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Finite orthogonal series in the design of digital devices, by M. G. Karpovsky, John Wiley and Sons, New York, 1976, 251 pp., $35.00 with bibliography and subject index.

The subject of this book is an interesting mathematical approach to the design of digital logic, with the intended application of the results to be used in the design of digital computers. The central problem of the book concerns the methodology for designing networks that realize arbitrary boolean functions. In the most elementary case, the problem is to realize a single boolean valued function $f(x_1, x_2, \ldots, x_n)$ of the n boolean variables $x_1, x_2, \ldots, x_n$. The design objective is to obtain minimum cost designs where cost is measured in terms of the costs of the primitive functions used to construct the given function. More complex problems derived from the basic one include the design of networks that realize two or more boolean functions of the same arguments, the design of networks that realize sequential functions (functions of present and past values of arguments), and the design of networks for partially specified functions. In the latter case the design makes use of the freedom to complete the function specification arbitrarily, and picks a completion that achieves minimal cost. Yet another problem is the design of networks that exhibit error-correction properties in that failures of such a network result in a network that produces a different function from the desired one with low probability.

Traditional approaches taken by practitioners involve costly searches over many possible implementations to find the best one, or they rely on canonical realizations that are improved by hand on an *ad hoc* basis. Karpovsky exposes a very different mathematical viewpoint to the minimization process that has several interesting properties. His work is partially stimulated by work by Ninomiya and by Lechner on harmonic analysis of boolean functions.

Karpovsky reports that he and his colleague E. S. Moskalev advanced the work in the Soviet Union, and this book is the first English release of the comprehensive material.

Since boolean functions of n-variables operate on a finite domain with 2^n points, it is relatively straightforward to construct a canonical realization of a function from 2^n "delta" functions where each delta function is 0 everywhere except possibly at one of the 2^n points in the domain. Conventional design techniques find the nonzero coefficients in the function expansion, sum these, and obtain the desired realization. There are some algebraic manipulations used to obtain further simplifications, and there exist several techniques for applying essentially enumerative procedures to function realizations to obtain minimal cost realizations.

The methodology proposed by Moskalev, and earlier by Ninomiya and Lechner, is to construct a function by finding 2^n Fourier coefficients for an expansion over a basis of 2^n harmonic functions. Again, because the domain and range are both discrete, the appropriate transform to use is the discretized version of the Fourier transform, which is popularly known as the Walsh transform. Generalizations of this to multivalued algebras from the usual two-valued boolean algebra are studied in the book and are known as Chrestenson transforms.

The essential idea in the minimization process is very simple. Expand a function f in terms of 2^n Walsh functions that form a basis and find the 2^n Walsh coefficients in this expansion. These coefficients are stored in a memory, and a function is realized by multiplying each Walsh function by its corresponding coefficient and adding in a summing device. Since the number of coefficients for the Walsh basis and for the usual basis are the same, it might appear that there is no difference in the costs of the networks made in the two different ways.

The major of attraction of the Walsh analysis and synthesis is that it produces nearly identical networks for two functions that differ from each other by a linear transformation of the input variables. Suppose for example that $f(x_1, x_2, x_3) = g(x_1, x_3, x_2)$. In this case the input variables differ by a permutation, but more generally the theory applies when the one set of variables is obtained from the other by a linear transformation. If we have an efficient network realization for g and are asked to find one for f, we obviously can use the g network with variables interchanged. This is easily detectable from Karpovsky's methods because f and g will have the same transform. Linear transformations of inputs and translations applied to inputs do not alter the spectrum obtained by a transform.

Karpovsky has been able to develop applications for Walsh transforms of boolean functions for several of the problems mentioned above. He has also developed techniques for using Haar transforms to solve problems related to realizing functions that are "near" one another. Unlike the Walsh basis functions, the Haar basis functions are "local" in nature. That is, a change in single Haar coefficient has an effect limited to a small locality of a single domain element, whereas a change in a single Walsh coefficient is felt throughout the domain.

At this point in time the work appears to be highly developed and rich in mathematical elegance. It is not clear what the long term directions of the research are, nor what the present implications are. Since the research is largely stimulated by the need to solve practical problems in computer design, one might measure the impact of the research on present design. The impact, unfortunately, has been quite small, and is not likely to improve over time. The cost functions on which the research is predicated have turned out largely to be unrealistic characterizations of present technology, although they were reasonable characterizations of past technology. Practitioners today are able to use canonical realizations with or without small improvements from *ad hoc* analysis to design computers, and the costs of nonminimal circuits have been very close to the costs of absolutely minimal circuits. The theory no longer has to satisfy past constraints and may be driven in new innovative directions.

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Global variational analysis: *Weierstrass integrals on a Riemannian manifold*, by Marston Morse, Mathematical Notes, Princeton University Press, Princeton, New Jersey, 1976, ix + 255 pp., $6.50.

The first thing that comes to mind in reviewing a new book by Marston Morse on the calculus of variations is that he wrote a book, *The calculus of variations in the large*, forty years ago. The early book gave the foundations of what is now called Morse theory. The publication of a new book by Morse on the same subject presents an occasion to give some personal perspectives on how this mathematics has developed in the last few decades. I say "personal perspectives" and indeed, I, myself, have been involved in, and inspired by, Morse's mathematics. For example, three of my papers contain the word Morse in the title. Another mathematician much influenced by Morse, Raoul Bott, was my adviser, and even work of Morse (but not variational theory) suggested to Bott the thesis problem he gave me (leading eventually to my work in immersion theory).

Another factor in writing a review like this is that, today, global analysis is very much alive, both in mathematics and other disciplines. It may give us some perspective to trace the development of one of the main roots of the subject.

Let us see what Morse, in 1934, had to say about global analysis (he used the word macro-analysis, then). I quote the full first paragraph of the Foreword of his book.

"For several years the research of the writer has been oriented by a conception of what might be termed macro-analysis. It seems probable to the author that many of the objectively important problems in mathematical physics, geometry, and analysis cannot be solved without radical additions to