the authors' most significant results, Theorem 12 of *On strong product integration* (Journal of Functional Analysis, vol. 28), was not included. This theorem, in which equation (1) is solved when the operators $A(t)$ are generators of contraction semigroups on a Banach space, notably simplifies and generalizes previous results of Kato and Yosida and would have fit nicely into Chapter 3.

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Nonnegative matrices in the mathematical sciences, by Abraham Berman and Robert J. Plemmons, Academic Press, New York, 1979, xviii + 316 pp., $32.00.

Matrices with nonnegative entries occupy a very special niche in matrix theory, because of their natural importance in a wide variety of applications and because of a long list of very aesthetic mathematical properties, which, among other things, establish their role as a natural generalization of nonnegative real numbers (along side positive semi-definite matrices). Important properties are still being discovered, and, typical of much of modern research in matrix theory, the work often involves an attractive marriage of algebra, analysis, combinatorics, and geometry. It is neither primarily linear nor primarily algebraic. In addition, the properties and applications of nonnegative matrices inspire an array of generalizations both inside and outside of the finite dimensional setting.

The most fundamental facts about nonnegative matrices, established by Perron [**5**] and Frobenius [**1**] about 70 years ago, are mathematically difficult enough that, despite their extreme importance, they do not generally find their way into even advanced undergraduate matrix theory courses. (This is in part due to the fact that a rigorous author considering a chapter in a book for such a course faces a difficult choice between a long drawn out proof or the use of external tools—or no chapter.) Since there are regrettably few graduate courses in matrix theory, the number of serious expository treatments of the subject is sadly limited (to a few survey papers and a few brief book chapters, all showing signs of age, and Seneta's 1973 book [**6**] which is highly specialized). The standard sources until now have been [**2, 7, 8, 9**], the most recent of which is nearly 20 years old. This is doubly unfortunate because there is a significant number of applied topics whose fundamental features are essentially obvious, given just a knowledge of the more basic theory of nonnegative matrices (such as elementary Markov processes, input-output analysis, parts of stability analysis and iterative methods, two-person zero-sum game theory and linear programming). Thus, it is fair to say that a comprehensive book on the subject was long overdue.

The book under review satisfies a good portion of the existing need, and the authors have done the mathematical and applied community a service in preparing what will be a standard reference for several years. But like any research survey, it would not have been written in the same way by other experts. The book is divided into two major parts, the first presenting a development of the mathematical theory (matrices which map a cone into itself, nonnegative matrices, nonnegative matrix semigroups, symmetric nonnegative matrices, and inverse positivity), and the second a collection of chapters each treating a major distinct type of application (M-matrices, iterative methods, Markov chains, input-output analysis, and the linear complementarity problem). The style and level of the book are difficult to classify. In the mathematical part the style is a mixture of self contained rigorous progression and Marcus-Minc [3] shotgun survey, and the level is moderately sophisticated. A very large number of ideas is mentioned, but frequently only with a brief comment and reference. The book contains essentially no previously unpublished work in the field, and portions of the book very closely parallel previous expository and research pieces (with varying degrees of integration). For the reader primarily interested in applications this may not provide an efficient development of the mathematical background (see further comments below), but a reading of the progression of results in chapter 2 should quickly provide a useful working knowledge of the facts. For mathematically interested readers and workers in this field, the mathematical portion of the book offers undoubtedly, at this point, the most complete accessible collection of results and references on the subject.

The primary mode for the theoretical development is cone theoretic. The basic features of the Perron-Frobenius theory are that a nonnegative matrix has an eigenvalue equal to its spectral radius and a corresponding eigenvector which is component-wise nonnegative. If the matrix is irreducible, then the spectral radius has multiplicity one as a root of the characteristic polynomial and the eigenvector is component-wise positive, and, further, if the matrix is primitive (some power is component-wise positive), then the spectral radius is the only eigenvalue of maximal absolute value. Furthermore, in the irreducible case, the spectral radius is the only eigenvalue possessing even a nonnegative eigenvector, the spectral radius is a strictly increasing function of the entries, and the occurrence of ties for the spectral radius is neatly classified (with the basic circulant permutation matrix being the archtypal example). Much of the entire theory depends only upon the fact that the matrix leaves a cone invariant (and the nature of the cone left invariant allows more or less strong conclusions), as has long been known, so that the authors choose the setting of cones as an appropriate level of mathematical generality. The authors' Herculean effort at being encyclopaedic and up to date is largely, but not totally, successful. The field is so massive and so diverse, however, that it would be surprising to do better. Among the missing, though, are discussion of the eigenvector (of primary importance in many applications) as a function of the matrix entries, any use of Gershgorin's theorem (a handy tool in this field), and careful discussion of L-U factorizations (only brief mention is made). The chapter headings (including the symmetric nonnegative inverse eigenvalue problem) indicate some of the specialized topics which are well-treated.

In the applications part of the book, the developments are more elementary, more fully discussed and more self-contained (modulo reference to the mathematical development). These provide surveys with pleasant points of view and could be used as textual material in a number of contexts (e.g. undergraduate applied mathematics students in some cases) where there would be definite benefits. Additional less standard, more modern types of applications (such as the analysis of pair-wise ratio comparisons to establish cardinal rankings or in graph theory and combinatorics [4]) would also have been welcome in this part of the book.

Any treatment of nonnegative matrices would be highly author-dependent, and the present work, which uniquely reflects the authors' tastes, makes a definite contribution. However, there are other possible developments of this subject and a further book in the field should not be precluded. An alternative would have been a work less general but more primarily, deeply and directly about nonnegative matrices and more internally developed. Such might be more useful to the casual applier and also allow fuller internal development of the fancier topics for the mathematician. Perhaps that will come sometime.

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Minimal factorization of matrix and operator functions, by H. Bart, I. Gohberg and M. A. Kaashoek, Operator Theory: Advances and Applications, Birkhäuser Verlag, Basel, Boston, Stuttgart, 1979, v + 227 pp., $17.50.

This is a monograph on system theory and a branch of operator theory known as operator models. It makes basic contributions to both subjects and should be widely read by people in both areas. System theory is a branch of theoretical engineering while model theory evolved as pure mathematics. In the early 1970's it was shown that the two independent subjects are fundamentally equivalent. "Equivalent" is a blurry word to use in comparing two large highly developed fields, and there were discrepancies. Roughly speaking, system theory was primarily finite dimensional, while model theory was well developed in infinite dimensions. On the other hand, model theory worked smoothly