*Complex function theory*, by Maurice Heins, New York, Academic Press, 1968.

In this book Maurice Heins has written what he feels should be included in a first course in complex function theory and also supplementary material. The result is a fairly standard treatment with the infusion of a rich variety of examples and special problems which reflect the author's long and fruitful experience in the field. The text is difficult with important material being included in the exercises; moreover, there are comparatively few routine exercises. Consequently it is hard to see how the book could be used at the undergraduate level except in an honors course. An ordinary graduate student without an undergraduate background in complex variables will also find the book difficult. For the qualified student, however, the book is admirably suited for a stern graduate course.

The book is divided into two parts, the first part according to the author being "a reasonable and, in fact, mathematically appealing program that can serve as a basis for the requirements in complex function theory for all doctoral candidates in mathematics at present." The author makes the book as self-contained as possible relying on the student's undergraduate experience for background rather than for foundations. Consequently, the first three chapters (I. The real field, II. The complex field. Limits, III. Topological and metric spaces) form a rigorous treatment of the foundations. Chapter IV (Complex differential analysis) introduces complex functions. The next four chapters (V. Cauchy theory, VI. Laurent expansions. Meromorphic functions, VII. Further applications of the Cauchy theory, VIII. The zeros and poles of meromorphic functions) develop the core of the subject. If one looks beyond the difficulty of the exercises in these chapters, one sees that Heins is constantly challenging the reader with imaginative problems whereby important and interesting theorems are proved. The good student (and many instructors) should find a course based on this book very stimulating. Chapter IX (The gamma and zeta function. Prime number theorem) does for Part I what the exercises do for the individual chapters; that is, the previous theory is applied to an interesting theorem.

The second part is a collection of topics designed to use and extend the experience gained in the first part. Here the treatment of topics is uneven. Some chapters are very short while others are fairly detailed so an instructor contemplating using the text should see how well his favorite advanced topic is covered. One can argue with the author as to what is essential and what is supplementary in complex function theory for a doctoral candidate. That is, one might say that some topics (analytic continuation, the Riemann mapping theorem, Riemann surfaces) might better have been included in Part I while some topics in Part I (Isomorphism theorem of Bers, Runge's theorem, the prime number theorem) might better have been in Part II. But this is hardly an important criticism since one can always adapt the text to one's needs after the first eight chapters are covered.

As is usual when a specialist in functions of one complex variable writes a text on complex functions, the subject is developed as a practicing analyst sees it. Since most instructors using the book will have similar interests, they will find Heins' treatment satisfactory. But it seems that certain, but not all, algebraic aspects of the subject are passed by. For instance, no homology theory is introduced. Perhaps more important, elliptic functions are treated rather lightly. But most important, the subject of Riemann surfaces is treated without once mentioning an algebraic curve. Of course, Heins is not alone in this last omission. Moreover, his treatment of Riemann surfaces is more complete than is usual in elementary texts. For closed surfaces he includes an existence theory based on the Perron method and closes with the theorem that birationally equivalent function fields yield conformally equivalent Riemann surfaces. Still, the fact that one can find in many elementary texts treatments of polynomials in two variables where a plane curve is never mentioned testifies to a certain inbreeding among such treatments. And this uniformity of treatment has consequences. The reviewer has talked with good mathematicians who were surprised to discover that a knowledge of plane curves leads to fruitful insights into the nature of closed Riemann surfaces.[1] One can well ask which is more important for a doctoral candidate: that he knows when two of those odd constructs (Riemann surfaces) obtained by analytic continuation are the same (conformally equivalent) or that the whole subject has something to do with curves he studied in analytic geometry and elementary calculus.

The discussion of the preceding paragraph perhaps makes it pertinent to ask just what is the subject matter of complex function theory. If one feels that it is a certain collection of methods to be used on functions meromorphic on Riemann surfaces, then one concentrates on the methods and the problems that can be solved by these methods. According to Heins: "The student should ultimately

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[1] Not so long ago things seem to have been the other way around. In the introduction to his treatise on algebraic curves, Coolidge hopefully asserts "It is assumed that the reader will not have heart failure at the mention of a Riemann surface." (J. L. Coolidge, *A treatise on algebraic plane curves*, Dover, 1959, p. x.)

recognize that four principal methods dominate complex function theory, methods closely associated with the founders of the subject: the powers series approach, the complex integral approach embracing the Cauchy theory in its full range, the approach based on the connection with the theory of harmonic functions, and the mapping approach."

But one might say that the subject matter is a collection of functions, including most of those which the student has previously met, and any method which throws light on the nature of these functions is appropriate. From this point of view plane algebraic curves have as much place in a course in complex functions as do canonical products. If the study of abelian integrals is fit for a first year calculus student, why should one next encounter them after having mastered the (Grothendieck?) Riemann-Roch theorem?

On a slightly less heretical note, let the reviewer now mention one of his pet peeves. Why is it that everybody, well almost everybody, insists on giving students only the Goursat proof of the Cauchy integral theorem? Is it that if the secret is let out of the bag an average student of advanced calculus might understand it? Even without going into the theory of double integrals, one can at least mention that the theorem is an easy (though hardly trivial) consequence of Green's theorem and the Cauchy-Riemann equations. But such a concession to previous courses in advanced calculus seldom occurs.

The point is that the subject of functions of one complex variable (not to mention several variables) can meaningfully employ some aspect of just about any mathematical technique around. This should not be surprising since many modern "branches" of mathematics have germinal ideas in some aspect of the study of complex functions. It would be impossible, of course, to incorporate in a text or course all the different ideas inherent in complex function theory. Certainly one cannot fault Heins for not writing a text that would be alien to him. However, it would be stimulating to see a text written by, say, an algebraic geometer. Hopefully, such a book would be nonstandard and would help us all to see this venerable subject from a different point of view.

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