

## RECENT DEVELOPMENTS IN INFINITE DIMENSIONAL HOLOMORPHY

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**1. Introduction.** There has been a recent and growing interest in holomorphy in infinite dimensional spaces; that is, the study of holomorphic mappings on open, or compact, or more general subsets of complex Banach spaces, or even of complex topological vector spaces which are locally convex, as well as of manifolds modeled after such spaces. This is an area of convergent attention of present day analysts and geometers.

Actually, such a field of research has old roots in the past. They stem from the investigation of holomorphic mappings in infinitely many variables and from the consideration of Taylor series-like expansions of functionals, discussed during the final quarter of the last century and the first quarter of this century. I refer you to the expositions in monograph form by Volterra, Pincherle, Paul Lévy and Hille-Phillips, for instance.

There is a huge literature on the subject covering the old-fashioned, the classical and the recent periods of the creation of this theory. An interesting historical survey has been made recently by Angus Taylor.

In this one-hour lecture, I shall try to describe some current progress, part of which is as yet unpublished. In so doing and by reason of limitation of time, I must be choosy. Needless to say, I will today confine myself to results which are closer to my heart.

**2. Holomorphic mappings.** Let me start with some notation and terminology.

$E$  and  $F$  will denote complex topological vector spaces which are locally convex. During most of this lecture, you can think of  $E$  and  $F$  as just being in particular Banach spaces.

Passing from the finite dimensional situation to Banach spaces presents us with challenging problems. Passing from Banach spaces to locally convex spaces leads us to new challenges.

$U$  will denote a nonvoid open subset of  $E$ .  $K$  will be a compact subset of  $E$ .

$\mathcal{L}_s(^m E; F)$  will denote the vector space of all continuous symmetric  $m$ -linear mappings  $A: E^m \rightarrow F$ , where  $E^m$  is the Cartesian  $m$ -power of  $E$ .

If  $A \in \mathcal{L}_s(^m E; F)$  and  $x \in E$ , I will write

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$$\underbrace{A(x, \dots, x)}_m = A \cdot x^m$$

for short. This notation makes formulas look the same way they do in one complex variable analysis, which has psychological advantages.

$\mathcal{P}(^m E; F)$  will denote the vector space of all continuous  $m$ -homogeneous polynomials  $P: E \rightarrow F$ .

To every  $A \in \mathcal{L}_s(^m E; F)$  there corresponds  $P = \hat{A} \in \mathcal{P}(^m E; F)$  given by  $P(x) = A \cdot x^m$  for every  $x \in E$ . The mapping

$$A \in \mathcal{L}_s(^m E; F) \mapsto P = \hat{A} \in \mathcal{P}(^m E; F)$$

is linear and bijective.

If  $E$  and  $F$  are Banach spaces, then  $\mathcal{L}_s(^m E; F)$  and  $\mathcal{P}(^m E; F)$  are naturally Banach spaces which are homeomorphic but not necessarily isometric under the aforementioned mapping.

In all this, we have  $m = 0, 1, \dots$  with the usual interpretation for  $m = 0$ .

A mapping  $f: U \rightarrow F$  is said to be *holomorphic* on  $U$  if, for every  $\xi \in U$ , there is a sequence of coefficients  $A_m \in \mathcal{L}_s(^m E; F)$ ,  $m = 0, 1, \dots$ , such that, for every continuous seminorm  $\beta$  on  $F$ , we can find an open subset  $V$ ,  $\xi \in V \subset U$ , such that

$$\lim_{M \rightarrow +\infty} \beta \left[ f(x) - \sum_{m=0}^M A_m \cdot (x - \xi)^m \right] = 0$$

uniformly for  $x \in V$ .

If  $F$  is a Hausdorff space, for each given  $f$  and  $\xi$ , the sequence  $(A_m)$  is unique. I will then write

$$d^m f(\xi) = m! A_m \in \mathcal{L}_s(^m E; F), \quad \hat{d}^m f(\xi) = m! \hat{A}_m \in \mathcal{P}(^m E; F),$$

for the  $m$ th differentials of  $f$  at  $\xi$  viewed as a continuous symmetric  $m$ -linear mapping and as a continuous  $m$ -homogeneous polynomial, respectively.

I shall denote by  $\mathcal{H}(U; F)$  the vector space of all holomorphic mappings  $f: U \rightarrow F$ .

For the sake of simplicity, let us assume from now on that  $E$  and  $F$  are Banach spaces, unless the contrary is explicitly stated.

**3. Domains of holomorphy.** The classical Cartan-Thullen theorem about domains of holomorphy in finite dimensions suggests the following definition.

A subset  $X$  of  $U$  is said to be  $\mathcal{H}(U; C)$ -bounding if every  $f \in \mathcal{H}(U; C)$  is bounded on  $X$ .

**THEOREM 1.** *Assume that from every bounded sequence in the dual space  $E'$ , it is possible to extract a weak-star convergent subsequence, in particular that  $E$  is separable or reflexive. Then every closed  $\mathcal{H}(E; C)$ -bounding subset must be compact.*

This result was noticed for  $E$  a Hilbert space independently by Alexander and Dineen. In the preceding general form, it is due independently to Dineen and Hirschowitz.

There is an example due to Dineen of a closed  $\mathcal{H}(E; C)$ -bounding subset which is not compact, when  $E = l^\infty$ , the Banach space of all bounded sequences of complex numbers.

A subset  $X$  of  $U$  is said to be  $U$ -bounded if  $X$  is bounded in  $E$  and  $X$  is bounded away from the boundary of  $U$ , that is  $\text{dist}(X, \partial U) > 0$ .

The mapping  $f \in \mathcal{H}(U; F)$  is said to be of *bounded type* if  $f$  is bounded on every  $U$ -bounded subset.

Let  $\mathcal{H}_b(U; F)$  be the vector subspace of  $\mathcal{H}(U; F)$  of all such  $f$  of bounded type.

On  $\mathcal{H}_b(U; F)$  we use the topology  $\mathcal{T}_b$  of uniform convergence on the  $U$ -bounded subsets, which is given by the family of seminorms

$$f \in \mathcal{H}_b(U; F) \mapsto \sup_{x \in X} \|f(x)\| \in \mathbb{R}_+,$$

where  $X$  is  $U$ -bounded. Then  $\mathcal{H}_b(U; F)$  is a Fréchet space, that is, a complete metrizable locally convex space.

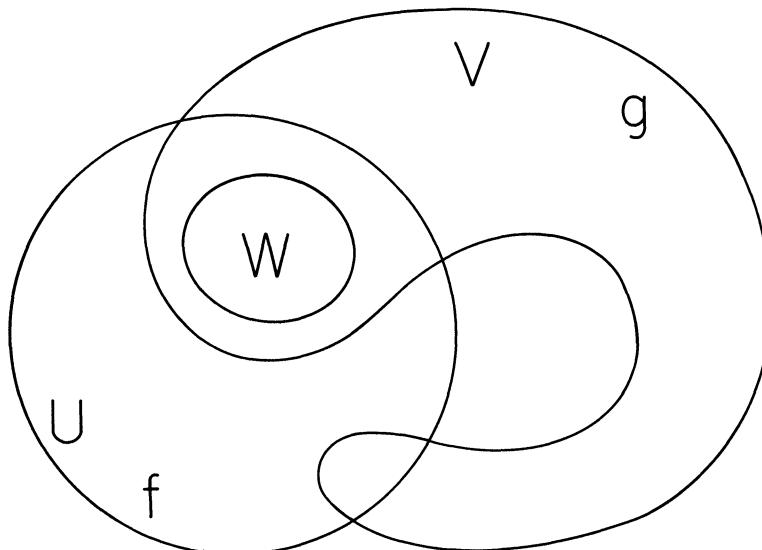


FIGURE 1

It is clear that  $\mathcal{H}_b(U; F) = \mathcal{H}(U; F)$  if  $E$  is finite dimensional or in the trivial case  $F = 0$ . It is conjectured that these spaces are different if  $E$  is infinite dimensional and  $F \neq 0$ . As noticed by Dineen, these spaces are indeed different if  $F \neq 0$  and if there is a sequence in  $E'$  which is weak-star convergent but not norm convergent. Actually it is conjectured that every infinite dimensional space  $E$  satisfies this last stated condition.

$U$  is said to be an  $\mathcal{H}_b$ -domain of holomorphy if  $U$  is connected and it is impossible to find open, connected and nonvoid subsets  $V, W$  in  $E$ ,  $W \subset U \cap V$ ,  $V \neq U$ , such that, for every  $f \in \mathcal{H}_b(U; \mathbf{C})$ , the restriction of  $f$  to  $W$  has a continuation to some  $g \in \mathcal{H}_b(V; \mathbf{C})$ . (See Figure 1.)

$U$  is said to be an  $\mathcal{H}_b$ -domain of existence if  $U$  is connected and there is  $f \in \mathcal{H}_b(U; \mathbf{C})$  having  $U$  as its natural domain of  $\mathcal{H}_b$ -existence, that is,  $f$  is everywhere  $\mathcal{H}_b$ -singular on the boundary of  $U$ . This means that, for every open, connected and nonvoid subsets  $V, W$  in  $E$ ,  $W \subset U \cap V$ ,  $V \neq U$ , the restriction of  $f$  to  $W$  has no continuation to some  $g \in \mathcal{H}_b(V; \mathbf{C})$ . (See Figure 1.)

$U$  is said to be  $\mathcal{H}_b$ -holomorphically convex if, for every  $U$ -bounded set  $X$ , the hull of  $X$  with respect to  $\mathcal{H}_b(U; \mathbf{C})$ ,  $\hat{X}_{\mathcal{H}_b(U; \mathbf{C})}$ , is also  $U$ -bounded; this hull is defined as the set of all  $t \in U$  such that the maximum modulus style estimate

$$|f(t)| \leq \sup_{x \in X} |f(x)|$$

holds for every  $f \in \mathcal{H}_b(U; \mathbf{C})$ .

A subset  $X$  of  $U$  is said to be  $\mathcal{H}_b(U; \mathbf{C})$ -bounding if every  $f \in \mathcal{H}_b(U; \mathbf{C})$  is bounded on  $X$ . This is analogous to a previous definition we gave with  $\mathcal{H}_b(U; \mathbf{C})$  now in place of  $\mathcal{H}(U; \mathbf{C})$ .

We then have the following theorem of the Cartan-Thullen style, which was noticed independently by Dineen and Matos.

**THEOREM 2.** *Let  $U$  be connected. The following conditions are equivalent:*

(1)  $U$  is an  $\mathcal{H}_b$ -domain of holomorphy;

(2)  $U$  is  $\mathcal{H}_b$ -holomorphically convex;

(3) a subset  $X$  of  $U$  is  $U$ -bounded if and only if  $X$  is  $\mathcal{H}_b(U; \mathbf{C})$ -bounding.

Moreover, if  $E$  is separable, the above are also equivalent to

(4)  $U$  is an  $\mathcal{H}_b$ -domain of existence;

(5) the complement in  $\mathcal{H}_b(U; \mathbf{C})$  of the set of all  $f \in \mathcal{H}_b(U; \mathbf{C})$  having  $U$  as its natural  $\mathcal{H}_b$ -domain of existence is meager, that is, of Baire first category.

Hirschowitz has found an example showing that the hypothesis of separability for  $E$  cannot be dropped in the above theorem. In his example,  $U$  is an open ball in the Banach space  $E = \mathcal{C}(X; \mathbf{C})$  of all continuous complex-valued functions on the compact space  $X = [0, \Omega]$  of the cardinal

numbers between 0 and  $\Omega$ , the first uncountable cardinal.

The full analogue of the Cartan-Thullen theorem with the space  $\mathcal{H}(U; C)$  in place of  $\mathcal{H}_b(U; C)$  is still an open question which is being investigated by various mathematicians.

This has to do with characterization of domains of holomorphy, domains of existence, holomorphic convexity, pseudoconvex domains, holomorphic continuation and envelopes of holomorphy as Riemann domains or manifolds spread over Banach or locally convex spaces, plurisubharmonic functions, Runge sets, etc., in infinite dimensions.

Several concepts and conditions which coincide in the finite dimensional case fail to do so in infinite dimensions.

There are plenty of closely related candidates for the wanted results.

We have to distinguish carefully between the collection of all bounded subsets, and the collection of all open covers, as well as between holomorphic functions and holomorphic mappings.

A description of these developments goes beyond the limits of this one-hour lecture.

Recent work on this type, and other aspects, of infinite dimensional complex analysis has been carried on mostly by Alexander, Bremermann, Coeuré, Dineen, Earle, Greenfield, Harris, Hirschowitz, Lelong, Matos, Noverraz, Rickart, Schottenloher and Wallach.

**4. Topological properties.** The question arises of what topology to use on  $\mathcal{H}(U; F)$ .

In the finite dimensional situation, there is the natural candidate, the compact-open topology  $\mathcal{T}_0$  of order zero, which coincides with the compact-open topology  $\mathcal{T}_\infty$  of infinite order.

In the infinite dimensional situation, there are still the compact-open topology  $\mathcal{T}_0$  of order zero defined by the family of seminorms

$$f \in \mathcal{H}(U; F) \mapsto \sup_{x \in K} \|f(x)\| \in \mathbf{R}_+,$$

where  $K \subset U$  is compact, as well as the compact-open topology  $\mathcal{T}_\infty$  of infinite order, defined by the family of seminorms

$$f \in \mathcal{H}(U; F) \mapsto \sup_{x \in K} \|d^m f(x)\| \in \mathbf{R}_+,$$

where  $K \subset U$  is compact and  $m = 0, 1, \dots$ . These two topologies may be different and in several respects they lack the right properties which they have in the finite dimensional case. This calls for further investigation.

A seminorm  $p$  on  $\mathcal{H}(U; F)$  is said to be *ported* by a compact subset  $K$  of  $U$  if, for every open subset  $V$ , where  $K \subset V \subset U$ , there is a real number  $c(V) > 0$  such that we have the estimate

$$p(f) \leq c(V) \sup_{x \in V} \|f(x)\|$$

for all  $f \in \mathcal{H}(U; F)$ . The simple intuitive idea behind this definition is that, if a variable mapping  $f \in \mathcal{H}(U; F)$  tends to 0 uniformly on some neighborhood of  $K$ , then  $p(f)$  should tend to 0.

The topology  $\mathcal{T}_\omega$  on  $\mathcal{H}(U; F)$  is defined by all seminorms on  $\mathcal{H}(U; F)$  each of which is ported by some compact subset of  $U$ .

We have  $\mathcal{T}_0 = \mathcal{T}_\infty = \mathcal{T}_\omega$  if  $\dim E < \infty$  or in the trivial case  $F = 0$ , whereas  $\mathcal{T}_0 < \mathcal{T}_\infty < \mathcal{T}_\omega$  if  $\dim E = \infty$  and  $F \neq 0$ .

The three topologies  $\mathcal{T}_0$ ,  $\mathcal{T}_\infty$  and  $\mathcal{T}_\omega$  define the same notion of bounded subset of  $\mathcal{H}(U; F)$ .

The following nice and basic result is due to Dineen.

**THEOREM 3.** *For “many” separable Banach spaces  $E$ , if  $U$  is balanced with respect to some point, then  $\mathcal{T}_\omega$  is the largest locally convex topology on  $\mathcal{H}(U; C)$  having the same collection of bounded subsets as  $\mathcal{T}_0$ . Hence  $\mathcal{T}_\omega$  is bornological.*

In the above theorem, it is assumed that the space  $E$  has a basis  $(u_n)$ ,  $n = 0, 1, \dots$ , such that, for every

$$\alpha = (\alpha_0, \dots, \alpha_n, \dots) \in c_0,$$

the Banach space of all sequences of complex numbers tending to zero, and for every

$$x = \sum_{n=0}^{+\infty} \beta_n u_n \in E,$$

it results that

$$\alpha \cdot x = \sum_{n=0}^{+\infty} \alpha_n \beta_n u_n$$

converges in  $E$  and the bilinear mapping

$$(\alpha, x) \in c_0 \times E \mapsto \alpha \cdot x \in E$$

is continuous.

It is conjectured that the preceding theorem holds for every separable Banach space  $E$  and every  $U$ . Dineen showed that this theorem breaks down for  $E = l^\infty$ .

On the other hand, Coeuré subsequently found another instance in which the conclusion of the above theorem is true.

If  $K \subset E$  is compact, we have the vector space  $\mathcal{H}(K; F)$  of all germs of holomorphic mappings around  $K$  into  $F$ .

Consider the big set

$$\bigcup_{U \supset K} \mathcal{H}(U; F)$$

where the union is taken over all  $U$  containing  $K$ . An equivalence relation on this big set is defined by saying that  $f$  and  $g$  in that big set are *equivalent modulo  $K$*  when  $f$  and  $g$  coincide in a neighborhood of  $K$ . The quotient of that big set by such an equivalence relation is the set  $\mathcal{H}(K; F)$ .

We have the natural mapping  $\mathcal{H}(U; F) \rightarrow \mathcal{H}(K; F)$  if  $U \supset K$ , coming from the quotient mapping. There is one and only one way of making  $\mathcal{H}(K; F)$  into a vector space so that this natural mapping is linear for every  $U \supset K$ .

The natural topology  $\mathcal{T}_\omega$  on  $\mathcal{H}(K; F)$  is obtained by considering  $\mathcal{H}(K; F)$  as a locally convex inductive (or direct) limit.

$$\mathcal{H}(K; F) = \lim_{\overrightarrow{U \supset K}} \mathcal{H}_b(U; F) = \lim_{\overrightarrow{U \supset K}} \mathcal{H}(U; F)$$

where  $\mathcal{H}_b(U; F)$  has its topology  $\mathcal{T}_b$  and  $\mathcal{H}(U; F)$  has its topology  $\mathcal{T}_\omega$ . Both inductive limits give the same topology  $\mathcal{T}_\omega$  on  $\mathcal{H}(K; F)$ .

The question arises of finding out whether, conversely, we have the topological projective (or inverse) limit relation

$$\mathcal{H}(U; F) = \varprojlim_{K \subset U} \mathcal{H}(K; F),$$

where  $\mathcal{H}(U; F)$  is endowed with its topology  $\mathcal{T}_\omega$  and  $\mathcal{H}(K; F)$  is endowed with the topology  $\mathcal{T}_\omega$ .

To answer this question, we define the compact subset  $K$  to be  $U$ -Runge if  $K \subset U$  and the image of  $\mathcal{H}(U; C)$  in  $\mathcal{H}(K; C)$  by the natural mapping  $\mathcal{H}(U; C) \rightarrow \mathcal{H}(K; C)$  is  $\mathcal{T}_\omega$ -dense in  $\mathcal{H}(K; C)$ .

Then, if every compact subset of  $U$  is contained in some  $U$ -Runge compact subset of  $U$ , the preceding topological projective limit relation holds, as shown by Chae.

Actually, it is conjectured that every compact subset of  $U$  is contained in some  $U$ -Runge compact subset; to my knowledge, however, this statement remains unsettled even in the finite dimensional case. This conjecture may be phrased in more general partial differential operator terms.

It is curious to realize that the topological study of  $\mathcal{H}(K; C)$  was considered as being more sophisticated than that of  $\mathcal{H}(U; C)$  for a finite dimensional space  $E$ . It was only after a classical article by Dieudonné and Schwartz on inductive limits that  $\mathcal{H}(K; C)$  could be treated as a locally convex space. However, when we jump to an infinite dimensional space  $E$ , it is the topological study of  $\mathcal{H}(U; C)$  which becomes more sophisticated than that of  $\mathcal{H}(K; C)$ .

A lot of work on the topological aspects of spaces of holomorphic mappings in infinite dimensions has been done by Alexander, Aron, Barroso, Chae, Coeuré, Dineen, Hirschowitz, Matos and Pisanelli.

### 5. Convolution operators.

We now turn to convolution operators.  
Let  $\mathcal{P}_N(^m E; \mathbf{C})$  be the Banach space of all *nuclear*, complex-valued,  $m$ -homogeneous polynomials on  $E$ , as defined by Grothendieck.

An entire function  $f \in \mathcal{H}(E; \mathbf{C})$  is said to be *nuclearly entire of bounded type* if

$$\hat{d}^m f(0) \in \mathcal{P}_N(^m E; \mathbf{C})$$

for  $m = 0, 1, \dots$ , and

$$\left( \left\| \frac{1}{m!} \hat{d}^m f(0) \right\|_N \right)^{1/m} \rightarrow 0$$

as  $m \rightarrow +\infty$ , where the index  $N$  denotes the nuclear norm on the space  $\mathcal{P}_N(^m E; \mathbf{C})$ .

We represent by  $\mathcal{H}_{Nb}(E; \mathbf{C})$  the vector subspace of  $\mathcal{H}(E; \mathbf{C})$  of all such  $f$ .

We shall use on  $\mathcal{H}_{Nb}(E; \mathbf{C})$  the topology  $\mathcal{T}_{Nb}$  defined by the family of seminorms

$$f \mapsto \sum_{m=0}^{+\infty} r^m \cdot \left\| \frac{1}{m!} \hat{d}^m f(0) \right\|_N$$

for every real number  $r \geq 0$ . This topology  $\mathcal{T}_{Nb}$  is locally convex, metrizable and complete; that is  $\mathcal{H}_{Nb}(E; \mathbf{C})$  is a Fréchet space.

A convolution operator  $\mathcal{O}$  on  $\mathcal{H}_{Nb}(E; \mathbf{C})$  is a continuous linear mapping

$$\mathcal{O}: \mathcal{H}_{Nb}(E; \mathbf{C}) \rightarrow \mathcal{H}_{Nb}(E; \mathbf{C})$$

which commutes with translations by elements of  $E$ . Such a convolution operator in this case is actually a constant coefficient, linear, differential operator of finite or infinite order.

A nuclear exponential-polynomial is an element of  $\mathcal{H}_{Nb}(E; \mathbf{C})$  of the form  $p \exp(\varphi)$ , where  $p$  is a nuclear complex-valued polynomial on  $E$  and  $\varphi \in E'$  is a continuous linear form on  $E$ .

The following approximation and existence theorem of the Malgrange type was proved by Gupta.

**THEOREM 4.** *If  $\mathcal{O}$  is a convolution operator on  $\mathcal{H}_{Nb}(E; \mathbf{C})$ , then the vector subspace  $\mathcal{O}^{-1}(0)$  where  $\mathcal{O}$  vanishes is the closure of its vector subspace formed by the finite sums of the nuclear exponential-polynomials belonging to  $\mathcal{O}^{-1}(0)$ . Moreover*

$$\mathcal{O}[\mathcal{H}_{Nb}(E; \mathbf{C})] = \mathcal{H}_{Nb}(E; \mathbf{C})$$

provided  $\mathcal{O} \neq 0$ .

An extension of this result to the space of nuclearly entire functions  $\mathcal{H}_N(E; \mathbf{C})$  not necessarily of bounded type has been made.

The bounded nuclear case has been extended by Matos to a locally convex space  $E$  whose strong dual space  $E'$  is metrizable.

Dwyer has done work along this line when  $E$  is a Hilbert space and we consider entire functions of the Hilbert-Schmidt type.

Boland has been investigating this kind of a result by using weights on the Banach space  $E$ .

The cases of the current type  $\mathcal{H}_b(E; \mathbf{C})$  and  $\mathcal{H}(E; \mathbf{C})$  remain untouched and at present seem hard.

It was the success of these studies in the nuclear type cases  $\mathcal{H}_{Nb}(E; \mathbf{C})$  and  $\mathcal{H}_N(E; \mathbf{C})$ , and the existence of the current type, that prompted the introduction of the concept of holomorphy type from  $E$  to  $F$ , such as the current type, the nuclear type, the Hilbert-Schmidt type, etc.

We notice that theorems of the Cartan-Thullen style, or of the Malgrange style, in infinite dimensions are harder for the current type, but become more accessible for the bounded case, or the bounded nuclear case.

**6. Uniform holomorphy.** Let us now give up the restriction that  $E$  and  $F$  are Banach spaces and assume that they are locally convex spaces.

I shall state next a result which is typical of this more general situation and does not have significance in the normed case.

To start with, we set the following simple notation. If  $\alpha$  is a seminorm on  $E$ , I denote by  $E_\alpha$  the vector space  $E$  seminormed by  $\alpha$ , and represent by  $E/\alpha$  the quotient normed space  $E_\alpha/\alpha^{-1}(0)$  of  $E_\alpha$  by the vector subspace  $\alpha^{-1}(0)$  where  $\alpha$  vanishes. The same is done for the space  $F$ .

First I shall assume that the space  $F$  is seminormed. Then I say that the mapping  $f \in \mathcal{H}(U; F)$  is *uniformly holomorphic* if there is a cover  $\mathcal{C}$  of  $U$  by nonvoid open subsets of  $U$  and if there is a continuous seminorm  $\alpha$  on  $E$  such that, for every  $W \in \mathcal{C}$ , there is  $V \subset E$  which is open with respect to  $\alpha$ , where  $W \subset V$ , and the restriction of  $f$  to  $W$  has an extension  $g$  to  $V$  which is holomorphic with respect to  $\alpha$  on  $V$  with values in  $F$ . (See Figure 1.)

More generally, when the space  $F$  is allowed to be locally convex, I say that the mapping  $f \in \mathcal{H}(U; F)$  is *uniformly holomorphic* if the mapping  $f \in \mathcal{H}(U; F_\beta)$  is uniformly holomorphic in the above sense for every continuous seminorm  $\beta$  on  $F$ .

**THEOREM 5.** *Assume that the space  $E$  satisfies the following condition: The set of seminorms  $\alpha$  on  $E$ , rendering the quotient mapping  $E \rightarrow E/\alpha$  continuous and open, is directed and defines the topology  $E$ . Then every*

mapping  $f \in \mathcal{H}(U; F)$  is uniformly holomorphic.

When  $F$  is seminormed, this result can be reformulated as meaning that every mapping  $f \in \mathcal{H}(U; F)$  is globally holomorphic with respect to some continuous seminorm on  $E$ ; but then we must use Riemann domains as manifolds spread over normed spaces, in order to avoid *multivalent* holomorphic mappings.

The preceding theorem is actually a generalization of the following classical remark: if  $I$  is an infinite set, then every holomorphic function  $f \in \mathcal{H}(\mathbb{C}^I; \mathbb{C})$  of infinitely many variables actually depends only on finitely many such variables.

In the case of a locally convex space  $E$ , use of uniform holomorphy has been made by Dineen and Noverraz to discuss mutual implications of various forms of conditions for  $U$  to be a domain of holomorphy, or a domain of existence, or holomorphically convex, or plurisubharmonically convex, or pseudoconvex, or polynomially convex.

**7. Concluding remarks.** Well, I hope to have succeeded in giving all of you an impression of a variety of directions of the current efforts to push ahead investigations on infinite dimensional holomorphy.

There are also other tendencies that I was not able to touch upon, such as the study of infinite dimensional analytic geometry carried on by Douady, Ramis and Ruget; or of infinite dimensional analytic manifolds; or applications of complex analysis in infinite dimensions to physics; etc.

In all these areas, the problems outnumber the solutions.

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