 pp., $78.00.

Introduction. In his book *Bases in Banach spaces*. II (BBS II), Ivan Singer takes all knowledge of bases and their generalizations to be his province. More precisely, he states in the preface that "this volume attempts to present the results known today on generalizations of bases in Banach spaces and some unsolved problems concerning them". *Bases in Banach spaces*. I (BBS I) was published in 1970 and BBS II in 1981. During the writing of these books, basis theory and its generalizations began to develop very rapidly. The task of the author became not that of describing a theory already essentially developed, but of presenting a theory in a very rapid state of development. Thus, in order to achieve his goal of a complete account of basis theory, its generalizations, and its applications, Ivan Singer is working on a third volume on applications, bases in concrete spaces, and perhaps some loose ends.

The book under review, BBS II, is encyclopaedic (at the Banach space level) with respect to its subject. It consists of twenty-one sections plus a section entitled *Notes and remarks*. The review will discuss the section on the solution of the basis problem and the section on basic sequences separately. The remaining nineteen sections of the book are on the known generalizations of bases. From these nineteen sections the reviewer will extract only one of several possible themes for discussion, namely: What are some of the main generalizations of bases that have arisen over the years, how general are they, and how are they related to one another?

The basis problem. Recall that a (*Schauder*) *basis* of an infinite dimensional Banach space (B-space) E is a sequence $\{x_n\}$ in E such that, for each x in E, there exists a unique sequence of scalars $\{a_i\}$ such that $x = \sum_{i=1}^{\infty} a_i x_i$, convergence in the norm topology. For almost forty years the question of Banach ([**2**], 1932), "Does every separable B-space possess a basis?" stood open. A B-space E is said to have the *approximation property* if the identity operator on E can be approximated uniformly on every compact subset of E by continuous linear operators of finite rank. If each of the above continuous linear operators has norm no greater than some positive number λ, the space E is said to have the *bounded approximation property*. When P. Enflo ([**10**], 1973) showed the existence of a separable reflexive B-space without the approximation property he also settled the basis problem in the negative. Enflo's result and his methods inspired a flow of papers on separable B-spaces which fail to have the approximation property. BBS II begins with a section zero which gives two different proofs of the Pełczyński-Figiel ([**29**], 1973) theorem on the existence of subspaces of c_0 and l_p $(2 < p < \infty)$ which fail to have the approximation property. These proofs are refinements of the proofs of Figiel [**11**] and Davie ([**5**], 1973), respectively. Section zero ends with the Lindenstrauss-James ([**24**], 1971) example of a B-space E with a basis whose dual space E^* is separable but does not have the approximation property. This example is used later to construct the decisive example of Figiel and Johnson ([**12**], 1973) of a separable B-space with the approximation property but without the bounded approximation property. Thus there are separable B-spaces with the approximation property which do not possess a basis. Singer's detailed and clear exposition of the proofs of the results mentioned above together with his copious notes and remarks on these outstanding achievements are a valuable contribution to the literature.

Basic sequences. The negative solution of the basis problem not only stimulated research on bases but also provided motivation for studying their generalizations. The first of these is that of basic sequence. A sequence $\{x_n\}$ in a Banach space is a *basic* sequence if it is a basis of its closed linear span $[x_n]$. Banach ([**2**], 1932) observed without proof that such sequences exist in every B-space. Recall that a basis is *unconditional* if the series's convergence in the expansion of each element is unconditional. Bessaga and Pełczyński ([**4**], 1958) asked the question, "Do unconditional basic sequences exist in every B-space?" This remains as probably the most important of the open questions in basis theory. A related question which goes back to Banach's book is: "Does every infinite dimensional B-space have a subspace isomorphic to either c_0 or l_p, for some $1 \leqslant p < \infty$?" Since c_0 and the l_p spaces have unconditional bases, a positive solution to this question would have provided a positive solution to the existence of unconditional basic sequences. However, Tsirelson ([**37**], 1974) gives an example of a reflexive B-space which contains no subspace isomorphic to c_0 or l_p, $1 \leqslant p < \infty$. In contrast to our ignorance about the existence of unconditional basic sequences, it was shown by Pełczyński and Singer ([**30**], 1964) that every B-space with a basis has a conditional basis and from this it follows that every B-space contains a conditional basic sequence.

Bessaga and Pełczyński ([**3**], 1958) wrote a pioneering paper in which sufficient conditions are given for a sequence in a B-space E to contain a basic subsequence. Johnson and Rosenthal ([**19**], 1972) discovered a dual to the main result of Bessaga and Pełczyński. It gives sufficient conditions for a sequence in E^* to contain a subsequence which is a w^*-Schauder basic sequence. Yet another interesting existence theorem is due to Davis, Dean, and Lin ([**6**], 1973) who show that *for every B-space E there exists a biorthogonal system $\{x_n, f_n\}$, ($\{x_n\} \subset E$, $\{f_n\} \subset E^*$) such that $\{x_n\}$ is basic in E, $\{f_n\}$ is basic in E^* and each sequence is norm bounded*. Perhaps the most interesting result in this area is due to Kadec and Pełczyński ([**21**], 1965) who showed that *a sequence $\{x_n\}$ in a B-space contains a basic subsequence if and only if one of the following holds*: (i) *$\{x_n\}$ is not conditionally weakly compact*; (ii) 0 *is a weak limit point of* $\{x_n\}$.

Biorthogonal systems. Recall that if E is a B-space a *biorthogonal system* is a pair of sequences $(\{x_n\}, \{f_n\})$, $\{x_n\} \subset E$, $\{f_n\} \subset E^*$, such that $f_i(x_j) = \delta_{ij}$, $i, j = 1, 2, \ldots$. A sequence $\{x_n\}$ is part of a biorthogonal system if and only if $x_n \notin [x_1, x_2, \ldots, x_{n-1}, x_n, x_{n+1}, \ldots]$, $n = 1, 2, \ldots$. Such a sequence is called a *minimal* sequence. Minimal sequences exist in every B-space. For a minimal sequence $\{x_n\}$ there is a unique sequence of functionals (the coefficient functionals) $\{f_n\}$ which form a biorthogonal pair with $\{x_n\}$ if and only if $\{x_n\}$ is complete. In each separable B-space E, every minimal sequence has an extension to a minimal sequence which is complete in E.

Arsove and Edwards ([**1**], 1960) called a biorthogonal system $\{x_n, f_n\}$, ($\{x_n\} \subset E$, $\{f_n\} \subset E^*$) with $\{f_n\}$ total, a *generalized basis* of E. A space need not be separable to have a generalized basis, e.g. l_∞ has one. In fact a B-space E has a generalized basis if and only if E^* contains a total sequence $\{f_n\}$.

A biorthogonal system $\{x_n, f_n\}$, ($\{x_n\} \subset E$, $\{f_n\} \subset E^*$) with $\{x_n\}$ complete and $\{f_n\}$ total is called an M-basis (Markuševič-basis). Banach ([**2**], 1932) observed, without proof, and Markuševič ([**27**], 1943) proved that every separable B-space has an M-basis. Banach ([**2**], 1943) asked if, for every separable B-space E, there exists an M-basis $\{x_n, f_n\}$ of E such that both $\{x_n\}$ and $\{f_n\}$ are norm bounded? This was finally answered in the affirmative by Ovsepian and Pełczyński ([**28**], 1975). With the help of others their result has been sharpened to the following: *In every separable B-space E and for all $\varepsilon > 0$ there exists an M-basis $\{x_n\}$ with coefficient functionals $\{f_n\}$ such that* $\sup\|x_n\| < 1 + \varepsilon$ *and* $\sup\|f_n\| < 1 + \varepsilon$. It is interesting to note that Davis and Johnson ([**7**], 1973) have shown that if "M-basis" is replaced above by "complete minimal sequence" the conclusion is "$\|x_n\| \leqslant 1 + \varepsilon$ and $\|f_n\| = 1$ for each n". On the other hand, Singer ([**36**], 1971) and Davis and Johnson ([**7**], 1973) have shown that if "M-basis" is replaced by "biorthogonal system $\{x_n, f_n\}$ with $\{f_n\}$ total (i.e. $\{x_n, f_n\}$ is a generalized basis)" then the conclusion is "$\|x_n\| = 1$ and $\|f_n\| \leqslant 1 + \varepsilon$ for each n".

An M-basis can be very far from being a basis since every separable B-space has an M-basis but some separable B-spaces do not have the approximation property. Ruckle ([**33**], 1970; [**34**], 1974) has studied three subclasses of M-bases, two of which he introduced. An M-basis $\{x_n\}$ of E with coefficient functionals $\{f_n\}$ is *strong* if for each $x \in E$ there is a triangular matrix of scalars $(\lambda_{nm}(x))$ such that $x = \lim_n \Sigma_{i=1}^n \lambda_{ni}(x) f_i(x) x_i$. An M-basis $\{x_n, f_n\}$ of E is *strongly series summable* if there exists a triangular matrix (λ_{nm}) (independent of x) such that $x = \lim_n \Sigma_{i=1}^n \lambda_{ni} f_i(x) x_i$ $(x \in E)$. The first notion is due to Ruckle and the second to Frink ([**13**], 1941). Ruckle has introduced a third class of M-bases called *series summable* M-bases which he shows is a subclass of the strong M-bases. It is not known whether the classes of strong and series-summable M-bases coincide. Ruckle shows that the class of strongly series summable M-bases is a proper subclass of the class of series summable M-bases. Furthermore he shows that a B-space, which has a series summable basis, has the approximation property. It is not known whether a B-space with a strongly series summable basis has a basis.

Let $T = (t_{mn})$ be a consistent infinite matrix. A sequence $\{x_n\}$ in a B-space E is a *T-basis* of E if for every $x \in E$ there exists a unique sequence of scalars (a_i) such that the series $\Sigma_{i=1}^\infty a_i x_i$ is T-summable to x. It is interesting to note that Gelbaum ([**14**], 1950) and Kozlov ([**22**], 1950) independently introduced the notion of T-basis in the same year. Furthermore, every T-basis of a B-space E is a strong M-basis of E, and every triangular T-basis of a B-space E is a strongly series summable M-basis of E.

A pair of families $(x_i, f_i)_{i \in I}$ where $\{x_i\}_{i \in I} \subset E$ and $\{f_i\}_{i \in I} \subset E^*$ is called an *extended biorthogonal system* if $f_i(x_j) = \delta_{ij} (i, j \in I)$. An *extended M-basis* of E is an E-complete, total extended biorthogonal system. The results here have been interesting and even surprising. On the one hand not every B-space admits an E-complete extended biorthogonal system, e.g., l_∞ does not. On the other hand every B-space admits a total extended biorthogonal system. If a B-space admits an extended M-basis then it contains no subspace isomorphic to $l_\infty(I)$, I uncountable. It is an open question whether $l_\infty(I)$ can be replaced by l_∞ in the above. On the positive side, if a B-space is weakly compactly generated each of its subspaces has an extended M-basis.

Bases of operators. In the preceding generalizations of the basis concept, each element of the space had an expansion but convergence requirements of the expansions were relaxed. We now examine generalizations in which each element of the space has an expansion which is required to converge to the element. For a basis $\{x_n\}$ of E, two sequences of finite rank linear operators may be defined:

$$P_n\colon E \to E \quad \text{by} \quad P_n(x) = a_n x_n \qquad \left(x = \sum_{i=1}^{\infty} a_i x_i\right) \quad \text{and}$$

$$S_n(x) = \sum_{i=1}^{n} P_i(x), \qquad x \in E, \quad n = 1, 2, \ldots.$$

Both $\{P_n\}$ and $\{S_n\}$ are uniformly bounded sequences of projections satisfying $P_i P_j = \delta_{ij} P_i = \delta_{ij} P_j$ and $S_n S_m = S_m S_n = S_{\min(n,m)}$, respectively. Furthermore, it is true that $x = \lim_n S_n(x) = \Sigma_{i=1}^{\infty} P_i(x)$, $x \in E$. Various selections of these properties have been used as generalizations of the basis concept.

A sequence of finite rank endomorphisms $\{u_n\} \subset L(E, E)$ is called an *approximative basis of operators* of E if $x = \lim u_n(x)$ $(x \in E)$. Approximative bases go back to Banach [**2**, p. 237] who used "compact operators" instead of "finite rank endomorphisms", and he asked whether every separable B-space admitted such a "generalized" basis. The notion of approximative basis of E has an equivalent formulation in terms of a sequence of elements in E and a row finite matrix of functionals in E^*.

A sequence of nonzero endomorphisms of finite rank $\{v_n\} \subset L(E, E)$ is a *finite dimensional expansion of the identity* I_E of E if $x = \Sigma_{i=1}^{\infty} v_i(x)$ $(x \in E)$. A main result of the theory is the following theorem: *For a B-space E the following are equivalent*: (i) *E has an approximative basis.* (ii) *There exists a finite dimensional expansion of the identity I_E.* (iii) *E is separable and has the bounded approximation property.* Coupling this with the fact that there are separable B-spaces without the bounded approximation property it follows that there are separable B-spaces which do not admit approximative bases.

Kadec ([**20**], 1961) calls a complete minimal sequence $\{x_n\}$ in a B-space E an *operational basis* of E if there exists a sequence of endomorphisms v_n: $[x_1,\ldots,x_n] \to [x_1,\ldots,x_n]$ such that $x = \lim_n v_n(\Sigma_{i=1}^{n} f_i(x)x_i)$ $(x \in E)$ where $\{f_n\}$ are the coefficient functionals. Kadec ([**20**], 1961) noted that every operational basis of a B-space E is a norming M-basis of E. Johnson ([**18**], 1970) showed that a B-space has an operational basis if and only if it has a approximative basis.

Schauder decompositions. An infinite sequence $\{G_n\}$ of (not necessarily closed) linear subspaces of a B-space E is called a *decomposition* (or basis of subspaces) of E if, for each $x \in E$, there exists a unique sequence $\{x_n\}$, $x_n \in G_n$ $(n = 1,2,\ldots)$ such that $x = \Sigma_{n=1}^{\infty} x_n$. If each G_n is closed, or, equivalently, if each of the projections $P_n(x) = x_n$ $(n = 1,2,\ldots)$ where $x = \Sigma_{i=1}^{\infty} x_i$ $(x_i \in G_i)$ is continuous, the decomposition is called a *Schauder decomposition*. A finite dimensional decomposition, i.e., one in which each subspace of the decomposition has finite dimension, is automatically a Schauder decomposition. This very natural generalization of the basis concept was introduced by M. M. Grindblium ([**15**], 1950). In the early 1960's Schauder decompositions were studied by this reviewer and his doctoral students, Sanders ([**35**], 1965), Retherford ([**31**], 1966), Ruckle ([**32**], 1964) and others. Although some nonseparable B-spaces admit Schauder decompositions it was shown (with partial results by Sanders and Retherford) by Dean ([**8**], 1967) that l_∞ does not admit a Schauder decomposition. Nevertheless this particular generalization of the basis concept has proved its importance. There are results in terms of Schauder decompositions which do not have analogues for bases or whose analogues for bases are not known or are false. For example, an open question for bases is: If E is an infinite dimensional separable B-space, does there exist a subspace F of E so that both F and E/F have a basis? Johnson and Rosenthal ([**19**], 1972) have shown that *if E is an infinite dimensional separable B-space, there exists a subspace F of E such that both F and E/F have a finite dimensional decomposition.*

Maddaus ([**26**], 1938) first generalized the notion of basis by replacing the sequence $\{x_n\} \subset E$ by a function $X(t)$ on some appropriate space T into a B-space E and by representing each element $x \in E$ as an integral rather than the sum of a series. R. E. Edwards ([**9**], 1960) introduced integral bases of inductive limits of Fréchet spaces. Hale ([**16**], 1969) showed that if a B-space admitted an integral basis in the sense of Edwards then it also possessed a Schauder decomposition. Thus we conclude that l_∞ does not have an integral basis.

If, in the definition of approximative basis of E one replaces the term "finite rank endomorphisms" by "finite rank projections" one obtains the definition of what Lindenstrauss ([**23**], 1964) called a *Π-basis* of E. It is not known whether a B-space with an approximative basis must also have a Π-basis. However, it can be shown that if a B-space has a Π-basis it also has a Π-basis with the additional property $u_m u_n = u_n$ $(n \leqslant m)$. A Π-basis with this additional property is called a *π-basis*. Thus a B-space has a Π-basis if and only if it has a π-basis. Johnson ([**17**], 1976) called a Π-basis $\{u_n\}$ of E with the additional property that $u_m u_n = u_m$ $(m \leqslant n)$ a *dual π-basis*. In all three of these projection type bases if there exists a positive λ such that $\|u_n\| \leqslant \lambda$ for all n, the basis is called a Π_λ-basis, a π_λ-basis, or a dual π_λ-basis. The following main theorem connects these concepts. *For a B-space E the following statements are equivalent*: (i) *E has a finite dimensional decomposition*. (ii) *E has a dual π-basis*. (iii) *E is isomorphic to a space which has a $\Pi_{1+\varepsilon}$-basis for each* $\varepsilon > 0$. (iv) *E has a π-basis $\{u_n\}$ with the property that $u_n u_m = u_m u_n = u_{\min(n,m)}$*, $n, m = 1, 2, \ldots$. It is not known whether the word *dual* may be omitted in the above theorem. It is also not known whether a B-space with a finite dimensional decomposition must also have a basis.

Conclusion. Since there exist B-spaces which do not have integral bases, Schauder decompositions, or even extended M-bases and there exist separable B-spaces which do not possess approximative bases, the quest for more effective generalizations continues. Sections seventeen through twenty of BBS II describe this quest with topics like transfinite bases, extended approximative bases, transfinite Schauder decompositions, and ordinal resolutions of the identity.

BBS I and BBS II by no means exhaust the literature on bases. Singer's project will not be complete until BBS III is written. It is to consist of a chapter on *Applications of bases and their generalizations to the study of Banach spaces* and a chapter on *Bases in concrete spaces*. In a sense, the best (that is, the applications), has been saved until the last. To mention two such applications, basis theory played a decisive role in showing that all separable B-spaces are homeomorphic and in showing that if each subspace of a separable B-space is complemented the space is isomorphic to Hilbert space. It also should be noted that a considerable body of significant work exists on bases and their generalizations in nonlocally convex spaces and in locally convex spaces of importance which are not Banach spaces.

Every expert in basis theory and every student of basis theory should have ready access to BBS I and BBS II. They are the most complete reference books on the subject. Furthermore, the proofs of theorems have been given in readable detail well within the scope of graduate students. Singer likes proofs. He savors them. For some theorems more than one proof is given and in one instance three are given. In order to help the student, the necessary tools from functional analysis have been carefully explained, either by giving them as lemmas with proofs or by giving references to books containing their proofs. Theorems are amplified and clarified by copious remarks and examples which are usually worked out in detail. In each section, open questions in the theory are displayed as problems. At the end of the book is a large, valuable section entitled *Notes and remarks*. The Notes and remarks section gives the author or authors of each theorem, remark, example, and problem, except in the numerous cases when the result is due to Singer himself and has not been elsewhere published. The Notes and remarks section is used also to present recent results, together with their proofs, not included in the text or in numerous cases to mention important results which are to be developed in BBS III—promissory notes as it were.

BBS II contains essentially everything known on each subject at the time of writing and each of its sections could be made the basis of a special topics seminar or course. However, encyclopedias are not good textbooks. A beginning student of basis theory, its generalizations and applications, should have access not only to BBS I and BBS II but also to Classical Banach Spaces I (CBS I) by Lindenstrauss and Tzafriri ([**25**], 1977). In CBS I there is an excellent, concise selection and presentation of main results on bases and their generalizations together with some of their applications. The books BBS (I and II) and CBS I nicely supplement each other. The first is complete with respect to subject matter, and the proofs are truly accessible to graduate students and nonexperts. The second is selective and concise, but the proofs are often sketches.

It is this reviewer's opinion that BBS I and BBS II are a major, even monumental, contribution to basis theory and its generalizations. In this review it is pointed out how Banach's open questions of fifty years ago have directed research and continue to do so. Singer, in taking all basis theory, its generalizations, and its applications as his province has achieved an overview and understanding of the theory which enables him to see the gaps. He has filled in many of these himself and has stimulated other researchers to do likewise. In his unique role as scribe for the whole theory he has faithfully recorded in BBS I and BBS II open questions that remain. Ivan Singer's efforts in the writing of BBS I and BBS II have been a big factor in the acceleration of research in this field and the books with their open questions will influence research in basis theory for years to come.

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