A CHARACTERIZATION OF CONWAY'S GROUP .3[1]

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Statement of result. Let G_0 be the group .3 discovered by J. H. Conway [1], and let s_0 be an involution in the center of a Sylow 2-subgroup of G_0. A direct examination of G_0 shows $(C_{G_0}(s_0)/\langle s_0\rangle \simeq \mathrm{Sp}(6, 2)$, a nonsplit extension.

THEOREM. *Let G be a finite group and s an involution in G, such that $C_G(s)\simeq C_{G_0}(s_0)$. Assume $G\neq C_G(s)O(G)$ ($O(G)$ is the maximal normal odd order subgroup of G). Then $G\simeq G_0$. In particular, G has the following properties*:

(i) *G has order* $2^{10}\cdot 3^7\cdot 5^3\cdot 7\cdot 11\cdot 23$, *and is simple.*

(ii) *G has two conjugacy classes of involutions. One class is represented by the involution s. A representative t of the second class has centralizer $C_G(t)\simeq\langle t\rangle\times M_{12}$ (M_{12} is the Mathieu group).*

(iii) *The normalizer of a Sylow* 23-*subgroup is a Frobenius group of order* $11\cdot 23$.

(iv) *The normalizer of a Sylow* 11-*subgroup is a direct product of Z_2 (the group of order* 2) *and a Frobenius group of order* $5\cdot 11$.

(v) *The normalizer of a Sylow* 7-*subgroup is a direct product of* Sym_3 (*the symmetric group*) *and a Frobenius group of order* $6\cdot 7$ *with kernel of order* 7.

(vi) *A Sylow* 5-*subgroup is nonabelian of exponent* 5. *There are two classes of elements of order* 5, *with centralizers of orders* $2^2\cdot 3\cdot 5^3$ *and* $2^2\cdot 3\cdot 5^2$. *The normalizer of a Sylow* 5-*subgroup has order* $2^4\cdot 3\cdot 5^3$.

(vii) *G has no outer automorphisms.*

The character table of G is obtained in the course of the proof, and will appear elsewhere with details of the proof.

Outline of proof. Property (ii) is proved using group-theoretic methods and Wong's characterization of M_{12} [2]. The order follows using a formula of Thompson's requiring only knowledge of the centralizers of involutions and conjugacy of involutions. Properties (iii)–(vi) are then straightforward. This provides sufficient informa-

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tion to determine the character table, with the aid of a computer. The most important techniques used are these:

(i) The p-block structure for $p=23$, 11, and 7.

(ii) Equations involving the values of characters on involutions, expressing the nonrealness of certain classes.

(iii) Orthogonality relations.

In particular, the group is seen to have a rational, absolutely irreducible character of degree 23. We then use the following result due to Feit: if a group H has a rational, absolutely irreducible, faithful representation of degree 23, and no subgroup of index 23 or 24, then H acts on one of three 23-dimensional lattices [3]. These lattices can be found as sublattices of the Leech lattice, and are the orthogonal complements, respectively, of vectors of types 2, 3, and $6_{3,2}$ (see [1]). The automorphism groups of these lattices are determined to be, respectively, $.2\times Z_2$, $.3\times Z_2$, and $M_{24}\times Z_2$. The orders of these groups now provides the conclusion $G\simeq G_0$. Property (vii) follows from the corresponding fact for Sp(6, 2) and the characterization of G_0.

Some remarks on the characters of G_0. G_0 has 42 irreducible characters. The degrees are as follows: 1, 23, 253, 253, 275, 896, 896, 1771, 2024, 3520, 3520, 4025, 5544, 7084, 8855, 9625, 9625, 20608, 20608, 23000, 26082, 31625, 31625, 31625, 31878, 40250, 57960, 63250, 73600, 80960, 91125, 93312, 129536, 129536, 177100, 184437, 221375, 226688, 246400, 249480, 253000, 255024. The characters of degrees 896, 3520, 9625, and 20608 occur in complex-conjugate pairs; the remaining 34 characters are rational.

For $p=23$, 11, and 7, the blocks are as described by Brauer [4]. For $p=5$, there is the principal block, one block of defect 1, and blocks of defect 0. For $p=3$, there is the principal block and one block of defect 1; there are no 3-blocks of defect 0. For $p=2$, there is the principal block, one block of defect 1, and one block of defect 3 (which has 8 characters, all of height 0, and an elementary abelian defect group); there are no 2-blocks of defect 0.

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