ON THE SEMISIMPLICITY OF INTEGRAL REPRESENTATION RINGS

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For a finite group G and a ring R, define the integral representation ring $a(RG)$ as the abelian group generated by the isomorphism classes of RG-lattices, with

$$[M] + [M'] = [M \oplus M'],$$

and

$$[M][M'] = [M \oplus_R M'].$$

The integral representation algebra $A(RG)$ is $C \otimes_Z a(RG)$. When does $a(RG)$ contain nontrivial nilpotent elements?

Let $|G| = p^\alpha n$, where $p \nmid n$, p prime. Denote by Z_p the p-adic valuation ring in Q, and by Z_p^* its completion. Reiner has shown

(i) If $\alpha = 1$, then $A(Z_pG)$ and $A(Z_p^*G)$ have no nonzero nilpotent elements (see [1]).

(ii) If $\alpha \geqq 2$, and G has an element of order p^2, then both $A(Z_pG)$ and $A(Z_p^*G)$ contain nonzero nilpotent elements (see [2]).

We have been able to settle the open case as to what happens when G has a (p, p)-subgroup. Our main result is

THEOREM 1. *Whenever $\alpha > 1$, both $A(Z_pG)$ and $A(Z_p^*G)$ contain nonzero nilpotent elements.*

As a matter of fact, the construction used shows

THEOREM 2. *If $|G|$ is not squarefree, then $a(ZG)$ and $a(Z'G)$ contain nonzero nilpotent elements, where*

$$Z' = \{a/b : a, b \in Z,\ b \text{ coprime to } |G|\}.$$

In the other direction, Reiner proved

(iii) If $|G|$ is squarefree, then $a(Z'G)$ has no nonzero nilpotent elements (see [1]).

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All of our results generalize to the case where Z is replaced by the ring of algebraic integers in an algebraic number field.

As a consequence of Theorem 1, we have Theorem 3. Let k be a field of characteristic p, p an odd prime. If G has a noncyclic p-Sylow subgroup, $a(kG)$ contains nonzero nilpotent elements.

References

1. I. Reiner, *Integral representation algebras*, Trans. Amer. Math. Soc. **124** (1966), 111–121. MR **34** # 2722.

2. ———, *Nilpotent elements in rings of integral representations*, Proc. Amer. Math. Soc. **17** (1966), 270–274. MR **32** #5745.

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