needed in the study of stochastic control problems. For some readers Chapter V will serve to fill in gaps in their backgrounds. For others it will serve as a good outline of the homework that they will have to do if they become serious about stochastic control theory.

As already noted, the stochastic control problems are treated via dynamic programming in a mathematically rigorous way. For the class of processes considered, the study of the optimal control problems is reduced to the study of certain second order nonlinear partial differential equations. The existence of solutions of the partial differential equations and the properties of the solutions are then investigated with considerable success.

The style is lean and clean. Proofs and major developments are broken up into easily digestible pieces. At appropriate places references are given to the literature for further development of topics or to alternate developments.

This is definitely a book that both the specialist and the person interested in an uncluttered introduction to some of the major aspects of deterministic and stochastic control will want to read and own.

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Combinatorial algorithms, by Albert Nijenhuis and Herbert S. Wilf, Academic Press, New York, San Francisco, and London, 1975, xiv + 253 pp., $19.50.

Combinatorial algorithms are computational procedures which are designed to help solve combinatorial problems. Combinatorial problems are problems involving arrangements of elements from a finite set *and* selections from a finite set. These problems can be divided into three basic types: (1) enumeration problems, (2) existence problems, and (3) optimization problems. In enumeration problems the goal is *either* to find how many arrangements there are satisfying the given properties *or* to produce a list of arrangements satisfying the given properties. In existence problems the goal is to decide whether or not an arrangement exists satisfying the given properties. In optimization problems the goal is to find where a given function of several variables takes on an extreme value (maximum or minimum) over a given finite domain. Graph theoretic algorithms are included in the above definition of combinatorial algorithms.

In this book Nijenhuis and Wilf discuss various combinatorial algorithms. Their enumeration algorithms include a chromatic polynomial algorithm and a permanent evaluation algorithm. Their existence algorithms include a vertex coloring algorithm which is based on a general backtrack algorithm. This backtrack algorithm is also used by algorithms which list the colorings of a graph, list the Eulerian circuits of a graph, list the Hamiltonian circuits of a graph and list the spanning trees of a graph. Their optimization algorithms include a network flow algorithm and a minimal length tree algorithm. They give 8 algorithms which generate at random an arrangement. These 8 algorithms can be used in Monte Carlo studies of the properties of random arrangements. For example the algorithm that generates random trees can be used to study the distribution of the degree sequence of a random tree with N vertices.

At first glance *Combinatorial algorithms* may appear to be just a collection of over 30 algorithms. A more careful study of this book reveals that these algorithms may be placed into classes of similar algorithms. The largest class of algorithms contains those algorithms which generate a random combinatorial arrangement. For example algorithms from this class generate a random subset of an n-set, a random composition of the integer n into k-parts, a random permutation of n letters, and a random unlabeled rooted tree. Nijenhuis and Wilf present interesting discussions of what they mean by choosing an arrangement at random when introducing each algorithm in this class. There is a basic method behind these random algorithms; this method is discussed in the section called "Deus ex Machina". This section indicates how computing and mathematics interact.

A major development in the theory of combinatorial algorithms was the discovery of the class of NP-complete combinatorial problems by S. A. Cook and R. M. Karp. (See R. M. Karp, "Reducibility among combinatorial problems" in *Complexity of computer computations*, edited by R. E. Miller and J. W. Thatcher, Plenum, New York, 1972.) Many of the Nijenhuis-Wilf algorithms attack problems that belong to this NP-complete equivalence class. These problems are equivalent in that *either* each of these problems has an algorithm that is bounded by a polynomial in the length of the input describing the problem *or* none of these problems have a polynomial-bounded algorithm. Deciding whether or not these NP-complete problems are polynomially bounded is a major unsolved problem in the theory of automata.

The reviewer could not find any difference in the typeface used for the letter "oh" and for the number "zero". This lack of difference makes it difficult to use the FORTRAN program listings printed in this book. Also, certain features of FORTRAN used in the program listings are not available on all compilers. Thus the reader may find it necessary to slightly modify these programs.

Since *Combinatorial algorithms* by Nijenhuis and Wilf bridges the gap between computer science and mathematics, it should be included in both computer science libraries and mathematics libraries. This book can be used by readers interested in building a collection of combinatorial programs *and* by readers interested in studying the mathematics associated with the algorithms. Like a novel, this book may be read on several levels.

EARL GLEN WHITEHEAD, JR.