appeared in the last couple of years concerning the finite element approximation, the few pages devoted to this subject seem to be either superfluous, or completely inadequate even as an introduction to this topic. On the other hand, some topics seem to stop short of mentioning some important recent results. For example, the only recent results on continuation of solutions reported in this book seem to be restricted to the theorems of G. H. Hile and M. Protter.

It was a surprise to find that Gårding's inequality was never mentioned in a book on elliptic partial differential equations.

Some minor misprints exist. For example, Carleman's name is misspelled on p. 212. Also, the spelling of all Russian names leaves a lot to be desired. Olga Ladyženskaja is spelled as "Ladyshenkaja" in the index. Lavrentjev is spelled "Lavrentieff" on p. 40 and "Lavrentjev" in the index. Lopatinskiĭ is spelled in two different ways on consecutive lines in the index. The index seems to be a minor disaster area. Rellich's theorem is supposed to be on p. 379. It turns out to be on p. 369. The estimate of Astrahancev is supposed to be on p. 366, but it actually is quoted on p. 367. The reviewer selected 10 items at random. Four turned out to be misnumbered in the index. On the positive side, this book leaves a general impression of competence, it is well written, contains a lot of information, and a vast amount of bibliography. Each chapter contains references and a list of additional references, plus remarks concerning the additional references. For example, the chapter on elliptic boundary value problems contains 116 references, and the chapter on singular integral equations contains 119 references. The reviewer recommends this book as a supplement to courses in partial differential equations and as a useful addition to the library of any analyst.

VADIM KOMKOV

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 2, Number 1, January 1980

0002-9904/80/0000-0015/$03.00

Introduction to modular forms, by Serge Lang, A Series of Comprehensive Studies in Mathematics, No. 222, Springer-Verlag, Berlin and New York, 1976, ix + 261 pp.

Modular forms are generalizations of the trigonometric functions which have proved to be very useful in number theory, physics, and geometry. In their most obscene generality they go by the name of automorphic forms, and this name refers to functions on symmetric spaces, such as the Poincaré-Lobachevskii upper half plane H, which satisfy certain differential equations (usually those of Cauchy and Riemann) and exhibit invariance properties under a discrete group of isometries of H, such as the modular group $\mathrm{SL}(2, \mathbf{Z})$ of 2×2 matrices of determinant one and integer entries, acting on H by fractional linear transformation. Number theorists, as will be seen, are often interested in congruence subgroups such as

$$\Gamma_0(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}(2, \mathbf{Z}) \middle| c \equiv 0 \pmod{N} \right\}.$$

One of the oldest and most useful examples of a modular form is one of Jacobi's theta functions:

$$\theta(z) = \sum_{n \geq 1} \exp(\pi i n^2 z), \quad \text{for } z \text{ in } H.$$

Theta is easily seen (using the Poisson summation formula) to have the invariance properties: $\theta(z+2) = \theta(z)$ and $\theta(-1/z) = (z/i)^{1/2}\theta(z)$. And $\theta(2z)$ can be shown to be a modular form for $\Gamma_0(4)$. Many of the first applications of theta were in physics. Fourier, for example, needed theta to solve the heat equation, as is demonstrated in the book of Dym and McKean [7, p. 64]. Another reason that Jacobi's four basic theta functions are discussed in physics texts such as Morse and Feshbach [**17**, Vol. I, pp. 430–432; Vol. II, p. 1251] is that they provide computable expressions for the elliptic integrals which give conformal mappings solving Dirichlet problems in the plane. Theta functions also appear in the solutions by Euler, Lagrange and Poisson of two special cases of the problem of describing the motion of a solid body rotating about a fixed point. The third known case of this problem was solved by Sonya Kovalevsky in work for which she was awarded the Prix Bordin in 1888. Her paper (Acta Math. **12** (1889), 177–232) shows that she needed generalizations of elliptic integrals known as abelian integrals and therefore theta functions of more than one variable. Many other physical problems lead to these hairier integrals and thus to the higher degree theta functions which were studied by Riemann, Weierstrass, Hermite and others in the 1800s. The higher degree theta functions live on the symmetric space H_n of the symplectic group. This space H_n of symmetric $n \times n$ complex matrices with positive imaginary part is known as Siegel's upper half space as a result of Siegel's important work on the subject. The reader can find a nice treatment of these things in Siegel's books [**23**]. The theta functions on H_n are also intrinsic to Siegel's remarkable work on quadratic forms as developed in papers 20, 22, 26 of [**22**, Vol. I]. This work has recently been connected with quantum mechanics via the Segal-Shale-Weil representation discussed by Cartier in [**3**, pp. 361–386] and Wallach in [**27**]. Evidently as great a mathematician as Picard told Sonya Kovalevsky in 1886 that he was sceptical that theta functions on H_2 "can be useful in the integration of certain differential equations". But 90 years later in the paper of Dubrovin, Matveev, and Novikov (Russian Math. Surveys **31** (1976), 59–146), which begins with the quote from Picard, theta functions are used to solve the Korteweg-de Vries type partial differential equations. It often helps to think of algebraic geometry and Riemann surface theory to understand these things. Algebraic geometers view theta functions as providing projective embeddings of abelian varieties. This point of view is expressed in Baily's paper [**3**, pp. 306–311], for example. The book of Lang under review does not mention theta functions, perhaps because this vast subject demands a book of its own. And certainly Lang has treated some of these matters in other books.

Perhaps the main reason that number theorists love theta functions is that number theorists are enamored of zeta functions. The connection between the two types of functions is exemplified by the following Mellin transform result–a result used by Riemann during the 1850s in his work on the zeta function that bears his name (if Re $s > 1$):

$$\Lambda(s) = 2\pi^{-s}\Gamma(s)\zeta(2s) = \int_0^\infty y^{s-1}(\theta(iy) - 1)\,dy, \qquad \zeta(s) = \sum_{n\geq 1} n^{-s}.$$

Riemann showed how to use this formula to obtain the analytic continuation of $\zeta(s)$ to a meromorphic function in the complex s-plane with a simple pole at $s = 1$. The idea is to use Jacobi's transformation formula $\theta(1/z) = (z/i)^{1/2}\theta(z)$ to deal with the integral over small y, where the series for theta converges very slowly. As a result, Riemann found a formula for $\Lambda(s)$ as a sum of 2 pole terms and a series of incomplete gamma functions. The latter series converges exponentially faster than the original Dirichlet series for $\zeta(s)$, to an entire function of s. Moreover the transformation formula for theta implies that zeta satisfies the functional equation $\Lambda(s) = \Lambda(\frac{1}{2} - s)$. This method has also been used since the early part of this century in crystal physics to compute potentials of crystal lattices–the potentials being realized as Epstein zeta functions. More details on this can be found in the reviewer's paper **[26]**. It was one of Hecke's original impressive contributions to algebraic number theory to note that the same sort of argument works for the Dedekind zeta function of an algebraic number field, once one finds the appropriate theta function of several variables.

In the 1930s Hecke made the discovery (see his lectures **[10]**) that Riemann's idea can be turned around and vastly generalized. The Mellin transform and its inverse thus provide a dictionary (called the Hecke correspondence) translating from zeta and L-functions of number theory to modular forms for congruence subgroups and conversely. Weil revitalized this subject with his 1967 paper **[28]**. Number theorists are interested in the Dedekind zeta function because its residue at $s = 1$ contains the basic invariants of the number field. Thus many open problems in algebraic number theory are often approached by trying to figure out what the Dedekind zeta function is doing near $s = 1$. This, in turn, leads one (as Stark described in **[24]**) to investigate the Artin L-function of a representation of a Galois group of an extension of number fields. The big open problem here is to prove Artin's conjecture that the Artin L-function is entire if the representation does not contain the unit representation. Suppose one knew the truth of Artin's conjecture for the Artin L-function of RR_1, with R a fixed irreducible 2-dimensional representation of a Galois group G of a number field $K/\mathbf{Q}$, and for all 1-dimensional representations R_1 of G. Then the Hecke-Weil-Langlands theory would say that the inverse Mellin transform of the Artin L-function for R (times its gamma factors) is a modular form of weight 1 for some congruence subgroup. Deligne and Serre have also proved a converse result. Thus recent progress on Artin's conjecture involves modular forms. References are the Corvallis conference lectures on base change **[4]**, Gelbart's paper in **[6**, Vol. VI, pp. 241–276], Serre's paper in **[8**, pp. 193–268], Tate's talk on Hilbert's 9th problem **[5**, pp. 311–322].

The preceding was an attempt to give a number theorist's explanation for the fact that Lang's book concerns in large part modular forms for congruence subgroups of SL(2, **Z**). (However, we should note that the book does not really treat the Hecke-Weil correspondence between modular forms and

Dirichlet series except in the case SL(2, **Z**).) A complex analyst might wish instead to look at arbitrary subgroups Γ of SL(2, **C**), since all analytic functions whose Riemann surface is D can in fact be realized as automorphic forms on some D/Γ. But a number theorist wants to look at the arithmetic groups like $\Gamma_0(N)$ and the specific modular forms which help to solve number-theoretic problems.

Another important example of a modular form is the Eisenstein series

$$G_k(z) = \sum_{(m,n)\neq(0,0)} (mz + n)^{-k}, \qquad k = 4, 6, 8, \ldots,$$

which satisfies

$$G_k\left(\frac{az + b}{cz + d}\right) = (cz + d)^k G_k(z), \quad \text{for} \begin{pmatrix} a & b \\ c & d \end{pmatrix} \text{ in } \mathrm{SL}(2, \mathbf{Z}).$$

Thus G_k is a modular form of weight k for SL(2, **Z**). For some mysterious reason Lang creates this function in Chapter 1 of his book but chooses not to name it until the last page. The Eisenstein series G_4 and G_6 are building blocks for all modular forms for SL(2, **Z**). For example, they can be combined to get the modular invariant j first constructed by Dedekind and Klein in the 1870s:

$$j = 1728\,(60G_4)^3/\Delta, \quad \text{with } \Delta = (60G_4)^3 - 27(140G_6)^2.$$

The invariant j maps $H/\mathrm{SL}(2, \mathbf{Z})$ conformally one-to-one onto **C**. It is called the Hauptmodul, since every modular form of weight one (or modular function) is a rational function of j. A reference for these things is Serre's book [**18**]. The mapping given by j can be used to obtain a quick proof of the small Picard theorem. Moreover, j characterizes elliptic curves and its special values yield class fields of imaginary quadratic fields, as is shown in the seminar on complex multiplication [**2**]. Class fields are extensions of number fields with abelian Galois group. Their theory was developed in the early part of this century by Takagi, Artin, Hasse et al. The theory includes a reciprocity law due to Artin, which vastly generalizes the quadratic reciprocity law of Gauss. Hilbert's 12th problem asks for a generalization of the construction of class fields of imaginary quadratic fields via modular forms–a construction which would include an explicit version of the reciprocity law for class fields over any base field. This problem has stimulated much of the work on modular forms and higher dimensional generalizations such as Hilbert and Siegel modular forms. Hecke's first papers with their unfortunate errors sought a theory for real quadratic fields. The main problem seems to be that the appropriate generalization of the discriminant function, $\Delta(z)$ from the denominator of j, does not exist. The most success on Hilbert's 12th problem has no doubt been achieved by Shimura (see his book [**20**]). Langland's article in [**5**] gives his view of the subject. There is quite a different approach to Hilbert's 12th problem due to Stark, an approach which we will discuss later. A related application of modular forms comes in the solution of polynomial equations. For example, consider the famous result of Hermite, Kronecker, and Brioschi on the solution of the general equation of 5th degree via modular forms. It should also be noted that Hecke himself obtained a proof of the quadratic reciprocity law for number fields, using the transformation properties of theta functions.

There are three more themes in the subject that must be mentioned–Fourier expansions, Euler products, and values of L-functions corresponding to modular forms by Mellin transform. A modular form $f(z)$ will be periodic in the real part of z. Thus it must have a Fourier expansion, which is assumed by definition to look like:

$$f(z) = \sum_{n \geq 0} a_n \exp(2\pi i n z).$$

The form f is called a cusp form if the constant term $a_0 = 0$. The term cusp form indicates that the function will vanish at the cusp at infinity in the fundamental domain for $H/\mathrm{SL}(2, \mathbf{Z})$, illustrated in Lehner's book [**13**, p. 5]. One can show that the discriminant $\Delta(z)$ is a cusp form for $\mathrm{SL}(2, \mathbf{Z})$, which does not vanish at any finite point in H (the useful property for construction of j). The Fourier coefficients a_n are often very interesting integers, for a number theorist's obscure tastes anyway. For example, if $f = G_k$, the Eisenstein series, then a_0 involves $\zeta(k)$ and a_n, $n > 0$, involves the divisor function:

$$\sigma_{k-1}(n) = \sum_{0<d|n} d^{k-1}.$$

The fact that G_4 and G_6 can be normalized to have integer coefficients is important for Swinnerton-Dyer's recent theory of modular forms mod p, obtained by reducing the Fourier coefficients modulo p. Swinnerton-Dyer was seeking an explanation for Ramanujan's congruences involving the Ramanujan numbers $\tau(n)$ in

$$\Delta(z)(2\pi)^{-12} = \sum_{n \geq 1} \tau(n) \exp(2\pi i n z),$$

$\Delta(z)$ from the denominator of j. The congruences relate the $\tau(n)$ and divisor functions modulo powers of 2, 3, 5, 7, 23 and 691 (see Swinnerton-Dyer's article [**6**, III, pp. 1–56]). Lang's Chapters 10 and 11 are somewhat motivated by the desire to know why these congruences do not occur for large primes. There are also connections with Artin L-functions and Serre's work on p-adic modular forms. Before leaving the subject of Fourier coefficients, it seems imperative to mention that recently Deligne proved an old conjecture of Ramanujan which says that $|\tau(n)| \leq n^{11/2}\sigma_0(n)$, using the Weil conjectures on zeta functions of certain varieties over finite fields. In fact Deligne's proof of the Weil conjectures includes an argument motivated by some of Rankin's work on the Ramanujan conjecture. This is described by Katz in [**5**, pp. 286–288].

There is still more to say about Hecke's correspondence between modular forms and Dirichlet series via the Mellin transform and its inverse. Hecke also found a way of characterizing the modular forms corresponding to Dirichlet series with Euler products analogous to that known by Euler:

$$\zeta(s) = \prod_{p \text{ prime}} (1 - p^{-s})^{-1}, \qquad \operatorname{Re} s > 1.$$

Hecke defined some linear operators now called Hecke operators on the finite dimensional vector spaces of modular forms of weight k for $SL(2, \mathbf{Z})$ and similar groups. And he showed that the eigenfunctions of the Hecke operators are exactly the modular forms

$$f(z) = \sum_{n \geq 0} a_n \exp(2\pi i n z)$$

such that the corresponding Dirichlet series $L_f(s) = \sum_{n \geq 1} a_n n^{-s}$ has an Euler product. Lang discusses this in Chapters 2 and 3. Note for example that the Eisenstein series has to be an eigenfunction of the Hecke operators since $L_{G_k}(s) = \zeta(s)\zeta(s + 1 - k)$. The story of Hecke operators for congruence subgroups $\Gamma_0(N)$ was first fully told by Atkin and Lehner in 1970. One goal is to find a basis for the vector spaces of modular forms for $\Gamma_0(N)$ in eigenforms of the Hecke operators prime to N. Lang treats this in Chapters 7 and 8.

The final theme in the study of modular forms concerns values of the corresponding L-functions at integer arguments. Such results go back to Euler, who found that the values of the Riemann zeta function at even integers are given by:

$$2\zeta(2n) = -(2\pi)^{2n}(-1)^n B_{2n}/(2n)!, \qquad \text{for } n = 1, 2, 3, \ldots,$$

where B_n denotes the nth Bernoulli number (which is rational). There is a proof of this formula in Chapter 10 of Lang's book, with a generalization to Dirichlet L-functions in Chapter 14. Recently Shintani [**21**] has generalized this result to L-functions of totally real algebraic number fields, by cleverly obtaining a higher dimensional analogue of the complex variable proof in Lang's Chapter 14. An old and related result from algebraic number theory says that the residue of the Dedekind zeta function at $s = 1$ involves the product of the class number and the regulator (a determinant involving logarithms of the fundamental units of the number field). Motivated by results of this sort, Stark has made conjectures in [**25**] which say that the values of L-functions at $s = 1$, under certain hypotheses on the characters involved, contain generators of class fields, thus providing another method of attack upon Hilbert's 12th problem. There are also a vast array of conjectures of Lichtenbaum in [**14**] connecting values of zeta functions, étale cohomology, and algebraic K-theory. Lang's book looks at the study of some L-values by viewing them as periods of modular forms, following work of Eichler, Shimura, and Manin. It is proved, for example, that ratios of certain L-values in the critical strip lie in the field generated by the Fourier coefficients of the corresponding modular form. This fits into a general philosophy of Deligne, described by Zagier in [**6**, Vol. VI, pp. 118–120]. Lang also considers a p-adic interpretation of the values of Dirichlet L-functions, using ideas of Iwasawa, Manin, and Mazur. The motivation from cyclotomic fields can be found in Iwasawa's book [**12**].

Now that we have attempted to express the basic examples and themes in the subject of Lang's book, we should perhaps consider the book itself. The text should be a useful reference for anyone wishing to jump into the agitated waters mapped out by the 1970s conferences at Antwerp and Bonn and published in [**6**], since it is the only text on modular forms that covers many of the necessary topics. The average reader would probably also have to look at the books of Hecke, Lehner, and Shimura [**10**], [**13**], [**20**], and some readers would need the dozen or so other texts on the subject. We should thank the author for writing another valuable text and hope that more are to come.

The reviewer wishes that the author had included a discussion of nonholomorphic modular forms, which were first studied by Maass in 1949. However, some of these forms appear in Lang's SL(2, **R**). An example is the Eisenstein series on H, defined by:

$$E_s(z) = \sum_{(m,n)\neq(0,0)} y^s|mz + n|^{-2s}, \qquad \operatorname{Re} s > 1.$$

This function is an eigenfunction for the non-Euclidean Laplace operator on H and is clearly invariant under SL(2, **Z**). In fact $E_s(z)$ is an Epstein zeta function, which makes it the Mellin transform of a theta function (up to gamma factors). Therefore it has an analytic continuation to all s in **C** using Riemann's idea. The Eisenstein series play a leading role in harmonic analysis on $H/$SL(2, **Z**) and Selberg's trace formula, developed by Selberg in [**19**]. A special case of the trace formula is discussed by Zagier in an appendix to Lang's book (but see the correction in [**6**, Vol. VI, pp. 171–173]). The reader interested in a unified treatment of holomorphic and nonholomorphic modular forms could look at the Tata lecture notes [**15**] of Maass, for example. In particular, Hecke's correspondence between modular forms and Dirichlet series, as well as Hecke operators, are discussed in these notes, in a uniform way for both cases. There are, of course, many more open questions in the nonholomorphic case than in the holomorphic. In particular, no one has been able to find an example of a nonholomorphic cusp form; that is, an eigenfunction of the non-Euclidean Laplace operator on H, invariant under SL(2, **Z**), and having zero constant term in its Fourier expansion. Yet one knows that there are infinitely many such cusp forms–the discrete spectrum of the Laplacian on $H/$SL(2, **Z**). One might say that the existence of the discrete spectrum is as mysterious as the existence of quanta in quantum mechanics. However there are tables of eigenvalues of the Laplacian on $H/$SL(2, **Z**) in existence. In fact, such a table, which was made by H. Haas at Heidelberg, includes the eigenvalue

$$\lambda = s(s-1), \quad \text{with } s = 0.5 + 14.13473\, i,$$

a number very familiar to aficionados of the Riemann hypothesis. Recently several people have noticed that the Haas table also contains zeros of the Dirichlet L-function formed with the Kronecker symbol $(-3/*)$. D. Hejhal has found that these eigenvalues which are zeros of zeta and $L(s, (-3/*))$ are actually spurious, so that this eigenvalue interpretation does not prove the Riemann hypothesis. These conjectural connections with the Riemann hypothesis may have motivated Selberg in his work on the trace formula. There are also interesting connections between holomorphic and nonholomorphic modular forms and the representations of SL(2, **R**), as is discussed in the book of Gelfand, Graev, and Piatetskii-Shapiro [**9**]. Finally we should note that one can use the Hecke-Maass correspondence in the nonholomorphic case, to deduce the existence of nonholomorphic cusp forms for $\Gamma_0(N)$, $N \neq 1$, from the functional equations of Hecke L-functions for real quadratic fields.

It is certainly unreasonable to wish that Lang had included a discussion of modular forms for symmetric spaces of higher rank such as the Siegel upper half space H_n. Experience with an unfinished manuscript about harmonic analysis on symmetric spaces has taught the reviewer only too well the difficulties of discussing modular forms for the general linear and symplectic groups in a reasonably brief volume. However, it appears that many higher dimensional problems in number theory, physics, and geometry are begging for the right knowledge of higher rank automorphic forms (despite Hermite's remarks quoted earlier). The general situation is described in Borel's article [**3**, pp. 199–210] and Baily's text [**1**]. The special cases of the symplectic and general linear groups are treated in the books of Siegel and Maass [**23**], [**16**] and in some Göttingen lecture notes of U. Christian. However, the problem of generalizing Hecke's correspondence and its inverse to a mapping which takes Siegel modular forms to Dirichlet series in several variables with functional equations (via a higher dimensional Mellin transform) remained open as far as obtaining a converse was concerned. At least the problem was open until recently when Kaori Imai showed in [**11**] that harmonic analysis on $H/\mathrm{SL}(2, \mathbf{Z})$, which is often called Selberg's spectral resolution of the Laplacian, allows one to invert the higher dimensional Mellin transform correspondence for Siegel modular forms of degree 2 (i.e., on H_2). The forms of higher degree living on H_3, etc., require explicit and simple versions of harmonic analysis on $\mathrm{SO}(n)\backslash \mathrm{SL}(n, \mathbf{R})/\mathrm{SL}(n, \mathbf{Z})$, coming from work of Selberg, Langlands, and Harish-Chandra. The study of Euler products for L-functions corresponding to Siegel modular forms has been made by Andrianov in [**6**, Vol. VI, pp. 325–338], using the theory of Hecke operators which was begun by Maass in the 1950s. This also connects with a general philosophy of Langlands, which was discussed, for example, at the Corvallis conference [**4**]. One is reminded at this point of a remark in Morse and Feshbach [**17**, Vol. II, p. 1252]. Here the authors express their reluctance to leave the plane for higher dimensional problems. And indeed much less is understood in higher dimensions. But the time must certainly have arrived for several books on the subject, since Lehner said this already in 1964.

REFERENCES

1. W. Baily, *Introductory lectures on automorphic forms*, Princeton Univ. Press, Princeton, N. J., 1973.

2. A. Borel et al., *Seminar on complex multiplication*, Lecture Notes in Math., vol. 21, Springer-Verlag, Berlin and New York, 1966.

3. A. Borel and G. Mostow, eds., *Algebraic groups and discontinuous subgroups*, Proc. Sympos. Pure Math., vol. 9, Amer. Math. Soc., Providence, R. I., 1966.

4. A. Borel and W. Casselman, *Automorphic forms, representations and L-functions*, Proc. Sympos. Pure Math., vol. 33, Amer. Math. Soc., Providence, R. I., 1979.

5. F. Browder, (ed.), *Mathematical developments arising from Hilbert's problems*, Proc. Sympos. Pure Math., vol. 28, Amer. Math. Soc., Providence, R. I., 1976.

6. P. Deligne et al., eds., *Modular functions of one variable* I–VI, Lecture Notes in Math., vols. 320, 349, 350, 476, 601, 627, Springer-Verlag, Berlin and New York, 1973, 1975, 1977.

7. H. Dym and H. P. McKean, *Fourier series and integrals*, Academic Press, New York, 1972.

8. A. Fröhlich, ed., *Algebraic number fields* (*L-functions and Galois properties*), Academic Press, New York, 1977.

9. I. M. Gelfand, M. I. Graev and I. I. Piatetskii-Shapiro, *Automorphic functions and representation theory*, Saunders, Chicago, Ill., 1969.

10. E. Hecke, *Dirichlet series, modular functions and quadratic forms* (IAS Lectures), Edwards Bros., Ann Arbor, Mich., 1938.

11. K. Imai, *Generalization of Hecke's correspondence*, Ph.D. Thesis, Univ. of California, San Diego, 1979.

12. K. Iwasawa, *On p-adic L-functions*, Princeton Univ. Press, Princeton, N. J., 1972.

13. J. Lehner, *Discontinuous groups and automorphic functions*, Mathematical Surveys, no. 8, Amer. Math. Soc., Providence, R. I., 1964.

14. S. Lichtenbaum, *Values of zeta-functions, étale cohomology, and algebraic K-theory*, (preprint).

15. H. Maass, *Lectures on modular functions of one complex variable*, Tata Institute, Bombay, 1964.

16. ______, *Siegel's modular forms and Dirichlet series*, Lecture Notes in Math., vol. 216, Springer-Verlag, Berlin and New York, 1971.

17. P. M. Morse and H. Feshbach, *Methods of theoretical physics* I, II, McGraw-Hill, New York, 1952, 1953.

18. J.-P. Serre, *A course in arithmetic*, Springer-Verlag, Berlin and New York, 1970.

19. A. Selberg, *Harmonic analysis and discontinuous groups in weakly symmetric Riemannian spaces with applications to Dirichlet series*, J. Indian Math. Soc. **20** (1956), 47–87.

20. G. Shimura, *Arithmetic theory of automorphic forms and functions*, Princeton Univ. Press, Princeton, N. J., 1971.

21. T. Shintani, *On evaluation of zeta functions of totally real algebraic number fields at non-positive integers*, J. Fac. Sci. Univ. Tokyo, Sec. IA **23** (1976), 393–417.

22. C. L. Siegel, *Gesammelte Abhandlungen*, Springer-Verlag, Berlin and New York, 1966.

23. ______, *Topics in complex function theory*, Vols. I, II, III, Wiley, New York, 1969–1971.

24. H. M. Stark, *The analytic theory of algebraic numbers*, Bull. Amer. Math. Soc. **81** (1975), 961–972.

25. ______, *L-functions at $s = 1$*, I, II, III, IV, Advances in Math. **7** (1971), 301–343; **17** (1975), 60–92; **22** (1976), 64–84 (to appear).

26. A. Terras, *Applications of special functions for the general linear group to number theory*, Sém. Delange-Pisot-Poitou, 1977, exp. 23.

27. N. Wallach, *Symplectic geometry and Fourier analysis*, Math. Sci. Press, Brookline, Mass., 1977.

28. A. Weil, *Über die Bestimmung Dirichletscher Reihen durch Funktionalgleichung*, Math. Ann. **168** (1967), 149–156.

AUDREY TERRAS

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 2, Number 1, January 1980

0002-9904/80/0000-0016/$01.50

Georg Cantor, his mathematics and philosophy of the infinite, by Joseph Warren Dauben, Harvard Univ. Press, Cambridge, Mass., 1979, ix + 404 pp., $28.50.

A century and a half ago in 1831 Gauss, in a letter to Schumacher, wrote: "I protest against an infinite quantity as an actual entity; this is never allowed in mathematics. The infinite is only a manner of speaking."

Forty-one years later Georg Cantor, a young mathematician at Halle, was studying the uniqueness problem for trigonometric series. In 1870 he had proven that if a real function f was represented by a trigonometric series which converged for all x, then the series was necessarily unique; in fact, uniqueness was guaranteed even if the set of exceptional points, where convergence failed, was discrete. By the following year he had extended his