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# GLOBAL DEFINABILITY THEORY IN $L_{\omega_1\omega}$[1]

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**Introduction.** Results in definability theory which are about a fixed structure are called "local" by Reyes [**R**]. An example is Scott's definability theorem [**Sc**]. In contrast, "global" results are about the class of models of a sentence (theory); an example is Svenonius' theorem [**Sv**]. Note that the straight analogue for $L_{\omega_1\omega}$ of Svenonius' theorem, if true, would be a global generalization of Scott's theorem, i.e., the latter would be obtained by applying the former to the Scott sentence of the given structure. Although this generalization is false, Motohashi [**Mo**] has found a totally satisfactory global generalization of Scott's theorem (his result is explained below).

We give two distinct global generalizations of a local weak-definability theorem by Kueker [**Ku 1**] and Reyes [**R**] (Theorems 1 and 2 and Corollary (A)) and one for Kueker's local theorem in [**Ku 1**] on structures with only countably many automorphisms (Theorem 3 and Corollary (E)). In Theorems 2 and 3, we utilize Motohashi's work. Theorem 4 is related to [**Ku 2**].

1. **Results.** $L$ denotes a fixed countable language, $L_{\omega_1\omega}$ the finite-quantifier logic with countable conjunctions and disjunctions based on $L$. $\boldsymbol{P}$ is an additional predicate symbol, $L_{\omega_1\omega}(\boldsymbol{P})$ is the corresponding extension of $L_{\omega_1\omega}$. $\mathfrak{A}$ and $(\mathfrak{A}, P)$ denote structures for $L_{\omega_1\omega}$ and $L_{\omega_1\omega}(\boldsymbol{P})$, resp. Following [**Ku 1**], we write $M_\sigma(\mathfrak{A})$ for $\{P: (\mathfrak{A}, P) \models \sigma\}$ and $M(\mathfrak{A}, P)$ for $\{Q: (\mathfrak{A}, Q)$ is isomorphic to $(\mathfrak{A}, P)\}$. $|X|$ is the cardinality of $X$.

THEOREM 1. *For any sentence $\sigma$ in $L_{\omega_1\omega}(\boldsymbol{P})$*, (i) $\Leftrightarrow$ (ii).

(i) *For every countable* $\mathfrak{A}$, $|M_\sigma(\mathfrak{A})| \leqq \aleph_0$ (*or, equivalently*, $< 2^{\aleph_0}$).

(ii) *For some formulas* $\varphi_n(\vec{x}, \vec{u}^n)$ $(n < \omega)$ *of* $L_{\omega_1\omega}$,

$$\sigma \models \bigvee_{n<\omega} \exists \vec{u}^n \, \forall \vec{x}[P\vec{x} \leftrightarrow \varphi_n(\vec{x}, \vec{u}^n)].$$

Theorem 1 is a direct analogue of the weak-definability theorem for finitary logic of Chang [**C**] and the author [**Ma 1**], as improved by Reyes [**R**] for countable structures. In fact, our proof gives the result for all admissible fragments of $L_{\omega_1\omega}$ (with the whole formula after "$\models$" in (ii) being in the fragment). A similar remark applies for our subsequent

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results. Taking $\sigma$ to be the Scott sentence of $(\mathfrak{A}, P)$, we obtain

COROLLARY (A) (KUEKER [**Ku 1**], REYES [**R**]). *$|M(\mathfrak{A}, P)| \leqq \aleph_0$ iff $|M(\mathfrak{A}, P)| < 2^{\aleph_0}$ iff for some finitely many elements $\vec{a}$ of $A$, $P$ is definable in $(\mathfrak{A}, \vec{a})$ by an $L_{\omega_1\omega}$-formula with the parameters $\vec{a}$.*

Our next two theorems utilize work of Motohashi [**Mo**].

Let $X$, $Y$ be *disjoint* infinite sets of variables. $x, x_i, \ldots$ denote variables from $X$; $y, y_i, \ldots$ from $Y$; $\vec{x}, \vec{x}^i, \ldots$ vectors of $x$'s, similarly for $\vec{y}, \vec{y}^i$.

DEFINITION (MOTOHASHI [**Mo**]). A formula $\theta$ in $L_{\omega_1\omega}(\boldsymbol{P})$ is called a Motohashi formula ($M$-formula) if every atomic subformula of $\theta$ is of the form either $\pi(\vec{x})$ or $\pi(\vec{y})$ with $\pi(\cdot)$ in $L_{\omega\omega}$ or else $\boldsymbol{P}\vec{y}$.

The following are easily seen.

PROPOSITION (B) ([**Mo**]). *A finitary $M$-formula $\theta(\vec{x})$ is logically equivalent to a finitary formula of the form $\bigwedge_{i<n} [\sigma_i \rightarrow \varphi_i(\vec{x})]$, $\sigma_i$ sentences in $L_{\omega\omega}(\boldsymbol{P})$, $\varphi_i(\vec{x})$ in $L_{\omega\omega}$.*

(C) *For given countable $(\mathfrak{A}, P)$, $\theta(\vec{x}, \vec{y})$ an $M$-formula, $\vec{a}^0$ elements of $A$, $\theta(\vec{x}, \vec{a}^0)$ is equivalent in $(\mathfrak{A}, P, \vec{a}^0)$ to an $L_{\omega\omega}$-formula $\varphi(\vec{x})$ without parameters.*

To obtain $\varphi$ in (C), convert in $\theta$ each $y$-quantifier, $\forall y \cdots y \cdots$ into $\bigwedge_{a\in A} \cdots \underline{a} \cdots$, with $\underline{a}$ a new constant for $a \in A$, and similarly for $\exists y$. Then replace each resulting atomic formula $\pi(\underline{\vec{a}})$, $\boldsymbol{P}\underline{\vec{a}}$ by its actual truth-value in $(\mathfrak{A}, P)$.

THEOREM (D) (MOTOHASHI [**Mo**]). *For any $\sigma$ in $L_{\omega_1\omega}(\boldsymbol{P})$,* (i) $\Leftrightarrow$ (ii).

(i) *For all (or, for all countable) $(\mathfrak{A}, P) \models \sigma$, $|M(\mathfrak{A}, P)| = 1$.*

(ii) *$\sigma \models \forall\vec{x}[P\vec{x} \leftrightarrow \theta(\vec{x})]$ for some $M$-formula $\theta(\vec{x})$.*

By (C), (ii) obviously implies (i). (D) can be proved by an application of Feferman's many-sorted interpolation theorem [**F**]. This proof as well as Motohashi's proof in [**Mo**] gives the result for all admissible fragments of $L_{\omega_1\omega}$. Hence by (B), (D) implies Svenonius' theorem [**Sv**]. Also by (C), (D) implies Scott's definability theorem [**Sc**] (apply (D) for the Scott sentence $\sigma$ of $(\mathfrak{A}, P)$).

THEOREM 2. *For any sentence $\sigma$ in $L_{\omega_1\omega}(\boldsymbol{P})$,* (i) $\Leftrightarrow$ (ii).

(i) *For all countable $(\mathfrak{A}, P) \models \sigma$, we have $|M(\mathfrak{A}, P)| \leqq \aleph_0$ (or, $< 2^{\aleph_0}$).*

(ii) *$\sigma \models \bigvee_{i<\omega} \exists\vec{x}^i\vec{y}^i\, \forall\vec{x}[P\vec{x} \leftrightarrow \theta_i(\vec{x}, \vec{x}^i, \vec{y}^i)]$ for some $M$-formulas $\theta_i$ ($i < \omega$).*

By (C), (ii) obviously implies (i). For the same reason, Theorem 2 specializes to (A) if $\sigma$ is the Scott sentence of $(\mathfrak{A}, P)$. By (B), Theorem 2 for finitary logic is a form of the weak-definability theorem [**C**], [**Ma 1**], [**R**]. As Motohashi [**Mo**] shows, conditions (i) in Theorems 1 and 2 are not equivalent for $L_{\omega_1\omega}(\boldsymbol{P})$, unlike in the finitary case. In fact, even (i) in (D) does not imply (i) in Theorem 1.

THEOREM 3. *For any sentence $\sigma$ in $L_{\omega_1\omega}$,* (i) $\Leftrightarrow$ (ii).

(i) *For all countable $\mathfrak{A} \models \sigma$, $\mathfrak{A}$ has at most countably many (or equivalently, less than $2^{\aleph_0}$) automorphisms.*

(ii) $\sigma \models \bigvee_{i<\omega} \exists \vec{x}^i \vec{y}^i\, \forall y\, \forall x[x = y \leftrightarrow \theta_i(x, y, \vec{x}^i, \vec{y}^i)]$ *for some M-formulas $\theta_i$ ($i < \omega$) without $\boldsymbol{P}$.*

By *Proposition* (C), in any given $\mathfrak{A}$, the part after "$\forall y$" of the formula in (ii) implies that $y$ is definable in $\mathfrak{A}$ with the parameters $\vec{x}^i$. Hence Theorem 3 has the following

COROLLARY (E) (KUEKER [**Ku 1**]). *For any countable $\mathfrak{A}$, $\mathfrak{A}$ has at most countably many (less than $2^{\aleph_0}$) automorphisms iff there are some finitely many elements $\vec{a}$ of $A$ such that every element of $A$ is definable in $(\mathfrak{A}, \vec{a})$ by an $L_{\omega_1\omega}$-formula.*

The finitary version of Theorem 3 is, via (B), the well-known result that every finitary sentence with infinite models has a countable model with $2^{\aleph_0}$ automorphisms.

Our last result utilizes, and adds to, Kueker's work on "finite generalizations" of Beth's theorem [**Ku 2**].

THEOREM 4. *For any $\sigma$ in $L_{\omega_1\omega}(\boldsymbol{P})$,* (i) $\Leftrightarrow$ (ii).

(i) *For all (or, for all countable) $\mathfrak{A}$, $|M_\sigma(\mathfrak{A})| < \aleph_0$.*

(ii) $\sigma \models \bigvee_{n<\omega} [\exists \vec{v}^n \varphi_n(\vec{v}^n) \wedge \forall \vec{v}^n[\varphi(\vec{v}^n) \rightarrow \bigvee_{i<n} \forall \vec{x}[P\vec{x} \leftrightarrow \varphi_{n,i}(\vec{x}, \vec{v}^n)]]]$ *for some $\varphi_{n,i}(\vec{x}, \vec{v}^n)$ in $L_{\omega_1\omega}$.*

2. **Proofs.** The proofs use abstract consistency properties (see [**Ke**], [**Ma 2**], [**Ma 3**]) and in case of Theorems 2 and 3, approximation of automorphisms by finite pieces similarly as in the proofs in [**Ma 3**]. We will show the proof of Theorem 2 in some detail.

**Proof of Theorem 2.** Let $C$ be a countably infinite set of new individual constants. Define $\Gamma_2$ to be the collection of objects $\gamma = \langle s, f_i \rangle_{i \in I}$ such that $s$ is a finite set of sentences of $L_{\omega_1\omega}(\boldsymbol{P})(C)$ in negation normal form (n.n.f.) with only finitely many constants from $C$, $I$ is a finite set, each $f_i$ is a finite subset of $C \times C$, and such that (the main condition) *there is no* formula $\mu$ with $(\text{i})_2(\gamma, \mu)$ where:

$(\text{i})_2(\gamma, \mu)$ $\mu$ is of the form of the formula after "$\models$" in Theorem 2(ii) and *whenever* $(\mathfrak{A}, P, \bar{c})_{c \in C}$ is a model of $s$ and, for $i \in I$, $g_i$ is an automorphism of $\mathfrak{A}$ such that $\langle c, d \rangle \in f_i \Rightarrow \langle \bar{c}, \bar{d} \rangle \in g_i$, *then* $\mathfrak{A} \models \mu$.

Suppose $\sigma$ is in n.n.f. and it does not satisfy (ii) in Theorem 2. Then clearly $\gamma_0 =_{\text{df}} \langle \{\sigma\}, \varnothing \rangle$ belongs to $\Gamma_2$. We successively extend this element of $\Gamma_2$, always remaining in $\Gamma_2$, such that the limit of the procedure yields, in a natural way, a model $(\mathfrak{A}, P)$ with $|M(\mathfrak{A}, P)| = 2^{\aleph_0}$.

LEMMA (ii). *For fixed $I$ and $f_i$ $(i \in I)$, $\{s: \langle s, f_i\rangle_{i \in I} \in \Gamma_2\}$ is an abstract consistency property.*

(iii) *For any $\gamma = \langle s, f_i\rangle_{i \in I} \in \Gamma_2$, $j \in I$, $c \in C$, let $d \neq c$ and let $d$ not occur in $\gamma$. Then*

$$\langle s, f_i, f_j \cup \{\langle c, d\rangle\}\rangle_{i \in I - \{j\}} \quad \textit{and} \quad \langle s, f_i, f_j \cup \{\langle d, c\rangle\}\rangle_{i \in I - \{j\}}$$

*belong to $\Gamma_2$.*

*Comment.* (iii) will be used to make sure that the domains and ranges of purported automorphisms will indeed be the whole domain (in this case, essentially $C$) of the structure.

(iv) *Let $\gamma$ and $i$ be as in* (iii). *Let $c$, $d_1$, $d_2$ be distinct constants in $C$ but not in $\gamma$. Put $f'_j = f_j \cup \{\langle d_1, c\rangle\}$, $f''_j = f_j \cup \{\langle d_2, c\rangle\}$ and $s' = s \cup \{Pd_1, \neg Pd_2\}$. Then $\gamma' = \langle s', f_i, f'_j, f''_j\rangle_{i \in I - \{j\}} \in \Gamma_2$.*

*Comment.* (iv) is used to "split" a finite approximation $f_j$ into two. Eventually the infinite paths of the tree of such approximations will be the automorphisms and they will give us $2^{\aleph_0}$ images of $P$. Note that for "extensions" $g'_i$, $g''_i$ of $f'_i$, $f''_i$, resp., "$g'_iP \neq g''_iP$".

PROOF OF (iv). Introduce new operation symbols $\boldsymbol{g}_i$ $(i \in I)$. The assumption that $\gamma' \notin \Gamma_2$ leads to the existence of $\mu'$ with $(\mathrm{i})_2(\gamma', \mu')$. Let $\xi$ be the formula $\neg\mu' \wedge \bigwedge s \wedge \bigwedge_{i \in I}$ "$\boldsymbol{g}_i$ is an $L$-automorphism extending $f_i$". By $(\mathrm{i})_2(\gamma', \mu')$,

(v) $(\mathfrak{A}, P, \vec{c}, g_i)_{c \text{ in } \gamma, i \in I} \models \xi$ implies that every automorphism of $(\mathfrak{A}, \vec{c})_{c \in \operatorname{dom} f_j}$ leaves $P$ fixed.

Hence by (an inessential strengthening of) (D),

(vi) $\xi \models \forall\vec{x}[P\vec{x} \leftrightarrow \theta(\vec{x}, \vec{c})]$ for $\vec{c} = \operatorname{dom} f_j$ and for some Motohashi formula $\theta(\vec{x}, \vec{x}', \vec{y}')$. Hence $\xi \models \mu''$ where $\mu'' = \exists\vec{x}'\vec{y}'\,\forall\vec{x}[P\vec{x} \leftrightarrow \theta(\vec{x}, \vec{x}', \vec{y}')]$. It follows that

$(\mathrm{i})_2(\gamma, \mu' \vee \mu'')$ holds, contrary to $\gamma \in \Gamma_2$.

Now, let $I_n$ be the set of finite 0-1 sequences of length $n$. Let $C = \{c_n: n < \omega\}$. We construct a sequence $\gamma_n$ $(n < \omega)$ of elements of $\Gamma_2$ starting with $\gamma_0 = \langle\{\sigma\}, \varnothing\rangle$ such that $\gamma_n = \langle s_n, f_i^n\rangle_{i \in I_n}$, $s_n \subset s_{n+1}$, $f_i^n \subset f_j^{n+1}$ for $j = i \frown \langle 0\rangle, i \frown \langle 1\rangle$ and

(vii) $s_\omega = \bigcup_{n<\omega} s_n$ is pseudocomplete (see 1.3 Definition in **[Ma 2]**) or, what is the same, the $s_n$ satisfy (1)–(5) on p. 13 in **[Ke]** (here we use (ii)),

(viii) $c_n \in \operatorname{dom} f_i^{n+1} \cap \operatorname{rn} f_i^{n+1}$ $(i \in I_n)$ (here we use (iii)), and

(ix) for each $n$ and $i \in I_n$, there are $d_0 \in \operatorname{dom} f_{j_0}^{n+1}$, $d_1 \in \operatorname{dom} f_{j_1}^{n+1}$ and $c \in \operatorname{rn} f_{j_0}^{n+1} \cap \operatorname{rn} f_{j_1}^{n+1}$ (here $j_0 = i \frown \langle 0\rangle$, $j_1 = i \frown \langle 1\rangle$) such that $\{Pd_0, \neg Pd_1\} \subset s_{n+1}$ (here we use (iv)).

For the canonical model $(\mathfrak{A}, P, \bar{c})_{c\in C}$ of $s_\omega$ (see the proof of the model existence theorem in [**Ke**], or 1.4 in [**Ma 3**]) we have

(x) $\mathfrak{A} \models \sigma$,

(xi) the maps $f_\alpha = \{\langle\bar{c}, \bar{d}\rangle: \langle c, d\rangle \in \bigcup_{n<\omega} f^n_{\alpha\restriction n}\}$ for $\alpha \in {}^\omega 2$ are automorphisms of $\mathfrak{A}$ (mainly by (viii)) and

(xii) $f_\alpha P \neq f_{\alpha'} P$ for $\alpha \neq \alpha'$ by (ix). Q.E.D.

ON THE PROOF OF THEOREM 1. The collection playing the role of $\Gamma_2$ above, $\Gamma_1$, is defined as follows. Let $\boldsymbol{P}_i$ denote distinct predicate symbols of the same arity as $\boldsymbol{P}$, and let us write $s(\boldsymbol{P}_i)$ for a set of sentences in $L_{\omega_1\omega}(\boldsymbol{P}_i)(C)$. Let $\Delta_I$ be the set of sentences of the form

$$\bigvee_{i\in I}\ \bigvee_{n<\omega} \exists \vec{u}^n\, \forall \vec{x}[\boldsymbol{P}_i\vec{x} \leftrightarrow \varphi^i_n(\vec{x}, \vec{u}^n)]$$

where the $\varphi^i_n$ are in $L_{\omega_1\omega}$. We define $\Gamma_1$ to be the collection of objects $\gamma = \langle s_i(\boldsymbol{P}_i)\rangle_{i\in I}$ with similar finiteness conditions as for $\Gamma_2$ and such that there is no $\mu$ with $(\mathrm{i})_1(\gamma, \mu)$ where:

$(\mathrm{i})_1(\gamma, \mu)$ $\mu \in \Delta_I$ and $\bigcup_{i\in I} s_i(\boldsymbol{P}_i) \models \mu$.

The crucial fact analogous to (iv) above is that for $\gamma$ as above, and a fixed $j \in I$, if we put $s'_j(\boldsymbol{P}'_j) =_{\mathrm{df}} s_j(\boldsymbol{P}'_j) \cup \{\boldsymbol{P}'_j c\}$, $s''_j(\boldsymbol{P}''_j) =_{\mathrm{df}} s_j(\boldsymbol{P}''_j) \cup \{\neg \boldsymbol{P}''_j c\}$ with $c \in C$ a constant not in $\gamma$, then $\langle s_i(\boldsymbol{P}_i), s'_j(\boldsymbol{P}'_j), s''_j(\boldsymbol{P}''_j)\rangle_{i\in I-\{j\}}$ again belongs to $\Gamma_1$. The proof of this applies the Beth-Lopez-Escobar theorem.

ON THE PROOF OF THEOREM 3. It is very similar to that of Theorem 2 and applies a corollary to (D): if every model of $\sigma$ has no nontrivial automorphisms, then $\sigma \models \forall y\, \forall x[x = y \leftrightarrow \theta(x, y)]$ for some $M$-formula $\theta$ *without* $\boldsymbol{P}$.

ON THE PROOF OF THEOREM 4. Let us call a formula of the form after "$\models$" in Theorem 4 (ii) a $K$-formula. Consider $\sigma = \sigma(\boldsymbol{P})$ not satisfying (ii). Define $\Gamma_4 = S_4$ to be the set of sets $s(\boldsymbol{P}_0, \ldots, \boldsymbol{P}_{n-1})$ of sentences of $L_{\omega_1\omega}(\boldsymbol{P}_0, \ldots, \boldsymbol{P}_{n-1})$ (C) with the usual finiteness conditions such that for any $K$-formula $\kappa(\boldsymbol{P})$, $s \not\models \sigma(\boldsymbol{P}) \rightarrow \kappa(\boldsymbol{P})$. The crucial property of $S_4$ is that if $s \in S_4$ is as above then $s \cup \{\sigma(\boldsymbol{P}_n)$, "$\boldsymbol{P}_n \neq \boldsymbol{P}_1$", "$\boldsymbol{P}_n \neq \boldsymbol{P}_2$", $\ldots$, "$\boldsymbol{P}_n \neq \boldsymbol{P}_{n-1}$"$\}$ belongs to $S_4$. Also, $S_4$ is an abstract consistency property.

ADDED IN PROOF (May 2, 1973). Jon Barwise noticed that Theorem 1 remains true if we replace $\sigma$ by a $\Sigma^1_1$-over-$L_{\omega_1\omega}(\boldsymbol{P})$ sentence $\exists\vec{\boldsymbol{S}}\sigma(\boldsymbol{P}, \vec{\boldsymbol{S}})$. A similar remark holds for the rest of the theorems too. In fact, no essential change is required in the proofs. Barwise also noticed that from the $\Sigma^1_1$ generalization of Theorem 1 in the "admissible version," the following strengthening of a theorem due to J. Harrison results immediately: If a $\Sigma^1_1$ set of reals does not contain a perfect subset, it is a subset of a set constructible below $\omega^K_1$ (Kleene's $\omega_1$) (notice that our proof gives in fact a perfect subset of $M_\sigma(\mathfrak{A})$). Subsequently, the author noticed that the $\Sigma^1_1$ generalization of Theorem 1 (formulated with "perfect subset") combined with an approximation theorem of Vaught (any constructible $\Pi_1^1$-over-$L_{\omega_1\omega}$ sentence is equivalent for countable structures to $\bigvee_{\alpha<\omega_1}\delta_\alpha$ with some constructible sequence $\langle\delta_\alpha:\alpha<\omega_1\rangle$ of $L_{\omega_1\omega}$-sentences) directly (and without the use of forcing) gives Mansfield's theorem: any $\Sigma_2^1$ set of reals not containing a perfect subset is constructible.

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