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MEASURES WHOSE TRANSFORMS VANISH AT INFINITY

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Let G be an LCA group with dual Γ, $M(G)$ the usual convolution algebra of finite Borel measures on G and ^ the Fourier-Stieltjes transformation. By $M_0(G)$ we mean the ideal of measures $\mu \in M(G)$ such that $\hat{\mu}$ vanishes at ∞. The purpose of this note is to announce the following results.

THEOREM. *Let G be a nondiscrete LCA group with Haar measure m_G. Let λ be a nonzero measure in $M_0^+(G)$ and D a σ-compact subset of G with $m_G(D) = 0$. Then there exists a nonzero measure σ in $M_0^+(\operatorname{supp} \lambda)$ such that*

(i) $\operatorname{supp} \sigma$ *is compact and has λ-measure zero,*

(ii) $m_G[D + G_p(\operatorname{supp} \sigma)] = 0$.

Here $G_p(\operatorname{supp} \sigma)$ denotes the subgroup of G which is algebraically generated by $\operatorname{supp} \sigma$.

COROLLARY 1 (VAROPOULOS [2]). *Every nondiscrete G contains a compact perfect set E such that $M_0^+(E) \neq \{0\}$ and $m_G[G_p(E)] = 0$.*

Let $M_s(G)$ denote the set of measures singular with respect to Haar measure on G.

COROLLARY 2. *Let B be a separable subset of $M_s(G)\backslash\{0\}$ such that $\hat{\mu}|\hat{F} \neq 0$ for all $\mu \in B$ and some σ-compact subset $\hat{F}$ of Γ. Then there exists a measure $\sigma \in M_s^+(G)$ such that*

$$\bigcup_{n=1}^{\infty} (B * \sigma^n) \subset M_s(G)\backslash\{0\}.$$

COROLLARY 3. *Suppose $\mu \in M(G)$ has the property that, $\forall \sigma \in M_0^+(G)$, $\exists n = n_\sigma \in N$ (the natural numbers) such that $\mu * \sigma^n \in L^1(G)$. Then $\mu \in L^1(G)$. In particular, we have $\mu * M_0(G) \subset L^1(G) \Rightarrow \mu \in L^1(G)$.*

As an application of Corollary 3 we give the solution to a question implicit in Meyer [1, p. 94]. Let E be a subset of Γ and define

$$M_{\hat{E}}(G) = \{\mu \in M(G): \operatorname{supp} \hat{\mu} \subset E\}.$$

We say that $\hat{E}$ *is a Riesz set of type* 0 if

(a) $$M_0(G)^\wedge|_{\hat{E}} = L^1(G)^\wedge|_{\hat{E}}.$$

Note that every Sidon set (or Helson set) has property (a).

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COROLLARY 4. *Let* $n = (n_1, n_2, \ldots, n_p) \in N^p$, $p \in N$, *and* $\hat{R} \subset \Gamma$ *satisfy*

(b)
$$\mu_j \in M_{\hat{R}}(G) \cap M_0(G) \quad \text{for } j = 1, 2, \ldots, p \text{ imply}$$
$$\mu_1^{n_1} * \mu_2^{n_2} * \cdots * \mu_p^{n_p} \in L^1(G).$$

Then $\hat{F} = \hat{R} \cup \hat{E}$ *has the following property* (c) *for every Riesz set* $\hat{E}$ *of type* 0:

(c)
$$\mu_j \in M_{\hat{F}}(G) \quad \text{for } j = 1, 2, \ldots, p \text{ imply}$$
$$\mu_1^{n_1} * \mu_2^{n_2} * \cdots * \mu_p^{n_p} \in L^1(G).$$

In particular we have for any compact abelian G the result: The union of a Riesz (or small) set and a Sidon set is a Riesz set.

Detailed proofs of the above will appear elsewhere.

REFERENCES

1. Y. Meyer, *Spectres des mesures et mesures absolument continues*, Studia Math. **30** (1968), 87–99. MR **37** #3281.

2. N. Th. Varopoulos, *Sets of multiplicity in locally compact abelian groups*, Ann. Inst. Fourier (Grenoble) **16** (1966), fasc. 2, 123–158. MR **35** #3379.

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