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Elements of homotopy theory, by George W. Whitehead, Graduate Texts in Math., vol. 61, Springer-Verlag, New York and Berlin, 1978, xxii + 744 pp.

Good news! George Whitehead has completed Volume 1 of the great American encyclopaedic treatise on homotopy-theory.

The approach adopted in this book is well described by the author.

"As the title suggests, this book is concerned with the elementary portion of the subject of homotopy theory. It is assumed that the reader is familiar with the fundamental group and with singular homology theory ....

"Anyone who has taught a course in algebraic topology is familar with the fact that a formidable amount of technical machinery must be introduced and mastered before the simplest applications can be made. This phenomenon is also observable in the more advanced parts of the subject. I have attempted to short-circuit it by making maximal use of elementary methods. This approach entails a leisurely exposition in which brevity and perhaps elegance are sacrificed in favour of concreteness and ease of application ....

"It is a consequence of this approach that the order is to a certain extent historical ....

"As I have stated, this book has been a mere introduction to the subject of homotopy theory. The rapid development of the subject in recent years has been made possible by more powerful and sophisticated algebraic techniques. I plan to devote a second volume to these developments."

Two quick comments take precedence over other business. One must agree that brevity is sacrificed; after all, we have a book of xxi + 744 pages. One need not agree that elegance is sacrificed; what is done (and there is a great deal of it) is done beautifully.

So what is done? To a first approximation, Chapters 1–12 contain all of homotopy theory that can be done without spectral sequences. Then Chapter 13 introduces spectral sequences; this sets the stage for the second volume, and incidentally it yields Serre's results on the finiteness of homotopy groups of spheres as the last major results in the book.

In this review there seems little point in summarizing material which most readers would believe belongs in a text on "elements of homotopy theory", up to and including Postnikov systems. However, the amount of homotopy theory that can be done without spectral sequences is more than you might think, particularly if the person doing it has a taste for concrete, down-to-earth methods which allow one to do sums. Some examples may help to give the flavour.

The author finds himself in a position to construct the Steenrod squares–because his suspension theorems are good enough to desuspend the cup-square. He proves the Jacobi identity for Whitehead products, by the method of his paper "On mappings into group-like spaces" (after all, it is the best way). He proves the Hilton-Milnor theorem, with an explicit equivalence whose components are the basic Whitehead products (so that the reader can calculate with Hilton-Hopf invariants). Particular attention is paid to the homotopy-theory of Lie groups and their homogeneous spaces; and on p. 701 the author starts a 14 page section on the exceptional Lie group F_4 (any friend of F_4 is a friend of mine).

One negative example may be indicative too. I did not notice anything about E. H. Brown's Representability Theorem; it may be elementary to prove it, but it doesn't help you to do sums.

The author also says "it is my hope that this approach will make homotopy theory accessible to workers in a wide range of other subjects . . . ". This may provoke further comments. First, I sympathise with the approach, and of course I would agree that homotopy-theory should justify itself by serving the needs of other subjects. But secondly, many of these "workers in a wide range of other subjects" will need material out of the second volume (maybe localisation, for example). And thirdly, some of them will need shorter presentations, to establish initial communication and transmit a few key ideas. I well remember that I learnt the basic ideas of obstruction-theory by word of mouth, which can be very efficient; as I recall, it took ten minutes. There must have been less in those ten minutes than there is in Chapter 6 of this book; but it was what I needed at the time. We shall go on needing lecture-courses, too. The teachers who give these courses have to select the material they present so as to produce the maximum enlightenment they can in 24 or 30 lectures; that might seem like 120 to 150 pages' worth. There may be a case for books written on the same selective principle, so as to convey a few key ideas in a limited space; that is not the task the present author has set himself.

Such consumer reaction as I have sampled is favourable; students who already have a sufficiency of the basic ideas like the book; they can see that while any book of this length may appear "formidable" globally, this particular one is not formidable locally. Therefore, this book is a success with them.

To conclude, I think George Whitehead proves at least one thing we already knew: in calibre as a mathematician, and for qualities of taste, style and judgement, he rates a lot higher than many who have written books on algebraic topology. I welcome this book most warmly, and I look forward to the second volume.

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Percy Alexander MacMahon: *Collected papers*, Volume I, *Combinatorics*, edited by George E. Andrews, Mathematicians of Our Time Series, The MIT Press, Cambridge, Massachusetts, 1978, xxx + 1438 pp., $75.00.

This is possibly a unique presentation of collected papers. First, papers are collected in chapters according to their consonance with the sections of MacMahon's treatise, Combinatory Analysis, whose order they follow. Each chapter begins with an (editorial) Introduction and Commentary, followed by a presentation of related current work, References (sizable bibliographies) and Summaries of the Papers (sometimes including alternative proofs of theorems). The editor's diligence and stamina are most impressive.

The papers, like the treatise, are motivated by MacMahon's ambitious objective (expressed in the preface to the second volume of the treatise) of "presentation of processes of great generality, and of new ideas, which have not up to the present time found a place in any book in any language".

Shorn of its context, this statement may appear more brashly confident than its author intended. It is preceded by a lengthy and glowing appreciation of Eugen Netto's *Kombinatorik*, possibly only to show that he has chosen a different path with due deliberation.

Nevertheless, the pursuit of originality and generality has its perils. For one thing, the current spate of combinatorial mappings has produced the feeling that multiplicity abounds. Perhaps the simplest example is the continuing appearances of the Catalan numbers ($(2n)!/n!(n+1)!$, $n = 0, 1, \ldots$), whose number sequence (No. 577 in Neil Sloane's *Handbook of integer sequences*, Academic Press, 1973) is 1, 1, 2, 5, 14, 42, ... Incidentally, these numbers are named after E. Catalan because of a citation in Netto's *Kombinatorik*, in relation to perhaps the simplest bracketing problem, proposed in 1838. An earlier appearance, which I first learned from Henry Gould, is due to the Euler trio, Euler-Fuss-Segner, dated 1761. There are now at least forty mappings, hence, forty diverse settings for this sequence; worse still, no end seems in sight. In this light, the Catalan (or Euler-Fuss-Segner) originality may be regarded as temporary blindness.

As for generality, MacMahon (paper 52, Chapter 7), gives a general solution of the Latin square problem–requiring the determination of coefficients in the expansion of powers of a multivariable sum. Since current