The chaper headings, which are self-explanatory, are: (1) Homotopy and the fundamental group; (2) Covering spaces and fibrations; (3) Polyhedra; (4) Homology; (5) Products; (6) General cohomology theory and duality; (7) Homotopy theory; (8) Obstruction theory; and (9) Spectral sequences and homotopy groups of spheres.

As a reviewer, I surely do not wish to suggest that editors be allowed to change or reject invited reviews. However, the review of this book which appeared in Mathematical Reviews was so unfair that the editors of that journal should have published a second review. A reform which might help would be for editors not to invite reviews from persons who have written a book which competes or pretends to compete with the book under review.

F. P. PETERSON

*Value distribution theory* by Leo Sario and Kiyoshi Noshiro. Van Nostrand, Princeton, N. J., 1966. xi+256 pp. $7.50.

This book is directed to the study of regular mappings from a Riemann surface $R$ into a Riemann surface $S$, particularly to the distribution of values of such mappings. Naturally, except for the simplest cases, $R$ will be open, $S$ may be closed or open. A priori, distribution of values might be construed in many ways but in analogy to the classical Nevanlinna pattern, where $R$ is the plane, $S$ the Riemann sphere, it has here the following context: $R$ is to be exhausted by a family of finite Riemann surfaces with smooth boundaries, on $S$ there is assigned a function to measure proximity of pairs of points and a related measure of area; the primary result is that for each value on $S$ the sum of terms representing the frequency with which this value is taken in an exhausting surface and the proximity of the values on its boundary to this value is equal to the integrated value of areas covered by the Riemann covering image over $S$. This is followed by a main theorem which represents an analogue of Nevanlinna's Second Fundamental Theorem.

The first two chapters are devoted to the explicit implementation of this program in the respective cases where $S$ is closed or open. The exhaustion of $R$ is obtained by starting with a parametric disc $R_0$ with boundary $\beta_0$, taking a finite Riemann subsurface $\Omega$ of $R$ with smooth boundary $\beta_0 \cup \beta_\Omega$ not containing $R_0$ and taking the harmonic function $u$ on $\Omega$ with boundary values 0 on $\beta_0$, $k(\Omega)$ on $\beta_\Omega$ such that $\int_{\beta_0} du^* = 1$. Further $\beta_h$ denotes the level line $u = h$, $\Omega_h$ the subset of $\Omega$ on which $0 < u < h(\leq k(\Omega))$ and $R_h = \overline{R}_0 \cup \Omega_h$. The construction of the proximity function on $S$ begins with the choice of points $\zeta_0$, $\zeta_1$ there together with corresponding fixed local uniformizing parameters.

Then $t_0$ is to be a harmonic function in $S-\{\zeta_0, \zeta_1\}$ with singularities $-2 \log |\zeta-\zeta_0|$, $2 \log |\zeta-\zeta_1|$ in the appropriate parameter neighborhoods. If $S$ is open there must be imposed here and in analogous situations a normalization at the ideal boundary which is implemented by a discussion on principal functions. If $S$ is closed $t_0$ is determined up to a constant fixed by a normalization at $\zeta_0$. Next is defined $s_0 = \log (1+e^{t_0})$. For $S$ closed the proximity function is defined as

$$s(\zeta, a) = s_0(\zeta) + t(\zeta, a)$$

the harmonic function $t(\zeta, a)$ with singularities $-2 \log |\zeta-a|$, $2 \log |\zeta-\zeta_0|$ being normalized so that

$$t(\zeta, a) - 2 \log |\zeta - \zeta_0| \to s_0(a)$$

as $\zeta \to \zeta_0$. For $S$ open a similar but more complicated construction is required. It is then shown that $s(\zeta, a)$ is uniformly bounded from below for all $\zeta$, $a$ on $S$ and is symmetric in $\zeta$, $a$. The metric on $S$ is defined to be $d\omega = \lambda^2 dA_\zeta$ with

$$\lambda^2 = e^{t_0}(1 + e^{t_0})^{-2} | \operatorname{grad} t_0 |^2.$$

In any case the total area of $S$ is $4\pi$. Apart from zeros the metric has constant Gaussian curvature 1.

Letting $\nu(h, a)$ denote the number of inverse images of $a$ in $R_h$ for the regular mapping $f$ of $R$ into $S$ we have the basic entities

$$A(h, a) = 4\pi \int_0^h \nu(h, a) dh,$$

$$B(h, a) = \int_{\beta_h - \beta_0} s(f(z), a) du^*,$$

$$C(h) = \int_0^h \int_{R_h} d\omega(f(z)) dh.$$

A simple application of Stokes' formula gives the primary result

$$A(k, a) + B(k, a) = C(k).$$

Integrating the corresponding result for $h$ twice (which is denoted by the subscript 2) and forming the sum for $q$ values $a_i$, $i=1, \cdots, q$, leads to the equality

$$A_2(k) + B_2(k) = qC_2(k)$$

where each of the first two terms denotes the appropriate sum. Estimation of the term $B_2(k)$ leads to the analogue of Nevanlinna's Second Fundamental Theorem:

$$(q-2)C_2(k) < \sum_{i=1}^{q} A_2(k, a_i) - A_2(k,f') - A_2(k,\lambda) + E_2(k) + O(k^3 + k^2 \log C(k))$$

where $A(k, f')$ counts zeros of the (locally defined) derivative of $f$ in $R_k$, $A(k, \lambda)$ counts zeros of $\lambda$ in $R_k$ and $E(k)$ is the integrated Euler characteristic of $\Omega_k$. In case $S$ is closed of Euler characteristic $e_S$, $\lambda$ has a finite number of zeros $e_S+2$ and the third term on the right-hand side can be omitted, replacing the left-hand side by $(q+e_S)C_2(k)$. It is observed that utilization of repeated integrations has obviated the need for the presence of exceptional intervals found in the usual formulation.

Applications of this result involve the exhaustion of $R$ by the surfaces $R_k$. There is no a priori natural limit in the ordinary sense so this has to be understood as a directed limit. Significant conclusions are confined to the case where

$$\lim_{R_k \to R} \frac{k^3 + k^2 \log C(k)}{C_2(k)} = 0.$$

It is observed that a sufficient condition for this to hold is the existence of $\alpha$, $0<\alpha<1$ with

$$\lim_{R_k \to R} \frac{k}{C(k)} = 0, \qquad \lim_{R_k \to R} \frac{\log C(k)}{C(\alpha k)} = 0.$$

Defining, for $a \in S$, the defect of $a$,

$$\alpha(a) = 1 - \limsup \frac{A_2(k, a)}{C_2(k)},$$

the ramification index

$$\beta(a) = \liminf \frac{A_2(k, f_a')}{C_2(k)},$$

($A_2(k, f_a')$ counting order of branch points above $a$)

$$\gamma(a) = \liminf \frac{A_2(k, \lambda_a)}{C_2(k)},$$

(numerator counting points of $f(R_k)$ covering a zero of $\lambda$ at $a$) and the Euler index

$$\eta = \liminf \frac{E_2(k)}{C_2(k)}$$

the following result is obtained (affinity relation)

$$\sum \alpha(a) + \sum \beta(a) + \sum \gamma(a) \leqq 2 + \eta.$$

If $S$ is closed this can be replaced by

$$\sum \alpha(a) + \sum \beta(a) \leqq \eta - e_S.$$

If $\eta$ is infinite no conclusion can be drawn from the affinity relation. It is observed that if $S$ is open and $\lambda$ has infinitely many zeros on $S$ there are no nondegenerate mappings with $\eta$ finite. This does not necessarily mean that some other version of value distribution might not provide information. Ozawa has studied a special situation involving such a phenomenon. The above inequalities lead to generalization of standard results on Picard values, defective values and totally ramified values.

Chapter I contains also a discussion of the sharpness of the bounds for Picard values for meromorphic functions ($S$ the sphere) and an alternative development of the main theorem for meromorphic functions on a class of surfaces $R$ endowed with conformal metrics possessing certain properties. In this case the usual exceptional intervals intervene.

Chapter II concludes with a result of Nakai that the set of points of $S$ for which $\liminf B_k(a)$ is infinite has vanishing capacity (in terms of an appropriate potential theory on $S$).

Chapter III deals with the class $MB$ of meromorphic functions of bounded characteristic. This involves using the concept of capacity function to derive an analogue of Jensen's theorem and a further development of Nevanlinna's First Fundamental Theorem. The analogue of the usual decomposition theorem is based on the class $LP$ of nonnegative functions harmonic on $R$ apart from logarithmic singularities with integral coefficients, the result being that $f \in MB$ if and only if $\log |f|$ is the difference of two functions in $LP$. The chapter concludes with the place in the classification of Riemann surfaces of the class $O_{MB}$ on which there are no nonconstant functions in $MB$ and results on sums of Green's functions including work of Heins, Parreau and Rao.

Chapter IV treats primarily the existence of the Evans-Selberg potential on a parabolic Riemann surface, a result due in this context to Nakai. This refers, given $a \in R$, to a function harmonic in $R - \{a\}$ which has a negative logarithmic singularity at $a$ and tends to infinity in approach to the ideal boundary. The relevant concepts introduced are the Čech compactification of $R$, Green's kernel, transfinite diameter and capacity of compact sets. The definition of transfinite diameter seems somewhat at variance with the usual one since it is a decreasing rather than an increasing function of sets as the conventional one is. The chapter closes with results on meromorphic functions defined in a neighborhood of the ideal boundary of a parabolic Riemann surface.

Chapter V is devoted to an exposition of constructions of Matsumoto dealing with Picard sets (i.e., the totality of locally omitted values). The first main result is that for every compact set $K$ of capacity zero in the $\zeta$-plane there exists a compact set $E$ of capacity zero in the $z$-plane and a meromorphic function $f$ in the complement of $E$ such that $f$ has an essential singularity at each point of $E$ and its Picard set at each singularity coincides with $K$. The principal technique is the cross-joining of copies of the complement $S$ of $K$ along slits associated with a suitable exhaustion of $S$. The method of cross-joining appears to have been first used in connection with Picard values by the reviewer. A second construction presents a meromorphic function with a set of essential singularities of vanishing linear measure and with a Picard set of positive capacity at each singularity. A final example gives a perfect plane set of positive capacity such that every meromorphic function in its complement with an essential singularity at each point of the set has at most three Picard values at each singularity. It is asserted that the same can be done with two instead of three.

The longest of the chapters, the sixth, deals with a generalization of the Ahlfors' theory of covering surfaces. The presentation is strictly the classical one due to Ahlfors with the exception that, at the point where Ahlfors restricts his covering surface to be simply-connected and his basic surface to be closed of genus zero, the exposition here follows through to allow a general open covering surface $\hat{S}$ and a closed basic surface $S_0$ of arbitrary genus. The proof requires only a straightforward extension of Ahlfors' technique and leads to the following generalization of his fundamental inequality. Let $S$ be a finite subsurface of $\hat{S}$ with mean sheet number $M$ above $S_0$, let $e^+(S) = \max(0, e(S))$ where $e(S)$ is the Euler characteristic of $S$, let $e_0$ be the Euler characteristic of $S_0$. Let $\Delta_\nu$, $\nu = 1, \cdots, q$, $q \geqq 2$, be disjoint simply-connected domains on $S_0$, $n(\Delta_\nu)$ the number of components of $S$ lying above $\Delta_\nu$ without relative boundary (islands),

$b(\Delta_\nu)$ the sum of the orders of branch points of all islands above $\Delta_\nu$. Let $L$ be the length of the relative boundary of $S$. Then

$$(e_0 + q)M \leqq \sum_{\nu=1}^{q} n(\Delta_\nu) - \sum_{\nu=1}^{q} b(\Delta_\nu) + e^+(S) + O(L).$$

This has the usual applications to defect and ramification. Applied to meromorphic functions the fundamental inequality leads to the so-called nonintegrated form of the Second Fundamental Theorem (exceptional intervals occur).

There are brief remarks on inverse functions, a localized Second Fundamental Theorem and localized Picard theorem. The chapter closes with the passage from the fundamental inequality to an integrated form of the Second Fundamental Theorem for a special class of surfaces ($R_p$ surfaces), some results of Noshiro on algebroid functions and manifestation of the sharpness of the nonintegrated defect relation.

There are two appendices. The first collects a number of results for Riemann surfaces appealed to in the main exposition. They are chiefly in the nature of tests for the classification of Riemann surfaces. The second appendix gives applications to minimal surfaces immersed in Euclidean 3-space through the medium of the Gaussian mapping. This is a mapping from the surface to the unit 2-sphere obtained by mapping each point to the end of the radius parallel to the normal there. The book closes with a quite extensive bibliography.

The exposition in the book is globally very smooth and the reader should have little difficulty following the general direction at all times. This is perhaps not true at all times locally and there are perceptible variations in style, perhaps to be ascribed to the role of the various collaborators. The reviewer is not entirely convinced that the standard Ph.D. curriculum would be adequate provision for reading the book without some additional study of the theory of Riemann surfaces, even though the authors are quite faithful in providing references for results quoted but not established.

Some open questions concerning minimal surfaces are indicated at the end of the second appendix. Elsewhere there is no indication of open questions even though some present themselves quite naturally. For example in the first two chapters the entities studied involve a number of arbitrary choices, in particular $R_0$ and the points $\zeta_0$ and $\zeta_1$. Nothing is said as to what effect these choices may have, say, on a value being defective although it is stated explicitly (e.g., p. 60) that $\zeta_0$, $\zeta_1$ are to be chosen distinct from the particular values to be studied. It is well known that in the simplest case of Nevanlinna theory of meromorphic functions in the plane the choice of the exhaustion is decisive in such questions but there the limits are taken with respect to a particular linearly ordered exhaustion.

JAMES A. JENKINS

*A Hilbert space problem book* by Paul R. Halmos. The University Series in Higher Mathematics, Van Nostrand, Princeton, N. J., 1967. xvii+365 pp. $11.50.

This book consists of 199 problems with hints and solutions, comprising 20 chapters. The chapter headings are: Vectors and spaces, Weak topology, Analytic functions, Infinite matrices, Boundedness and invertibility, Multiplication operators, Operator matrices, Properties of spectra, Examples of spectra, Spectral radius, Norm topology, Strong and weak topologies, Partial isometries, Unilateral shift, Compact operators, Subnormal operators, Numerical range, Unitary dilations, Commutators of operators, Toeplitz operators.

The book is well suited for graduate students who have already had a course in Hilbert space theory. One is expected to know the spectral theorem and Fuglede's Theorem for instance, and there is a short discussion of both, but no proofs. The problems include the very simple as well as the contents of recent papers. The hints range from the pithy exhortation "Polarize" to a paragraph of detailed instructions. Accompanying the solutions are references to the literature which will easily enable one to follow up a topic of interest. (These references are by no means complete, and in several cases the author has been forced by demands of space to omit not only sharper and more technical theorems, but even the mention thereof.)

The book succeeds admirably in two respects. First, it presents a diverse collection of tools, techniques, and tricks which should prove valuable to the Hilbert space apprentice. Second, there is a reasonable survey of operator theory in the space allotted. Included, with proofs, are

(i) a characterization of the invariant subspaces of the unilateral shift,

(ii) the coisometric extension of a contraction $T$, where $T^n$ converges strongly to 0,

(iii) the unitary dilation of a contraction,

(iv) von Neumann's Theorem that the unit disc is a spectral set for any contraction, and

(v) the F. and M. Riesz Theorem.