cation problems are treated by topological and analytical techniques. Applications are discussed. The final part is devoted to analysis in the large. The Leray-Schauder degree is developed as well as degree for C^2 Fredholm maps. These, and related notions, are used to investigate nonlinear boundary value problems. Critical point theory, with applications, completes the book.

Globally, I very much like the spirit and the scope of the book. The writing is lively, the material is diverse and yet maintains a certain unity, and the interplay between the abstract analysis and certain concrete problems is emphasized throughout. Locally, more attention could have been paid to detail; there are many misprints, some mistatements of results, and some proofs need tightening. On balance, the book is a very useful contribution to the growing literature on this circle of ideas, and I look forward to the author's promised companion volume.

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Combinatorial set theory, by Neil H. Williams, North-Holland Publishing Company, Amsterdam, New York, Oxford, 1977, xi + 202 pp., $26.75.

Combinatorial set theory is frequently distinguished from axiomatic set theory, although the distinction is becoming less and less clear all the time. If there is a difference, it is more one of method than substance. Axiomatic set theory uses the tools of mathematical logic, such as the method of ultrapowers and the theory of forcing and generic sets, while the methods of combinatorial set theory are purely "combinatorial" in nature. In practice, an argument or result is "combinatorial" if it is *not* overtly model-theoretic, topological, or measure-theoretic.

Both branches of set theory experienced explosions in interest at about the same time, in the middle 1960s, but at widely separated places. Combinatorial set theory grew up around Erdős and his school, in Budapest, while axiomatic set theory received its impetus from the work of Cohen, Scott and Solovay at

Stanford and Berkeley. During the last decade, beginning roughly with the work of Silver, the two subjects discovered each other, and the result has been a boom of activity which is still continuing.

The central notion of combinatorial set theory is the *partition relation*. Given cardinals κ, λ, and μ, and a positive integer n, the partition relation

$$\kappa \rightarrow (\lambda)^n_\mu$$

means that if A is a set of cardinality κ and the set $[A]^n$ of (unordered) n-element subsets of A is partitioned into μ pieces, then there is a set $B \subseteq A$ such that B has cardinality λ and all the elements of $[B]^n$ lie in the same piece of the partition. The set B is called *homogeneous* for the partition. The simplest infinite partition relation is Ramsey's theorem, which asserts

$$\aleph_0 \rightarrow (\aleph_0)^n_k \quad \text{for all finite } n \text{ and } k.$$

In their 1956 paper [**1**] Erdős and Rado found a generalization of Ramsey's theorem to large cardinals which proved to be useful in areas ranging from mathematical logic to set-theoretic topology. Perhaps more importantly for combinatorial set theory, however, they also discovered that the partition relation is capable of infinite variation. For $n = 2$ the relation $\kappa \rightarrow (\lambda)^n_\mu$ talks about partitioning the complete graph on κ vertices. Why not partition other graphs, or ask for other kinds of homogeneous sets? One can make the relation more precise, and require the cardinality of the homogeneous set B to depend on which piece of the partition the elements of $[B]^n$ lie in. One can make it weaker, and ask only that $[B]^n$ does not meet *all* the pieces of the partition. One can consider partition relations for arbitrary order types, partial or total, instead of cardinal numbers. And finally, what about the connection between these notions and older notions of combinatorial set theory, like almost-disjoint sets, transversals, and set systems?

These and similar problems precipitated the great thrust of activity in the early and middle 1960s by Erdős, Hajnal, Rado, and others. And while considerable progress was made, there emerged two kinds of problems which remained intractable, and which were ultimately solved using powerful methods from mathematical logic. The first kind dealt with "large" cardinals, i.e., cardinals which are at least inaccessible. The problem of determining the relative sizes of these cardinals was finally solved using the theory of ultrapowers and the notion of indescribability of cardinals.

The second kind of problem concerned the consistency of certain combinatorial assertions with the axioms of set theory. Combinatorial arguments naturally involve lots of cardinal arithmetic, and cardinal exponentiation is quite impossible without some simplifying assumptions. Until Cohen's method of forcing became available, the tactic universally adopted was to assume the generalized continuum hypothesis (GCH) whenever the going got sticky. Since then, there has been much reexamination of the old arguments to see if they can be improved and, if not, whether forcing arguments will show they are best possible.

It is understood now that most of these logical methods have combinatorial counterparts, and even forcing is considered as a combinatorial tool. Nevertheless the student of combinatorial set theory is obliged to master the logical techniques because of their superior intuitive content.

But that's the easy part. The hard part has always been to learn the fundamental combinatorial results, because they are scattered through many journals, in papers which are often immense and written in forbidding notation and appalling generality. For that reason Neil Williams' book, *Combinatorial set theory*, is particularly welcome.

Williams has collected the basic results, from the Erdős-Rado theorem and Hajnal's set-mapping theorem through the theory of almost-disjoint sets and graph colorings. There is enough variety so that even many experts will find something new. It is a pleasure to see these results presented in standard notation (the only exception I found was the use of κ' for cf κ). The proofs are, for the most part, quite well done. Many of them have been considerably shortened from their original form.

The book is entirely classical in the sense that arguments involving large cardinals and forcing do not occur. The GCH is assumed "whenever it leads to a simplification in the statement or proof of a result".

One particularly nice feature of the book is the way, when GCH has settled a question, the author takes a moment to review all the cases. There are usually references to the literature for further results, although the ones on consistency results tend to be rather spotty. The book is sprinkled with open problems, most of which seem to be culled from the original papers or from the problem papers of Erdős and Hajnal. The reader should be warned that a significant fraction of the problems has already been solved, and in many cases the solutions require techniques far beyond the scope of the book.

Nearly any specific combinatorial result can be generalized and, particularly if one is studying a partition relation with several cardinal parameters, the generalization may be quite incomprehensible. This extreme generality is typical of papers in combinatorial set theory and, perhaps inevitably, Williams' book suffers from it too. The reader is therefore advised to adopt the standard strategy for dealing with such generality: Discover the simplest nontrivial case to which the theorem applies, and read the proof with that case fixed firmly in mind. The generalization is usually obvious.

With a book as slender as this one, everyone is sure to miss some favorite results. I wish there were more material on trees. And while there is a nice treatment of the simplest case of Silver's theorem on the GCH at singular cardinals, it would have been nice to see the extension of this theorem due to Galvin and Hajnal. On the other hand, it was an unexpected pleasure to find included two of the most difficult and beautiful proofs in the field: Larson's proof of Chang's partition theorem for the ordinal ω^ω and Galvin's combinatorial proof of the relation $\omega_1 \to (\alpha)^2_k$ for all $\alpha < \omega_1$ and $k < \omega$.

There are several minor typographical errors, the only one of consequence being the transposition of pp. 67 and 71.

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