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Pattern synthesis, lectures in pattern recognition, Volume 1, by U. Grenander, Applied Mathematical Sciences, No. 18, Springer-Verlag, New York and Berlin, 1976, vii + 509 pp., \$14.80.

A generation or so ago, applied mathematics consisted primarily of the solution of partial differential equations subject to diverse geometrical constraints. The development of computers radically altered this emphasis: numerical methods superseded analytical approximations, and the rapidly expanding capacities of digital computers obviated the need for ingenious shortcuts.

Analog computers, with their versatile internal structures, would probably have given rise to investigations into structural configurations if they had not fallen behind digital automata because of their restricted speeds. Nevertheless, the design of digital computer circuitry itself has stimulated work on configurational mathematics, and the very scalar quality of the digital computer has necessitated an analytical approach to pattern recognition. The theory of self-organizing automata (Turing), the genetic code and research on neural nets have proven the power of an algorismatic approach to structure.

A pattern is an ordered array whose components bear a well-defined relation to each other. Pattern recognition amounts to the identification of these components, and the nature of their interrelationships. Such identification is to a certain extent subjective: the expression of the internal vibrations of material objects as a series of monochromatic mutually orthogonal functions is convenient, but not necessarily fundamental in acoustic theory. The description of a crystal in terms of stacked cubical unit cells may please the crystallographer, but the solid-state chemist will prefer a model based on stacking spheres of different diameters. Thus the same crystal will appear in different "gestalten" in different contexts.

A very effective way of reducing the apparent complexity of a pattern is the above-mentioned algorismatic approach, namely by synthesizing it according to a given generating rule, which relates each component to at least two other components. For example, an assembly of points defined by the relation that each point be at the center of a regular tetrahedron whose vertices constitute points belonging to this same assembly, constitutes the locations of carbon atoms in the diamond structures.

Ulf Grenander, of the Division of Applied Mathematics at Brown University, proposed ten years ago at a scientific meeting in Loutraki, Greece, that it would be possible to create a general theory of patterns; the present volume is the first in a projected series reporting on the progress of this study. The author cautions us in his Introduction that the first two volumes should be considered as only a preliminary presentation. He states that pattern theory is a fairly new branch of applied mathematics, so that it would be unreasonable to expect at this time a presentation as polished and careful as he would have liked to make it.

Grenander's approach is the algorismatic one: he defines a concept of *arity* as the maximum number of connections that may relate any structural component to other components. When every component bears an identical relation to all other components, the structure is regular. The above-mentioned diamond structure would have arity *four*: the arity is here analogous to a chemical valency. For directed graphs the arity equals the sum of the in-arity and out-arity.

It is difficult to know for what audience this volume is intended. The subtitle suggests that it is based on lecture notes for Grenander's course at Brown University; the introductory notes also indicate that it is a report on research in progress. The result is somewhat uneven: many applied mathematicians will find the first one hundred and forty pages tough going without attending the lectures. This is not surprising because of the rapid expansion of applied mathematics into new areas and the novelty of the theory presented here. The formalism of this first third of the book is uncompromising. In reading these pages this reviewer immediately recalled Conway's Game of Life, so popular with undergraduates taking computer courses. Nevertheless, this game is not discussed before page 313, and even then the generating rules of this game, as presented here, would not recruit many undergraduates to the study of this rather significant growth system.

Following the abstract image theory, there are applications to a wide variety of structures: fibres, slivers, etc., the heart, genealogical trees, river systems and motion studies. The chapters on space-time patterns are among the most fascinating in this book; their relevance is attested to by recent work on, for instance, morphogenesis by René Thom, catastrophe theory by James Callahan, on allometry by Gould, on branching systems by Woldenberg and on coastline structures by Mandelbrot. Nevertheless it is not likely that many experts in the widely ranging field of application of Grenander's theory will be able to draw directly upon this book and use it profitably in their own work. There is a need for translation: presumably this theory is expected to spin off applications in a manner analogous to the spinoffs of the switching and information theories. Indeed not every present-day systems engineer would have been able to cope directly with early Caldwell, Huffman, Shannon or Wiener; nevertheless he is now indirectly applying the results of those early investigations. It is hoped, however, that in its final form this prototype will become more accessible.

In warning the reader that his growth patterns should be seen as mathematical constructs rather than biological realities, Grenander quotes Rosen on biological morphogenesis: one investigates the *capability* of models. This reviewer has pointed out elsewhere that the similarity of patterns occurring at widely different scales is due to the fact that the specific nature of interactive forces is frequently superseded by the properties of three-dimensional space, which permit but a limited repertoire of patterns and connectivities. Therefore these mathematical constructs have a validity in equilibrium and steady-state systems, regardless of specific interactive forces.

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Spectral synthesis, by John J. Benedetto, Academic Press, Inc., New York, 1975, 278 pp., $27.50.

Let Φ be in $L^\infty(\mathbf{R})$. If Φ can be written as

$$\Phi(x) = \sum_{k=1}^{n} c_k \exp(ixy_k),$$

then the set of characters $\{\exp(ixy_k): k = 1, \ldots, n\}$ is called the spectrum of Φ and denoted $\operatorname{sp}\Phi$. The set of translates of Φ spans a finite-dimensional subspace $\mathfrak{T}_\Phi$ of $L^\infty(\mathbf{R})$, namely the linear span of $\operatorname{sp}\Phi$. In fact, $\operatorname{sp}\Phi$ is exactly the set of characters $\exp(ixy)$ belonging to $\mathfrak{T}_\Phi$. Thus the linear span of the translates of Φ is determined by its spectrum. The problem of spectral synthesis for bounded functions is to study suitable generalizations of this simple observation. That is, given Φ in $L^\infty(\mathbf{R})$, is the smallest translation-invariant subspace of $L^\infty(\mathbf{R})$ containing Φ and closed in some topology generated by the spectrum of Φ ? The problem has been studied with various topologies on $L^\infty(\mathbf{R})$, but for many purposes the most suitable is the weak-* topology. Also the setting is often generalized to a locally compact abelian group G with character group Γ. In our discussion above, $G = \mathbf{R}$ and $\Gamma = \{\exp(ixy): y \in \mathbf{R}\}$.

For the more general set-up, let Φ be in $L^\infty(G)$ and let $\mathfrak{T}_\Phi$ be the smallest weak-*-closed translation-invariant subspace of $L^\infty(G)$ containing Φ. For any weak-*-closed translation-invariant subspace $\mathfrak{T}$ of $L^\infty(G)$, we define its spectrum as $\mathfrak{T} \cap \Gamma$. And the spectrum of Φ is, by definition, the spectrum of $\mathfrak{T}_\Phi$. The spectrum is a closed subset of Γ and every closed subset E of Γ is the spectrum for at least one $\mathfrak{T}$. If there is exactly one $\mathfrak{T}$, i.e. if E determines $\mathfrak{T}$ in