BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 84, Number 1, January 1978

ASYMPTOTIC COMPLETENESS FOR A CLASS OF FOUR PARTICLE SCHRÖDINGER OPERATORS[1]

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Communicated by R. G. Douglas, May 4, 1977

1. Introduction. The purpose of this announcement is to state some new results in multichannel nonrelativistic quantum scattering theory.

The scattering theory of two particle nonrelativistic quantum mechanics is reasonably well understood for potentials which decrease at least as fast as $r^{-1-\epsilon}$ at infinity (see Reed and Simon [5] and the references therein). In comparison, relatively little is known about the general N particle problem, which for $N \geqslant 3$, involves multichannel scattering.

Asymptotic completeness and the absence of singular continuous spectrum in the three particle problem were first proved for a large class of two-body potentials by Faddeev [2]. Balslev and Combes [1] have proved the absence of singular continuous spectrum for N particles when the potentials are dilation analytic. However, no general asymptotic completeness results have previously been proved for $N \geqslant 4$, although Ginibre and Moulin [3], Thomas [9], and Howland [4] have simplified and extended Faddeev's asymptotic completeness results for $N = 3$.

Independent of the work being announced here, Sigal [7] has proved results which overlap with those stated below.

2. Main results. Let $\widetilde{H} = -\Sigma_{i=1}^4 \Delta_i/2m_i + \Sigma_{i<j} \lambda_{ij}V_{ij}$ be the Schrödinger operator for a system of four particles moving in $m \geqslant 3$ dimensions, and let $H = H_0 + \Sigma_{i<j} \lambda_{ij}V_{ij}$ denote the Schrödinger operator on $L^2(\mathbf{R}^{3m})$ for the same system with the center of mass motion removed.

For each pair i, j, $L^2(\mathbf{R}^{3m})$ decomposes into $L^2(\mathbf{R}^m) \otimes L^2(\mathbf{R}^{2m})$, where the first factor denotes functions of $x_{ij} = x_i - x_j$. Under this decomposition, $V_{ij} = v_{ij} \otimes 1$. It is assumed that $v_{ij} = u_{ij}w_{ij}$, such that both $u_{ij}(-\Delta + 1)^{-1/2}$ and $w_{ij}(-\Delta + 1)^{-1/2}$ are compact as operators on $L^2(\mathbf{R}^m)$ (here Δ denotes the Laplacian in the x_{ij} variable). U_{ij} and W_{ij} denote $u_{ij} \otimes 1$ and $w_{ij} \otimes 1$, respectively.

THEOREM. *Let $m \geqslant 3$, and suppose the potentials V_{ij} have been chosen so that*

(i) *each u_{ij} and w_{ij} is dilation analytic in some strip.*

AMS (*MOS*) *subject classifications* (1970). Primary 47A40, 81A48.

[1]Research for a doctoral thesis; supported by an NSF Graduate Fellowship.

(ii) $(1 + x_{ij}^2)^\gamma u_{ij}(x_{ij})$ *and* $(1 + x_{ij}^2)^\gamma w_{ij}(x_{ij})$ *belong to* $L^p(\mathbf{R}^m) + L^\infty(\mathbf{R}^m)$ *for some* $p > m$ *and* $\gamma > 1/2$.

(iii) *the bound state energies of the three body subsystems are nonpositive. Then for generic couplings* $\{\lambda_{ij}\}$, *the wave operators for H are asymptotically complete in the sense that* $P_{ac} = \Sigma_\alpha \Omega_\alpha^\pm \Omega_\alpha^{\pm *}$.

3. **Remarks.** 1. Simon [8] has given sufficient conditions for (iii) to hold, but it is hoped that hypothesis (iii) can be removed as it is required for only one estimate in the proof of the theorem.

2. Yukawa potentials satisfy all the hypotheses of the theorem.

3. For generic couplings, the two and three body subsystems have finitely many bound states, so there are finitely many thresholds for the four body system. Sigal's methods [6] also yield this result.

4. The generic couplings are precisely those for which no subsystem has a threshold resonance or threshold bound state. This set of couplings is large in the sense that its complement is a closed set of Lebesgue measure zero.

5. The proof of the theorem is quite similar to the asymptotic completeness proof given in [3]. The starting point is the derivation of a resolvent formula which is much simpler than the Yakubovskii formula [10]. Unfortunately, this formula must be modified in order to study scattering, and the modification gives rise to large matrices of operators.

I would like to thank Professor Barry Simon for his advice throughout all stages of this work.

ADDED IN PROOF. Hypothesis (iii) of the theorem has been studied in E. Balslev, *Absence of positive eigenvalues of Schrödinger operators*, Arch. Rational Mech. Anal. 59 (1975), 343–357.

REFERENCES

1. E. Balslev and J. M. Combes, *Spectral properties of many-body Schrödinger operators with dilatation-analytic interactions*, Comm. Math. Phys. 22 (1971), 280–294.

2. L. D. Faddeev, *Mathematical aspects of the three-body problem in the quantum scattering theory*, Israel Program for Scientific Translations, Jerusalem, 1965.

3. J. Ginibre and M. Moulin, *Hilbert space approach to the quantum mechanical three-body problem*, Ann. Inst. H. Poincaré Sect. A. 21 (1974), 97–145.

4. J. S. Howland, *Abstract stationary theory of multichannel scattering*, J. Functional Analysis 22 (1976), 250–282.

5. M. Reed and B. Simon, *Methods of modern mathematical physics*, Vol. III, Academic Press (to appear).

6. I. M. Sigal, *On the point spectrum of the Schrödinger operators of multiparticle systems*, Comm. Math. Phys. 48 (1976), 155–164.

7. ———, E. T. H. (preprint).

8. B. Simon, *Absence of positive eigenvalues in a class of multiparticle quantum systems*, Math. Ann. 207 (1974), 133–138.

9. L. E. Thomas, *Asymptotic completeness in two-and three-particle quantum mechanical systems*, Ann. Physics 90 (1975), 127–165.

10. O. A. Yakubovskii, *On the integral equations in the theory of N particle scattering*, Soviet J. Nuclear Phys. 5 (1967), 1312–1320.

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