here about simple, practical methods for bounding problems, for determining sensitivities, for encoding probability, or for approximating risk preference.

And so we may ask again, for whom is the book written? In the preface, we find that it should be suitable for business schools and for economic and applied mathematics courses at the upper undergraduate and lower graduate levels. The author has doubtlessly thought about some of the problems that will arise when a typical business school student encounters a Jacobian determinant. This may explain why he has made Appendix 1 a chart of the Greek alphabet. However, I think that the problem lies at a deeper level. For business school students I find the book too mathematical relative to the insights it produces. For the mathematically inclined student, it seems to avoid many problems of real mathematical interest, such as how to assess uncertain functions as well as uncertain variables. For the practical student, such as the engineer, it falls short in presenting the links between theory and practice that are essential in application.

Thus, what might have been an interesting mathematical text on statistical decision theory falls somewhat short of its title as a treatment of decision analysis. The book is an extensive, meticulous, and well-written elementary text on decision theory for the mathematically inclined, but it is not an effective guide to either the philosophical comprehensiveness or professional practice of decision analysis.

RONALD A. HOWARD

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Groupes et anneaux réticulés, by Alain Bigard, Klaus Keimel and Samuel Wolfenstein, Lecture Notes in Math., vol. 608, Springer-Verlag, Berlin and New York, 1977, xi + 334 pp.

The book under review is a study of groups and rings which carry a (necessarily distributive) lattice structure in such a way that the group operation distributes over the lattice operations, and in addition, in the case of rings, products of positive elements are positive.

The study of algebraic systems where an order relation is introduced, compatible with the algebraic operations, does not have a long history. While various algebraic generalizations of the real number field have been the subjects of extensive theories in the 19th century (quaternions, matrix and linear algebras etc.), the importance of order relation in algebra has been completely overlooked. The explanation for this might be found in the absence of total order in the complex number field and in the old opinion that inequalities serve to express continuity and as such they are alien to algebra.

Towards the end of the last century, the necessity of order relation in algebraic systems emerged in the foundations of plane geometry (D. Hilbert): the collection of coordinates on the line had to be made into a totally ordered field. In this respect, a decisive role was played by the archimedean axiom which was listed by Hilbert (along with the completion axiom) among the axioms of continuity. Shortly after the turn of the century, O. Hölder analyzed the archimedean property and proved his famous theorem that every archimedean, totally ordered group is order-isomorphic to a subgroup of the additive group of the real numbers. The nonarchimedean case was considerably more difficult: H. Hahn (1907) gave an extremely long proof to show that every totally ordered abelian group can be embedded in an appropriate lexicographic product of the reals. The question as to when a group can be furnished with a total order was raised by F. Levi (1913) who proved that, in the abelian case, a necessary and sufficient condition was torsion-freeness. A significant development here is the theory of formally real fields by E. Artin and O. Schreier (1926) which led to the solution of Hilbert's 17th problem (a positive definite real rational function is a sum of squares).

The study of divisibility in rings of algebraic integers is the origin of the theory of (not necessarily totally) ordered groups. The idea can be traced back to Dedekind; it simply consists of introducing a partial order relation $\leqslant$ in the multiplicative group of the field of quotients, reflecting the divisibility relation in the domain of integrity. For a long time, virtually no progress has been made in exploiting this idea, until the emergence of multiplicative ideal theory in the 20's and 30's of this century (E. Artin, H. Prüfer, P. Lorenzen) and W. Krull's general valuation theory (1932).

The greatest impetus to the theory of lattice-ordered groups was given by vector lattices, i.e., lattice-ordered vector spaces over the real field. Examples of vector lattices are abundant in real function spaces: one need look no farther than continuous functions on a topological space for a major example. In his lecture at the International Mathematical Congress in Bologna (1928), F. Riesz recognized that decompositions of linear operators can be successfully discussed by exploiting the natural order between them. Subsequently, a profound theory of vector lattices has been developed by L. Kantorovitch (1937), H. Freudenthal (1936); F. Riesz (1940), H. Nakano (1941) et al.

The fusion of the three independent lines into a unified theory of lattice-ordered groups is the creation of G. Birkhoff. In his paper in 1942, he lay down the foundations of a theory which after a slow start developed into a rich and diversified theory. The study of lattice-ordered rings was inaugurated in 1956 in a joint paper by Birkhoff and R. S. Pierce.

There have been several books or parts of books available on lattice-ordered groups and rings [the most up-to-date ones are P. Conrad's Lecture Notes at Tulane and at Sorbonne], but none is as exhaustive and informative as the book under review.

The authors' goal was to give an up-to-date and reasonably complete exposition of the main body of the theory of lattice-ordered groups and rings. They did, indeed, an excellent job in collecting a large amount of recent research in this volume and presenting it in a readable form. The material is well organized and carefully presented; many of the proofs have been shortened or simplified from their original form. Undoubtedly, the book represents the most exhaustive summary of the present state of affairs available on this subject. There is enough variety so that a serious reader can receive sufficient information to start his own research on the field, even many experts will find something new. Nice features are the remarks at the end of the chapters which bring the reader up to date in the literature. Occasionally, there are omissions: the results on linearly ordered groups and rings have not received due attention in these remarks, e.g., real closed fields are totally ignored.

Rather modest background is needed to read the book; basic group and ring theory with some knowledge in lattice theory should suffice in general, but from time to time, additional knowledge is required (e.g., in the chapter on sheaf representations of lattice-ordered rings). The frequent motivations and the readable style make this volume a good choice for a graduate text.

The total impression about the material covered by this book is that, though the major motivation seemed to be more internal than external, there has been a commendable effort by the authors to relate their subject to other fields of mathematics. The theory of real functions lends its flavor throughout the subject, abstract group theory has penetrated so far only into the theory of totally ordered groups, while the few problems studied recently under the influence of modern ring and module theory have not had a great impact on the development of lattice-ordered structures (except for the attractive theory of free lattice-ordered groups). It is hoped that this excellent text will enhance the interest in lattice-ordered groups and rings, and their applications in various other fields.

LASZLO FUCHS

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General lattice theory, by George Grätzer, Mathematische Reihe Band 52, Birkhäuser Verlag, Basel, 1978, xiii + 381 pp.

Lattice theory has come a long way in the last 45 years! Although Dedekind had written two penetrating papers on "Dualgruppen" before 1900, and insightful isolated theorems had been published in the 1920's by individual mathematicians such as by R. Baer, K. Menger, F. Riesz, Th. Skolem, and A. Tarski, it was not until the 1930's that lattice theory became studied systematically, and recognized as a significant branch of mathematics.

This recognition was largely due to realization that "many mathematical theories may be formulated in terms of [lattice-theoretic concepts], and the systematic use of these concepts gives a unification and simplification of the various theories"[1] In brief, it was due to the wide range of *applications* of lattice theory to other branches of mathematics, and emphasis on such applications pervaded the talks given at the first symposium on lattice theory [1], sponsored by the American Mathematical Society in 1938.

Indeed, success may have come too easily to lattice theory in the first decade of its renaissance. The very simplicity and pervasiveness of its basic concepts (greatest lower and least upper bounds of order relations), and the ready availability of general ('universal') algebraic techniques having well-known analogues for groups and rings, made some mathematicians (most

[1]O. Ore in Bulletin of the American Mathematical Society **48** (1942), p. 75.