TWO THEOREMS IN GEOMETRIC MEASURE THEORY

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The following two propositions give some new information about the structure of differentiable maps. We use the symbols R^m and H^s to designate m dimensional Euclidean space and s dimensional Hausdorff measure, respectively.

THEOREM 1. *If $m > r \geqq 0$ and $k \geqq 1$ are integers, Y is a normed real vectorspace, $f: R^m \to Y$ is k times continuously differentiable, and*

$$S = R^m \cap \{x: \dim \operatorname{im} Df(x) \leqq r\},$$

then

$$H^{r+(m-r)/k}[f(S)] = 0.$$

THEOREM 2. *If $f: R^m \to R^n$ is Lipschitzian, r is an integer, $0 \leqq r \leqq m$, and*

$$T = R^n \cap \{y: H^{m-r}(f^{-1}\{y\}) > 0\},$$

then H^r almost all of T can be covered by a countable family of r dimensional submanifolds of class 1 of R^n.

The first theorem optimally sharpens the results of [**4**], where the history of the problem is discussed; its proof uses a refinement of the key lemma in [**3**], which dealt with the case $r=0$. The second theorem is related to the coarea formulae obtained in [**2**] and [**1**]. Proofs of both theorems will appear in the author's book *Geometric measure theory*.

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