BOOK REVIEW

Abstract harmonic analysis. Vol. I: Structure of topological groups. Integration theory. Group representations. By Edwin Hewitt and Kenneth A. Ross. Die Grundlehren der Mathematischen Wissenschaften, Band 115. Springer-Verlag, Berlin-Gottingen-Heidelberg, 1963. viii+519 pp.

For a certain category of mathematicians, integration theory is, or at least used to be, restricted to its charming virtues on R^n. Measures however provide us with a powerful and elegant tool when used along with the intuition that keeps fertile company to them in a large quantity of algebraic, analytic and geometrical situations. André Weil's *L'intégration dans les groupes topologiques et ses applications* opened new perspectives in mathematics from that standpoint when it became available in 1940. Fourier series and integrals, finite and Lie groups representations, integral geometry, algebraic number theory, and so on, became interacting members of the same promising club. The influence of that masterpiece was retarded by a few years due to the natural circumstances created by World War II. Such a delay seems immaterial, now that more than 25 years have elapsed, except maybe for people who had a hard time in getting hold of Weil's monograph during 1940–1945 and remember the exceptional pleasure of being able at last to read it and wonder hopefully about the wealth of research and turns in teaching ideas that would come next. The book under review is a good representative of the quarter of a century started by Weil's IGTA.

In the present volume, the authors give the structure of topological groups needed for harmonic analysis; treat integration on locally compact groups; and introduce the readers to group representations. They promise formally that a second volume will go into the intimate life of compact groups and locally compact Abelian groups, in considerable detail. The authors "hope to have justified the writing of yet another treatise on abstract harmonic analysis by taking up recent work, by writing out the details of every important construction and theorem, and by including a large number of concrete examples and facts not available in other textbooks." The pleasant, useful and important features of this volume, however, go far beyond those claimed by the authors. The book is intended to be readable by students having a first year graduate training, and to be useful for specialists as well. As such it happens, although it may not have been meant, to be a twin brother with a different temperament of

Walter Rudin's *Fourier analysis on groups*, 1962 [See Jean-Pierre Kahane's excellent review, Bull. Amer. Math. Soc. **70** (1964), 230]. The fact that harmonic analysis is very much lively and subject to advances in its classical foundations was recently shown by Lennart Carleson's affirmative solution of Lusin's long open problem concerning convergence almost everywhere for partial sums of Fourier series for functions in L^2 (Acta Math. **116** (1966)).

Chapter 1 presents preliminaries about notation, terminology, group theory and topology.

Chapter 2 exposes the elements of the theory of topological groups leading to the structure theory for locally compact Abelian groups; the principal result is that every such group which is compactly generated (that is, contains a compact subset for which the subgroup generated is the whole group) is topologically isomorphic with a product of a compact Abelian group and a finite number of copies of the group R of the reals and the group Z of the integers.

Chapter 3 is an exposition of integration on locally compact spaces.

Chapter 4 is an introduction to the Haar integral; it follows the method of Henri Cartan in providing existence and uniqueness at one coup.

Chapter 5 goes into convolutions and group representations. It leads to the fundamental Gelfand-Raikov theorem, according to which for every element a in the locally compact group G different from the identity, there is a continuous, irreducible, unitary representation V of G such that V_a is different from the identity operator. It is to be combined with the results asserting that every such representation of a compact group is finite dimensional, and that every such representation of a locally compact Abelian group is 1-dimensional.

Chapter 6 is devoted to characters and duality of locally compact Abelian groups, that is the fine structure of them. The basic tool there keeps on being the Pontrjagin-van Kampen duality theorem asserting that the natural mapping of a locally compact Abelian group G into the character group of the character group of G is actually an isomorphism and a homeomorphism onto.

Three appendices assemble material on Abelian groups, topological linear spaces and normed algebras for the reader's convenience.

The authors did a very nice job in writing a book which strictly speaking is not exclusively a classroom text and is not either solely a reference treatise, but shares both features as a sort of a two-bladed knife in the good sense of the expression.

The above description of chapters is not a fair hint of the richness and completeness of the book within the ample boundaries set forth by the authors for the students and the specialists they aimed at.

Many interesting sections are devoted to the history of the major theorems and ideas, to illuminating examples, as well as to the partisanship that fortunately enough authors are still entitled to follow.

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