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The algebraic structure of group rings, by Donald S. Passman, John Wiley & Sons, New York, London, Sydney, Toronto, 1977, xiv + 720 pp., $34.95.

There appeared in 1976 an expository paper by the present author **[52]** entitled "*What is a group ring*?" This question, rhetorical as it is, may nevertheless be answered directly by saying that for a group G over an integral domain R the group ring $R(G)$ is a free unitary R-module over the elements of G as a basis and in which the multiplication on G is extended linearly to yield an associative multiplication on $R(G)$, $R(G)$ becoming a ring with an identity. While this may answer the question, the underlying aim of the author is evidently to draw attention to this particular ring $R(G)$ which, over the past decade and especially when G is infinite, has come to be intensively studied **[51]**. In the main $R(G)$ is studied under the assumption that R is a field K and so, although $K(G)$ is nowadays commonly called a group ring, $K(G)$ in an older and more informative terminology is a linear associative algebra.

The group ring $K(G)$ of a finite group G over a field of characteristic 0 is semisimple. Over a sufficiently large extension K of the rational field $\mathbf{Q}$ there is a well-known theory of group characters by whose means, for example, explicit characterisations of the primitive central idempotents of $K(G)$ are obtainable. Over a field of prime characteristic p and for G finite the Jacobson radical $JK(G)$ of $K(G)$ may be nontrivial but, since around 1940 **[7]**, the development of Brauer's theory of modular characters has again, for a sufficiently large extension of the prime field $GF(p)$, yielded characterisations of the primitive central idempotents **[44]**. All of this work, for which the text of Curtis and Reiner is a well-known reference **[10]**, depends heavily on the finiteness of G.

Passman's book is concerned with the case of a group ring $K(G)$ in which G is potentially infinite and for which, in consequence, ordinary or modular character theory is of little help. The bulk of the work on infinite group rings has been done in the period 1967–1977, a major, if not the major, contributor being the present author. Prior to the mid-1960s the earliest significant work was due to Jennings **[33]**, who for a finite p-group G over a field K of characteristic p obtained group-theoretic descriptions of the dimension subgroups $D_n(K(G))$ formally defined from the ring structure as

$$D_n(K(G)) = \{x \in G: x - 1 \in (JK(G))^n\} \qquad (n = 1, 2, \dots).$$

Modular theory, apart from confirming the equality of $JK(G)$ with the augmentation ideal $\omega(K(G))$, of $K(G)$, is here of no assistance and, not unexpectedly, the arguments of Jennings relating as they do to commutation properties of the group elements are appropriate also to his own later investigation of infinite nilpotent groups **[34]**. In view of the methods the work of Jennings is not expounded in 'Curtis and Reiner' but does appear in

'Passman', whose discussion incorporates Lazard's group-theoretic description of the dimension subgroups **[36]** and finally obtains Hartley's pretty result that $\bigcap_{n=1}^{\infty} [\omega(K(G))]^n = \{0\}$ if and only if $\bigcap_{n=1}^{\infty} D_n(K(G)) = \{1\}$ **[26]**.

Following Amitsur's work on algebras **[1]**, **[2]** interest has been stimulated in the semisimplicity problem for group rings. The nonexistence of nontrivial nil ideals in $K(G)$ whenever K has characteristic 0 or K has prime characteristic p but G has no nontrivial p-elements is fairly easy to prove. This result is however a long way from establishing a corresponding result for $JK(G)$. Combining the results of Amitsur and Passman **[2]**, **[45]** it follows that if K is uncountable then, under the previous hypotheses, $JK(G) = \{0\}$. Many attempts have been made to remove the assumption of uncountability but without some alternative assumption such as, for example, that G is free, or solvable or a suitable wreath product there has been no success. By the early 1960s there had emerged three major and easily comprehended problems, namely:

(1) For all groups G is $\mathbf{Q}(G)$ semisimple?

(2) For all groups G with no nontrivial p-elements is $GF(p)(G)$ semisimple?

(3) For all torsion-free groups G is it true that $K(G)$ has no proper zero divisors?

Of these three questions it is probably fair to say that group ring theorists believe, almost as articles of faith, in affirmative answers to (1) and (2). It is simple to demonstrate that if $K(G)$ has no proper zero divisors then necessarily G is torsion-free but the converse assertion seems to be viewed with caution. Like Fermat's Last Theorem, whose enunciation is easily comprehended and which remains to tantalise, these problems have been an impetus for other work. There had been in the early 1960s one further important and apparently more sophisticated problem, namely:

(4) If $K(G)$ satisfies a polynomial identity does G have an abelian subgroup of finite index?

From the work of Kaplansky **[35]** and Amitsur **[3]** it had been known that the existence in G of such a subgroup is sufficient to ensure that G does satisfy a polynomial identity. Isaacs and Passman **[29]**, **[30]** were to show that for characteristic 0 the problem had indeed an affirmative answer. For characteristic $p > 0$ the issue is more complicated and the first attack on it was made by Martha Smith **[61]** who introduced ring-theoretic methods utilising Posner's Theorem **[55]** on the embedding of a prime ring satisfying a polynomial identity in a matrix ring over a division ring. Denoting the set of elements of G consisting of all elements having at most a finite number of conjugates by $\Delta(G)$ Connell had previously shown that $K(G)$ is prime if and only if $\Delta(G)$ is torsion-free abelian **[9]**, the work of M. Smith revealed the need to bound $|G : \Delta(G)|$.

The above results on semisimplicity and polynomial identities together with further results on idempotents and annihilator and nilpotent ideals are summarised by Passman in an earlier book of 149 pages published in 1971. This book, the contents of which are essentially subsumed in the present volume, was recommended by the Reviewer for Mathematical Reviews **[42]** who, however, questioned 'whether the subject had sufficient maturity to warrant a book, especially because of its specialised nature and limited appeal for a subject in its infancy'. The infant has grown lustily in the brief span of six years into the healthy maturity of a comprehensive treatise of 720 pages. While the text is inevitably specialised the numbers of papers and research workers indicate that the appeal is much wider than was formerly the case. Passman has taken considerable pains to ensure that his text is self-contained so that, apart from one noteworthy exception, a beginning course in groups, rings and fields, in which there is included Sylow's Theorems, Artin-Wedderburn theory, the Jacobson Density Theorem and some Galois Theory of cyclotomic fields, is by and large an adequate prerequisite. The outstanding exception is the quoted result [31] that for a given monic irreducible polynomial $f(\zeta)$ with integral coefficients of degree strictly greater than one there exist infinitely many primes p such that $f(\zeta)$ (mod p) has a root which is not in $GF(p)$. This assumed result is employed to develop a systematic use of places, a place being a mapping ϕ: $K \to F \cup \infty$, F being a second field and ∞ a symbol such that ϕ is a homomorphism on $\phi^{-1}(F)$ and $\phi(k) = \infty$ if and only if $\phi(k^{-1}) = 0$. New proofs, using places, are given for the existence of nilpotent ideals. Applications are made to obtain Zalesskiĭ's theorem [71] that if e is an idempotent in $K(G)$ then the coefficient of e is in $\mathbf{Q}$ or $GF(p)$ and to obtain Formanek's theorem [16] that if G is a torsion-free group with ascending chain condition on cyclic subgroups and if K has characteristic 0 then $K(G)$ has no nontrivial idempotents.

A favorite gambit of group ring theorists has been to impose some well-defined ring-theoretic property on a group ring in the expectation that the group may, in some sense, be determined. Villamayor [66] and Connell [9] showed that $K(G)$ is von Neumann regular if and only if G is locally finite and has no nontrivial elements of order p in the event that K has characteristic p. Woods showed that $K(G)$ is perfect if and only if G is finite [69]. In the 1970s further results in this vein appeared. Following earlier work [9], [20], [40] Renault showed [56] and Farkas proved neatly [12] that $K(G)$ is self-injective if and only if G is finite. Extending this result the two conditions that $K(G)$ is von Neumann regular and that all irreducible $K(G)$-modules are finite-dimensional over their commuting rings were shown by Farkas and Snider [14], under the auxiliary assumption that G is countable, to be equivalent to the condition that all irreducible $K(G)$-modules are injective and the same two conditions were shown by Goursaud and Valette [23] to be equivalent to the condition that all irreducible $K(G)$-modules are Σ-injective. Passman always brings problems of this sort entirely up-to-date and, in this instance, he incorporates the recent (1977) work of Hartley [27] which, advancing previous work [14], proves that if G is a locally finite group with no elements of order p then all irreducible $K(G)$-modules are finite-dimensional over their commuting rings if and only if G has an abelian subgroup of finite index.

More complete results on polynomial identities are now available. There is, firstly, the straightforward result of Passman [49] that if $K(G)$ satisfies a polynomial identity of degree n then the derived subgroup $(\Delta(G))'$ of $\Delta(G)$ is finite and there holds the (sharp) bound $|G : \Delta(G)| \leqslant \frac{1}{2}n$. If K has prime characteristic p it is now known [48] that $K(G)$ satisfies a polynomial identity if and only if G has a subgroup H of finite index such that H' is a finite p-group, a result which had previously proved awkward as $K(G)$ is not, in general, semiprime. The author introduces central polynomials which, inverting chronological order, he uses to reprove Posner's Theorem **[55]** and thereby to rederive the original work of M. Smith **[61]** on prime polynomial rings with a polynomial identity.

Considerations of semisimplicity seem currently to devolve either into determining $JK(G)$ for some wide class of groups or into determining conditions for the primitivity of $K(G)$. In the first alternative the problem is to determine a nil-ideal which, in the class of groups under consideration, is 'big-enough' to be the Jacobson Radical. The author introduces his N^*-radical, which may be defined for any ring and which for $K(G)$ is denoted by $N^*K(G)$ and is defined as the set of all elements $x \in K(G)$ such that $xK(H)$ is nilpotent for all finitely generated subgroups H for which $x \in K(H)$ **[50]**. For group rings the desirable property that $N^*(K(G)/N^*K(G)) = \{0\}$ holds, the corresponding result for a general ring being false. He defines two subgroups $\Lambda^+(G)$ and $\Lambda^p(G)$, analogously to $\Delta^+(G)$ and $\Delta^p(G)$, by letting $\Lambda^+(G)$ be the set of all elements $x \in G$ such that x has finite order and x has at most a finite number of conjugates under the action of any finitely generated subgroup H of G and by letting $\Lambda^p(G)$ be the subgroup of $\Lambda^+(G)$ generated by all p-elements of $\Lambda^+(G)$. He shows that $N^*K(G) = JK(\Lambda^+(G))K(G)$ and that for characteristic p of K $N^*K(G) = JK(\Lambda^p(G))K(G)$. The author conjectures that always $N^*K(G) = JK(G)$, citing, as evidence, the cases of linear and solvable groups. A key issue is to determine the existence of a unique subgroup H for which $JK(G) = JK(H)K(G)$ and which is minimal with respect to this equality. The most recent work of Passman, not mentioned in present text **[53]**, shows that if G is locally solvable and if K has characteristic p then $JK(G) = JK(W)K(G)$ where W is a subgroup of $\Lambda^p(G)$ which, at least if G has no nontrivial normal p-subgroup, is generated by all finite subgroups of $\Lambda^p(G)$ each of which is itself generated by p-elements of $\Lambda^p(G)$ and each of which is subnormal in any larger finite subgroup of $\Lambda^p(G)$.

A necessary condition for a ring to be primitive, that is to have a faithful irreducible module, is that the ring is prime. Consequently for $K(G)$ to be primitive it is required that $K(G)$ is prime or, equivalently, that $\Delta(G)$ is torsion-free. It is comparatively easy to show that if G has a subgroup H of finite index then $K(G)$ is primitive if and only if $K(H)$ is primitive **[59]** but no actual examples of primitive group rings were known until the result of Formanek and Snider **[19]** proving that if G is locally finite and countable then $K(G)$ is primitive if and only if $K(G)$ is prime and semisimple. This result depends only marginally on group theory, the pertinent fact being that a countable locally finite group is a union of an ascending sequence of finite groups, Farkaš and Snider showing that a semisimple prime ring which is the ascending chain of artinian rings, all with the same identity, is necessarily primitive. The free product of two cyclic groups each of order two is isomorphic to the infinite dihedral group D and $K(D)$ is certainly not primitive, however Formanek **[17]** showed that if G is the free product of two nontrivial groups and if G is not isomorphic to D then $K(G)$ is primitive. The argument is here group-theoretic and depends on being able to assign a 'length' to the elements in a free product. Group-theoretic arguments also establish that the group ring of an algebraically closed group is primitive. The fore-going results are independent of the field K but it is also evident that the precise nature of K may be critical. Thus if G is a nontrivial polycyclic group for which $\Delta(G)$ is trivial and if the transcendence degree of K over the prime field of K is at least equal to the Hirsch number of G, then $K(G)$ is primitive **[47]** whereas if K is algebraic over $GF(p)$ then $K(G)$ cannot be primitive **[57]**. Results of a different sort, particularly for solvable groups, were proved by Zalesskiĭ **[70]**, **[72]** who has obtained various key intersection theorems, so-called because in each theorem there is, or there is constructed by induction, a well-defined subgroup H of G such that if I is an ideal of $K(G)$ then $I \neq \{0\}$ implies $I \cap K(H) \neq \{0\}$, the primitivity of $K(H)$ then implying the primitivity of $K(G)$. Such results move, rather than remove, difficulties and this area of group ring theory is perhaps best described in the words of Farkas and Passman **[13]** who remark that "the situation is so chaotic that no plausible necessary conditions uniting the host of examples have ever been conjectured".

It is natural to investigate in group rings the consequences of various chain conditions (left and right being in group rings usually indistinguishable). Connell showed in 1963 **[9]** that $K(G)$ is right artinian if and only if G is finite, there are now available the stronger results of Goursaud **[22]** and Valette **[64]** showing respectively that $K(G)/N^*K(G)$ is artinian if and only if either the characteristic of K is 0 and G is finite or the characteristic of K is p and G is a locally finite group having a normal p-subgroup of finite index and that if K is an uncountable algebraically closed field then $K(G)/JK(G)$ is artinian if and only if either the characteristic of K is 0 and G is finite or the characteristic of K is p and G has a normal p-subgroup H of finite index such that $JK(H)$ is the augmentation ideal of $K(H)$. These two similar results would simplify if it were always true that $JK(G) = N^*K(G)$. It is easy to verify, using augmentation ideals, that if $K(G)$ is a noetherian ring then G is a noetherian group but the determination of all noetherian groups is unresolved, however Hall **[24]** showed that if G is a finite extension of a polycyclic group then $K(G)$ is noetherian. This important result, whose proof has affinities with the proof of the Hilbert Basis Theorem, has enabled the machinery of noncommutative ring theory to be applied to group rings, the first applications being made by P. Smith in the early 1970s **[62]**, **[63]**. This work was extended by Roseblade, a typical result **[57]**, **[63]** being that if K is a field of characteristic 0 and if G is a polycyclic-by-finite group having a normal subgroup H then the three conditions, that H be a finite-by-nilpotent subgroup, that the augmentation ideal of $K(H)$ be polycentral and that the augmentation ideal of $K(H)$ should generate a right ideal of $K(G)$ having the Artin-Rees property, are mutually equivalent. The concept of polycentrality is, in fact, somewhat restrictive and Roseblade and Smith have been able to extend the notions to consider hypercentral group rings **[58]**. If K has characteristic 0 they show that $K(G)$ is hypercentral if and only if every nontrivial homomorphic image of G has a proper normal subgroup which is either finite or central, in characteristic p slightly more complicated equivalent conditions are obtained.

In 1946 Hirsch **[28]** proved that a polycyclic, and so a polycyclic-by-finite, group is residually finite and in 1959 P. Hall [25] having shown that a finitely generated nilpotent extension of an abelian group is residually finite, posed the problem of whether 'nilpotent' could be replaced by 'polycyclic', the crux of the problem being to show that if G is a polycyclic-by-finite group and if M is a finitely generated $\mathbf{Z}(G)$-module, the intersection of whose nonzero submodules is nonzero, then M is necessarily finite. An affirmative answer to Hall's problem is due to Roseblade [57] and Jategoankar [32] as the culmination of previous work including that of Malcev [39] and of Bergman [4]. Malcev had established that if a solvable group G acts faithfully on a finite-dimensional irreducible $K(G)$-module then G has an abelian subgroup of finite index, and Bergman had established an important dichotomy, reformulated by Passman, for invariant ideals of $K(G)$ where G is a finitely generated abelian group acted on by a group of automorphisms A where A acts so that A and all subgroups of A act irreducibly on $\mathbf{Q} \otimes_{\mathbf{Z}} G$.

Until quite recently one of the best results on the nonexistence of proper zero-divisors in $K(G)$ was due to Bondi [6] who, extending the notion of a polynomial ring over a field, showed that if G has a finite normal series of subgroups with torsion free abelian quotient groups then $K(G)$ has no proper zero divisors. If G is a free group or even an ordered group it had earlier been shown [38], [43] that $K(G)$ is embeddable in a division ring and so, immediately, such a $K(G)$ has no proper zero divisors. In 1972 Lewin [37] showed that if G is the free product of two subgroups A and B with an amalgamated normal subgroup N such that $K(A)$ and $K(B)$ have no proper zero-divisors and $K(N)$ is an Ore Domain then $K(G)$ is an Ore Domain, a result whose proof involves the idea of 'length' of an element and the use of the Ore condition to 'link' $K(A)$ and $K(B)$. Here the problem of zero-divisors rested until Brown [8] took the novel step of applying homological techniques and of invoking a zero divisor theorem of Walker [67] which, ostensibly at least, had no connection with group rings. Using a theorem of Grothendieck and Serre [60] that if G is a poly-infinite cyclic group then a finitely generated projective $K(G)$-module is a quotient of two finitely generated free modules, Farkas and Snider [15] showed that if G is a torsion free polycyclic group and if K has characteristic 0 then $K(G)$ has no proper zero-divisors, a comparable result being obtained for prime characteristic. The original proof of Farkas and Snider uses the concept of the reduced rank of a module but the proof given in the present text by Passman is based on the more direct notion of uniform dimension of a module [21].

The final topic treated by Passman is the so-called 'isomorphism question', namely, if G and H are two groups such that $K(G)$ and $K(H)$ are isomorphic does it follow that G and H are isomorphic? If G and H are two finite abelian groups of the same order then $\mathbf{Q}(G)$ and $\mathbf{Q}(H)$ are isomorphic if and only if G and H are isomorphic [54] but if K is a sufficiently large extension of $\mathbf{Q}$ then $K(G)$ and $K(H)$ are always isomorphic. This clearly suggests that the answer to the question is field-dependent but, on the other hand, there is the striking example of Dade [11] of two finite nonisomorphic metabelian groups G and H such that for all choices of the field K $K(G)$ and $K(H)$ are isomorphic. As an aside and drawing on Whitcomb's work, [68] regrettably never published except in a Chicago Ph.D. thesis, it happens to be true that if

G and H are two finite metabelian groups such that $\mathbf{Z}(G)$ and $\mathbf{Z}(H)$ are isomorphic then G and H are isomorphic. By means of Ulm's characterization of reduced countable abelian p-groups Berman [5] and May [41] showed that if G and H are two countable abelian p-groups and if K has characteristic p then the isomorphism of $K(G)$ and $K(H)$ implies the isomorphism of G and H.

The author has written a majestic account of the existing developments of the past few years and has contrived to write an exposition which is both encyclopedic and lucid. Each chapter has a wealth of exercises, many of which contain research work garnished with hints for the reader. It is not possible to indulge in the usual reviewer's privilege of comparing the present text with other similar texts since in Western European languages there are none (cf. [73]). In the main the material of the text has appeared previously only in research papers and, accordingly, in the limited space available, only a highly selective indication can be given of the contents but an indication which, it is to be hoped, has imparted their flavour. It appears to be well-nigh obligatory for a reviewer to find something at which he may cavil and the present reviewer would not wish to overlook a hallowed tradition. He is therefore constrained to record that, somewhat surprisingly, the author's first chapter seems to be unsatisfactory in its selection of material and it may mislead an unwary reader by introducing twisted group rings, thereby creating the unwarranted impression that these exotic objects are to be of subsequent significance. To make this criticism, however, is to focus attention on a minute blemish on an otherwise remarkable book, which has already become, and which is destined for many years to remain, a standard reference.

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