proof by analogy in the first paragraph on p. 292 is unconvincing and the proof in the following paragraph tacitly assumes the converse to the non-equivariant complex Adams conjecture, which is false.)

This volume is addressed to experts in algebraic topology. There is no general introduction and the individual chapters have at most a few sentences of introduction. There is no index and a quite inadequate list of notations. On the other hand, most chapters end with historical comments and a guide to the relevant literature, and there is a very useful bibliography (although several references in the text failed to reach it). The "exercises" tend to be just that early in the book but become references to deeper results and research problems later on. There are numerous misprints. In particular, symbols meant to be completed by hand rather than by typewriter are often incomplete. For example, $\in$ or $=$ may appear where $\notin$ or $\neq$ is intended (e.g., in the statements of Propositions 7.4.3 and 7.7.3). Nevertheless, the experts owe tom Dieck a considerable debt of gratitude, since they will be able to use the book to get some feel for this fascinating new direction in algebraic topology.

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J. P. MAY

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Integral representations, by Irving Reiner and Klaus W. Roggenkamp, Lecture Notes in Math., vol. 744, Springer-Verlag, Berlin, Heidelberg, 1979, 272 pp., $14.30.

Representations of a finite group G are finitely generated RG-modules, where R is a commutative ring. Thus representation theory is largely concerned with the commutative monoids $m(RG)$ where, for any ring Λ, $m(\Lambda)$ denotes the monoid of isomorphism classes of finitely generated Λ-modules with addition given by the direct sum.

Classically R is taken to be the complex numbers. The monoids $m(\mathbf{C}G)$ have a very simple description: they are freely generated by finitely many irreducible modules. Indeed for any field K whose characteristic does not divide the order $|G|$ of G the same is true of $m(KG)$, though the generators depend on K. In all these cases the relevant point is that the ring RG is semisimple, so that all short exact sequences of modules split.

When K is a field whose characteristic divides $|G|$ this is no longer true. However, the Krull-Schmidt-Azumaya theorem still guarantees that $m(KG)$ is freely generated by the indecomposable modules–now to be distinguished from the irreducible ones. It is this case which constitutes the theory of modular representations.

A next step in the generalization of coefficients is to take for R the ring of integers in a field of algebraic numbers: to study, then, the integral representations.

When the coefficients are not a field the representation theory, as might be expected, becomes more complicated. Rings R of integers, or more generally Dedekind domains, which have themselves a relatively simple module theory, provide a reasonable generalization of field coefficients, since one can bring to bear on the module theory of RG the techniques of algebraic number theory.

Prominent among these is localization. There is indeed a level of generality intermediate between field and integral coefficients, namely that in which they are complete P-adic rings. In the integral case the Krull-Schmidt-Azumaya theorem no longer holds in general for RG-modules: $m(RG)$ is not free. But for each P-adic completion $\hat{R}_P$, $m(\hat{R}_P G)$ is free. The use of these to study the more intractable $m(RG)$ exemplifies the application of localization in integral representation theory.

There appear to be two major strategies in the study of integral representations. One of these exploits the fact that $m(\Lambda)$ is functorial in Λ. If $\Lambda \to \Gamma$ is a ring homomorphism then an associated homomorphism $m(\Lambda) \to m(\Gamma)$ is defined by $[M] \to [\Gamma \otimes_\Lambda M]$, where $[M]$ denotes the isomorphism class of the module M. Thus for example if $R \to S$ we have $m(RG) \to m(SG)$; this provides the connection between $m(RG)$ and $m(\hat{R}_P G)$ alluded to above, and also relates $m(RG)$ to $m(KG)$ where K is the quotient field of a domain R. If on the other hand H is a subgroup of G then $m(RH) \to m(RG)$ gives the induced representations. A number of powerful induction theorems extract information about representations of a group from the module theory of more tractable subgroups.

Another application of this functoriality uses the fact that when R is a domain of characteristic 0 then RG is an order in the semisimple algebra KG and is thus contained in a maximal order Λ. Maximal orders have in general a simpler module theory than nonmaximal ones and thus the homomorphisms $m(RG) \to m(\Lambda)$ may also be used to provide information about $m(RG)$.

The other strategy is to study in parallel with $m(RG)$ associated objects which may be more accessible to computation and may in addition embody independently interesting information. For any R-order Λ for example the Λ-lattices, i.e., the finitely generated R-projective Λ-modules, form a submonoid $l(\Lambda) \subset m(\Lambda)$, while the Λ-projective modules form a submonoid $p(\Lambda) \subset l(\Lambda)$. All these objects may be further "stabilized", monoids being somewhat cumbersome, by substituting for them their abelian group reflections, in effect the abelian groups with the same generators and relations. The notations $m(\Lambda)$, $l(\Lambda)$, $p(\Lambda)$ are *ad hoc*; Reiner uses $a(\Lambda)$ for the group reflection of $l(\Lambda)$; the notation $K_0(\Lambda)$ for that of $p(\Lambda)$ is of course familiar.

In $a(RG)$ there is a multiplication given by $[M] \cdot [N] = [M \otimes_R N]$, with the diagonal operation of G on the tensor product, making $a(RG)$ a ring. It is interesting to divide this by the ideal generated by the elements $[M'] - [M] + [M'']$ associated to short exact sequences $0 \to M' \to M \to M'' \to 0$. The quotient is the Grothendieck ring $G_0(RG)$. As the subscripts are meant to indicate $K_0(\Lambda)$ and $G_0(\Lambda)$ are the first terms in sequences $\{K_n(\Lambda)\}$ and $\{G_n(\Lambda)\}$ of groups–$K_1(\Lambda)$ and $G_1(\Lambda)$ are associated with automorphisms of the modules in question. The study of these constitutes algebraic K-theory, with which rapidly growing field integral representation theory thus finds itself sharing a common border.

This *Lecture Notes* volume is made up of two parts: *Topics in integral representation theory* by Irving Reiner and *Integral representations and presentations of finite groups* by Klaus W. Roggenkamp.

The first of these is a general account of integral representation theory, describing the strategies outlined above as well as some of the major results, such as Swan's characterization of the projective modules over RG for suitable Dedekind domains R, Dade's theorem, which gives conditions under which $l(RG)$ is infinitely generated and Bass' computation of the rank of $K_1(\mathbf{Z}G)$. Its brevity prevents it from being self-contained: it depends on a certain amount of information about classical and modular representation theory, as well as about orders in algebras–all readily available in previous books by the author. At the other end its articulation with K-theory depends–no doubt inevitably–on references to the periodical literature. It provides nevertheless an admirable conspectus of the field, organizing a remarkably large body of material in coherent fashion and demonstrating the styles of argument which have dominated its progress.

"*Integral representations and presentations of finite groups*" in contrast undertakes a more exhaustive discussion of the narrower field indicated in its title, and in particular of the recent work of K. W. Gruenberg and the author.

The access to this area is to a large extent *via* the relation modules of a group. If $\mathfrak{R}$ is the kernel of a free resolution, i.e., an epimorphism $F \to G$ with F free, then $\overline{\mathfrak{R}} = \mathfrak{R}/[\mathfrak{R}, \mathfrak{R}]$ is a $\mathbf{Z}G$-module, the relation module associated to the resolution. These modules are involved in a variety of ways with the properties of G; a sampling of these may give some idea of the scope of this exposition.

If $d(G)$ is the minimal number of generators of G, $d_G(N)$ the minimal number of generators of $N \lhd G$ as a normal subgroup and $d_\Lambda(M)$ the minimal number of generators of a Λ-module M then $d_F\mathfrak{R}$ is the minimal number of relators in a presentation of G by the generators of a free resolution $F \to G$. Evidently $d_F\mathfrak{R} \geqslant d_{\mathbf{Z}G}\overline{\mathfrak{R}}$. But $d_F(\mathfrak{R})$ depends on the resolution even when it is minimal, i.e., when $d(F) = d(G)$, in particular $d(F) - d_F(\mathfrak{R})$ is not an invariant.

The presentation rank $\operatorname{pr}(G) = d(G) - d_{\mathbf{Z}G}(\mathfrak{g})$, where $\mathfrak{g}$ is the augmentation ideal (the kernel of $\mathbf{Z}G \to \mathbf{Z}$) is an invariant of appropriate type. Any relation module $\overline{\mathfrak{R}}$ can be nonuniquely factored as $A \oplus P$ where P is projective and A has no nontrivial projective summand; such an A is a "relation core". Given any relation core A^0 and any minimal relation module $\overline{\mathfrak{R}}$, $\overline{\mathfrak{R}}_{(G)} \approx A^{00}_{(G)} \oplus (\mathbf{Z}_{(G)}G)^{\mathrm{pr}(G)}$, where $(\ \ )_{(G)}$ denotes semilocalization at the primes dividing $|G|$. It is shown that $\mathrm{pr}(G) = 0$ if $d(G) \leqslant 2$ or G is solvable; on the other hand if G is perfect then $\lim_{n\to\infty} \mathrm{pr}(G^n) = \infty$.

Decomposibility of modules appears as a recurrent theme, as does also its generalization to what might be called stable decomposability (the author prefers to say that stably indecomposable modules are "Heller modules"). A module M is stably decomposable if for some projective P, $M \oplus P$ decomposes as the sum of two nonprojective modules. A typical result asserts that for solvable G, $\mathbf{Z}$ is stably indecomposable if and only if G is a special sort of iterated semidirect product (a 2-Frobenius group).

A notable technical innovation is the introduction of extension categories as an amplification of the module categories which underlie the characterization of representation theory advanced above. In fact relation modules appear in the character of group extensions $\overline{\mathfrak{R}} \to F/[\mathfrak{R}, \mathfrak{R}] \to G$ with abelian kernel, thus as objects in the category $(\frac{G}{\ })$ of such extensions or alternatively in the equivalent category $(\frac{\mathfrak{g}}{\ })$ of module extensions $A \to E \to \mathfrak{g}$.

From this point of view new representation–theoretic phenomena become prominent. For example Frattini extensions $A \to E \to G$, that is to say those for which $H < E$, $A \cdot H = E$ imply $H = E$, correspond to extensions $A \to M \xrightarrow{\varphi} \mathfrak{g}$ such that $\varphi\psi$ epic implies ψ epic. These essential epimorphisms φ are dual to the essential monomorphisms, such as the inclusions of modules into their injective envelopes. While injective envelopes always exist, the dual projective covers do not. Here once more is a typical result: $\mathfrak{g}_{(G)}$ has a projective cover if and only if G is p-primary or cyclic.

In this application the extension categories have been relativized by restricting the kernel to be a module over $\mathbf{Z}_{(G)}$. Other relativizations are also interesting: e.g., to modules with trivial operation of G, thus to central extensions. The fundamental theorem on Schur multiplicators appears in this context as a special case of a more general result.

In contrast to the first part of these *Lecture Notes*, the second is substantially a report on work in progress. No doubt as this work advances the organizing principles of the field will become more apparent. The reader should perhaps be reassured that the somewhat germanic grammar of this essay does not detract from its otherwise adequate readability.

ALEX HELLER