PRODUCTS OF FULLY COMPLETE SPACES

BY W. H. SUMMERS

The permanence properties of fully complete spaces were con-
sidered first by Collins, and in his paper [1] which appeared in 1955
it is shown that an arbitrary product of fully complete spaces need
not be fully complete. In 1958, Kelley [5] introduced the notion of
hypercomplete spaces (a priori, fully complete), and conjectured
that countable products of hypercomplete spaces would be hyper-
complete. The purpose of this paper is to resolve this conjecture.

LEMMA 1. *If X is a Fréchet space and if X_τ^* denotes the topological
dual of X endowed with the Mackey topology $\tau(X^*, X)$, then the direct
sum E of denumerably many copies of X_τ^* has the Krein-Smulian prop-
erty (for a definition, see [2, p. 364]), and hence is both hypercomplete
and fully complete.*

Since the strict topology β on l^∞ is $\tau(l^\infty, l^1)$ from [3], we have the
following result.

LEMMA 2. *The direct sum of denumerably many copies of l_β^∞ has the
Krein-Smulian property.*

THEOREM 1. *If E is the direct sum of denumerably many copies of
l_β^∞ and if F is the product of denumerably many copies of c_0 (c_0 is, of
course, endowed with the uniform topology), then both E and F have the
Krein-Smulian property, but $E \times F$ is not fully complete.*

That $E \times F$ is not fully complete was suggested by an example due
to Grothendieck [4, p. 92], and it should be noted that, since a
Fréchet space has the Krein-Smulian property, $E \times F$ can be con-
sidered as a product of spaces with the Krein-Smulian property of
any finite or denumerable order.

We conclude by noting that the question of whether or not B_r-
completeness is preserved by products is still open.

REFERENCES

1. H. S. Collins, *Completeness and compactness in linear topological spaces*, Trans.
Amer. Math. Soc. **79** (1955), 256–280.

2. ———, *On the space $l^\infty(S)$ with the strict topology*, Math. Z. **106** (1968), 361–373.

3. J. B. Conway, *The strict topology and compactness in the space of measures*, Bull.
Amer. Math. Soc. **72** (1966), 75–78.

4. A. Grothendieck, *Sur les espaces (F) et (DF)*, Summa Brasil. Math. **3** (1954),
57–122.

5. J. L. Kelley, *Hypercomplete linear topological spaces*, Michigan Math. J. **5**
(1958), 235–246.

UNIVERSITY OF ARKANSAS, FAYETTEVILLE, ARKANSAS 72701