In all the years that these newer phenomena were being discovered, people also continued working on more direct generalizations and refinements of the classical limit theorems listed in the first paragraph. Petrov's book is almost exclusively concerned with these refinements. The book stresses the situation where the independent summands are not necessarily identically distributed, an area to which Petrov himself has made significant contributions. After a derivation of the classical limit laws the book gives an excellent survey (up till 1972, the date of the publication of the Russian edition) of error estimates, asymptotic expansions, local limit theorems and large deviation estimates, all for the case of a normal limit law in one dimension. The last two chapters give conditions for the laws of large numbers and the law of the iterated logarithm. Each chapter ends with a supplement, listing (with references) technical results which should be of particular interest to experts. Since much of the recent work in all these areas is not easily accessible to western readers, the book provides a welcome service. Even though the reviewer's preferences are more in the direction of the newer aspects of sums of independent random variables, one should not undervalue the refined versions of the classical limit theorems. Many of the proofs require tremendous skill in classical analysis, especially Fourier analysis, and many of the results, such as the Berry-Esseen estimate, the Edgeworth expansion and asymptotic results for large deviations, are important for statistics and theoretical purposes.

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Mathematical developments arising from Hilbert problems, edited by Felix E. Browder, Proc. Sympos. Pure Math., vol. 28, American Mathematical Society, Providence, Rhode Island, 1976, xii + 628 pp., $37.60.

To begin this review I move a vote of thanks: to the AMS, to the organizing committee of the May, 1974 De Kalb symposium on Hilbert's problems (Bateman (Secretary), Browder (Chairman), Buck, Lewis, Zelinsky), and to all the authors. Admirers of Hilbert and his problems, a set which must be nearly identical with the set of mathematicians, will lovingly place the book on their shelves (as soon as their orders are filled).

Some readers may be curious concerning what was previously available by way of a survey of Hilbert's problems. Since these references do not appear in the present book, I shall furnish them here. Bieberbach **[1]** surveyed the status of the problems in 1930, as did Demidov **[2]** in 1966. In 1969 the Russian survey **[6]** appeared, followed by a translation into German **[7]**. There is a survey in Japanese by Sin Hitotumatu. Until now the only account in English was that of Fang **[3]**. There is some biographical information concerning Hilbert's talk on the problems in **[8]** and **[9]**.

Browder's preface asserts that the main thrust of the symposium was not toward the history of the problems and their current status but rather toward their significance for today's and tomorrow's mathematics. By and large the chapters have fulfilled this aim. But for a peek at the future, readers will, in addition, turn to the section entitled "Problems of present day mathematics". At the initiative of Dieudonné, the daring step was taken of assembling a new list of problems. They are divided into 27 sections and occupy a full 44 pages. You probably won't find your favorite problem here but you will surely agree that it is a stimulating collection. One problem (§9) has already been solved and it was fortunately possible to catch this in proof; in early 1976 Quillen and Suslin independently settled affirmatively Serre's problem on projective modules over polynomial rings.

There are quite a few threads of continuity between the old problems and the new, and there is even some continuity in the strongest possible sense: Montgomery's writeup on Hilbert's 8th problem (the distribution of primes and the Riemann hypothesis) is identical, except for typography, with §3 of the new problems.

I hereby wish for the new problems a success comparable to that of the old. To be sure, not all of Hilbert's 23 problems were resounding successes. But let me just mention the mathematics that was stimulated by four of my favorites. (i) The creation by Artin of a whole theory of formally real fields in order to solve the 17th (Is every positive polynomial a sum of squares?). This problem is covered by Pfister. (ii) The brilliant idea of Gleason that cracked the 5th (Is every locally Euclidean group a Lie group?). The report is by Yang. (iii) The Hasse principle for quadratic forms (no. 11, O'Meara). (iv) The charming proof by Dehn that polyhedra of equal volume need not be decomposable into congruent pieces (no. 3).

There are 23 problems and 23 authors. A one-to-one correspondence? Not quite. A number of "singularities" combine to cancel out. Those who heard Conway on the third problem will regret that there is no manuscript. We can, however, be grateful to Stampacchia for submitting a manuscript on no. 23 (calculus of variations) which he was unable to present orally. Problem 16 (the topology of the branches of a real algebraic curve) is not covered (but one should note Arnold's suggestions for work in the area in §13 of the new problems). Davis, Matijasevic, and Robinson coauthor no. 10 (Is there an algorithm for Diophantine equations?); the oral presentation was given by Julia Robinson. Three different aspects of no. 8 are treated by Bombieri, Katz, and Montgomery. Katz's contribution is an overview of Deligne's proof of the Riemann hypothesis for finite fields, and Katz does a second overview of Deligne for no. 21 (existence of differential equations with prescribed monodromy group). Bombieri is also a double threat man, his second presentation being no. 19 (variational problems). Readers will not be bothered too much by an interchange of numbers between this problem and no. 20 (boundary value problems); the two problems are closely related. Serrin is the reporter on no. 20.

There are still 12 problems and corresponding authors that I have not managed to mention: 1. Continuum hypothesis (Martin); 2. Consistency of mathematics (Kreisel); 4. Geometries in which straight lines give the shortest distance (Busemann); 6. Axioms for mathematical physics (Wightman); 7. Transcendence of a^b for a and b algebraic (Tijdeman); 9. General reciprocity (Tate); 12. Generalize Kronecker's Jugendtraum (Langlands); 13. Is every function of 3 variables expressible in terms of functions of 2 variables? (Lorentz); 14. Finite generation of subrings (Mumford); 15. Rigorous foundation of Schubert's enumerative calculus (Kleiman); 18. Crystalographic groups (Milnor); 22. Uniformizations (Bers).

With two further notes I conclude my task. The book reproduces the translation of Hilbert's address by Mary Winston Newson (1902, Bull. Amer. Math. Soc.). The German original is perhaps most readily available in the third volume of Hilbert's collected works [**5**]. In French there is in [**4**] a reprinting of the summary which appeared in the 1900 Congress Proceedings. To Paul Halmos we are indebted for 22 photographs (Conway is included and Matijasevic and Stampacchia are missing). I have cited so many integers in this review that I can't resist one more: there are 10 beards.

ERRATA. In the article entitled *Hilbert's tenth problem. Diophantine equations: Positive aspects of a negative solution,* by Martin Davis, Yuri Matijasevič and Julia Robinson, pages numbered 357–358 should precede pages numbered 355–356.

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9. C. Reid, *Hilbert,* Springer-Verlag, Berlin and New York, 1970, pp. 65–85. MR **42** #5767.

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Decomposition of multivariate probabilities, by Roger Cuppens, Probability and Mathematical Statistics, Series, Vol. 29, Academic Press, New York, San Francisco, London, 1975, xv + 244 pp., $26.50.

In 1929, B. de Finetti introduced the class of infinitely divisible probability