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VARIETIES OF GROUPS AND BURNSIDE'S PROBLEM

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If $\mathfrak{V}$ is a variety of groups and d is a positive integer, $\mathfrak{V}^{(d)}$ denotes the variety consisting of the groups whose d-generator subgroups are all in $\mathfrak{V}$. In a recent paper [3], B. H. Neumann formulated the Extended Burnside Problem:

Problem 7. Let $\mathfrak{V}$ be a locally finite variety and $d \geqq 1$ an integer. Is $\mathfrak{V}^{(d)}$ locally finite?

He went on to ask two related questions:

Problem 8. Is there, to each locally finite variety $\mathfrak{V}$, an integer $d = d(\mathfrak{V})$ such that $\mathfrak{V}^{(d)}$ is locally finite?

Problem 9. Do the locally finite groups in $\mathfrak{V}^{(d)}$, where $\mathfrak{V}$ is a locally finite variety, form a variety?

Neumann called the latter the Restricted Extended Burnside Problem. One might derive from it, like Problem 8 from Problem 7, the following:

Problem N. Is there, to each locally finite variety $\mathfrak{V}$, an integer $n = n(\mathfrak{V})$ such that the locally finite groups in $\mathfrak{V}^{(n)}$ form a variety?

The purpose of this note is to present reduction theorems for Problem 8 and Problem N, similar to the Hall-Higman reduction theorems [1] for the classical forms of Burnside's Problem.

THEOREM 1. *If $\mathfrak{V}$ is a locally finite and locally soluble variety, $\mathfrak{V}_{LN}$ is the variety consisting of the locally nilpotent groups of $\mathfrak{V}$, and $\mathfrak{V}_{LN}{}^{(d)}$ is locally finite for some integer d, then $\mathfrak{V}^{(d^*)}$ is locally finite for some integer d^*.*

This is a direct consequence of (c) of the forthcoming paper [2]. Theorem 2 will be derived from the following part of (b) of [2]:

LEMMA. *If $\mathfrak{U}$ is a locally finite variety which contains only finitely many (isomorphism classes of) finite simple groups, and if $\mathfrak{W}_1$ is the class of those groups whose nilpotent factors and simple factors all belong to $\mathfrak{U}$, then $\mathfrak{W}_1$ is a locally finite variety.*

In order to formulate Theorem 2 in full generality, some additional terminology is required. According to M. B. Powell (cf. Sheila Oates [4]), the complexity of a finite simple group S is 0 if S is abelian, and $k+1$ if S is not abelian and k is the largest complexity which occurs for the proper simple factors of S. In particular, the groups of complexity 1 are precisely the minimal simple groups of J. G. Thompson [5]. Attempts of Oates and Powell (cf. [4]) have raised the hope that the near future may bring a proof of the

CONJECTURE. For each nonnegative integer k, there exists an integer $m(k)$, depending on k, such that:

(k) Every finite simple group of complexity at most k can be generated by $m(k)$ elements.

Obviously, (0) is valid with $m(0)=1$, and Thompson's classification of the minimal simple groups [5] implies (1) with $m(1)=2$. It has, of course, been long conjectured that (k) is always true with $m(k)=2$; but no proof of this stronger claim is within sight, and for the present context it does not matter how generous one is in overestimating $m(k)$.

THEOREM 2. *Let $\mathfrak{V}$ be a locally finite variety such that the locally finite groups in $\mathfrak{V}_{LN}^{(d)}$ form a variety $\mathfrak{U}_1$ and the finite simple groups in $\mathfrak{V}$ all have complexity less than k. If* (k) *is valid, then the locally finite groups in $\mathfrak{V}^{(d^*)}$ form a variety where $d^*=\max\{2, d, m(k)\}$. In particular, if $\mathfrak{V}$ is locally soluble (so that k can be chosen as 1), then the locally finite groups in $\mathfrak{V}^{(d^*)}$ form a variety where $d^*=\max\{2, d\}$.*

PROOF. As $\mathfrak{V}_{LN}^{(d^*)}$ is obviously contained in $\mathfrak{V}_{LN}^{(d)}$, the finite nilpotent groups of $\mathfrak{V}^{(d^*)}$ all lie in $\mathfrak{U}_1$. Let S be a finite simple group in $\mathfrak{V}^{(d^*)}$. If S can be generated by d^* elements, then it lies in $\mathfrak{V}$, has complexity less than k, and the order of S is at most that of the $\mathfrak{V}$-free group of rank d^*. If S cannot be generated by d^* elements, then (k) implies that the complexity of S is greater than k. Thus $\mathfrak{V}^{(d^*)}$ has no groups of complexity precisely k, hence it cannot contain groups of complexity greater than k either, and it follows that $\mathfrak{V}^{(d^*)}$ contains only finitely many (isomorphism classes of) finite simple groups. Let $\mathfrak{U}$ be the variety generated by $\mathfrak{U}_1$ together with these simple groups: then $\mathfrak{U}$ is locally finite and, as $\mathfrak{U}_1$ is also locally nilpotent, $\mathfrak{U}$ contains only finitely many finite simple groups. (Use, for instance, (4) of [2].) Let $\mathfrak{W}_1$ be as in the Lemma. Then the finite groups of $\mathfrak{V}^{(d^*)}$ are all in

$\mathfrak{W}_1$, and hence so are all the locally finite groups of $\mathfrak{B}^{(d^*)}$. Thus the class of the locally finite groups of $\mathfrak{B}^{(d^*)}$ is precisely the variety $\mathfrak{W}_1 \cap \mathfrak{B}^{(d^*)}$.

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