BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 77, Number 6, November 1971

RESEARCH ANNOUNCEMENTS

The purpose of this department is to provide early announcement of significant new results, with some indications of proof. Although ordinarily a research announcement should be a brief summary of a paper to be published in full elsewhere, papers giving complete proofs of results of exceptional interest are also solicited. Manuscripts more than eight typewritten double spaced pages long will not be considered as acceptable. All research announcements are communicated by members of the Council of the American Mathematical Society. An author should send his paper directly to a Council member for consideration as a research announcement. A list of the members of the Council for 1971 is given at the end of this issue.

CAUSALLY ORIENTED MANIFOLDS AND GROUPS

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Communicated by I. M. Singer, June 7, 1971

A C^∞ manifold is said to be *causally oriented* if there is given in the tangent plane at each point p a nontrivial convex cone defined locally by C^∞ inequalities. A *time-like arc* is an oriented C^∞ curve whose forward tangent at each point lies in $C(p)$; the manifold is *strongly causal* if no nontrivial time-like arc is closed. There is then a partial ordering $x<y$ on M, defined by the existence of a nontrivial time-like arc with initial point x and terminal point y. If neither $x<y$ nor $y<x$, x and y are *incommunicable*; a *space-like submanifold* is a submanifold, any two of whose points are incommunicable. These notions are in part abstractions of some of those treated in [1].

A temporal displacement T is an automorphism of (M, C) such that either $x<Tx$ for all $x\in M$ ("forward displacement"), or $Tx<x$ for all $x\in M$, or $Tx=x$ for all $x\in M$. A causally oriented manifold (M, C) is said to be *homogeneous* if there exists a maximal space-like surface S, on which the subgroup of automorphisms leaving S fixed as a set is transitive, both on the points of S and on the directions at each point and a smooth one-parameter group T_t of temporal displacements such that $M=\bigcup_{t\in R^1} T_t(S)$.

THEOREM. *The finite coverings of the conformal compactification* [2] $\overline{M}$ *of n-dimensional Minkowski space-time M admit causal orientations compatible with that in Minkowski space, but are not strongly causal.*

AMS 1970 *subject classifications*. Primary 83C40, 35L99.

[1] Research supported in part by the NSF.

However, the universal covering space $\tilde{M}$ of $\overline{M}$ is strongly causal, and homogeneous with $S=S^{n-1}$.

The conformal automorphism group Γ ($\cong O(n, 2)$) of $\overline{M}$ is transitive, as a result of which $\overline{M}$, and hence also any covering space of $\overline{M}$, is conformally locally identical to M. Since Maxwell's and similar equations (wave, Dirac with zero mass, etc.) are uniquely determined by the conformal structure, it follows from [1] that

COROLLARY 1. *On the universal covering space of the conformal compactification of Minkowski space-time, Maxwell's equations (etc.) admit global retarded and advanced elementary solutions.*

Colloquially speaking, this means that these equations are "quantizable" on $\tilde{M}$, but not on finite coverings of $\overline{M}$.

A topological group G is said to be *causally oriented* if there is given in it a nontrivial subset C with the properties: $C^2 \subset C$, $a^{-1}Ca \subset C$ if a is in the component of e in G, and $C \cap C^{-1} = \{e\}$. In general, an open simple Lie group admits no causal orientation, but

COROLLARY 2. *The universal covering group of the conformal group* Γ *is causally oriented by the designation of C as the set of all forward displacements, together with the identity, in its action as a group of conformal automorphisms of M.*

REMARK 1. There are no presently known strongly causal homogeneous 4-manifolds other than the two involved in classical and relativistic mechanics, and $\tilde{M}$ (in the case $n=4$).

REMARK 2. The infinitesimal generator of the temporal development group involved in the homogeneity of $\tilde{M}$ has been studied in another connection in [3], and shown to have a discrete spectrum in certain unitary representations of Γ. It follows that it is not conjugate to the standard relativistic generator; it is however generic in a sense in which the standard generator appears as a singular special case.

REFERENCES

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3. I. Segal, *Positive-energy particle models with mass splitting*, Proc. Nat. Acad. Sci. U.S.A. **57** (1967), 194–197. MR **34** #5449.

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