ential inequalities in stability considerations, theorems on the ultimate boundedness of solutions and a concept they introduce called "practical" stability which fits more closely the requirements arising in technical applications than the usual notion of stability. This point is illustrated on van der Pol's equation with a small perturbation term.

Mathematicians may object to a few statements such as the definition of a closed set as the outside of some open set (p. 19). In the reviewer's opinion these are unfortunate attempts to please the potential reader by keeping the mathematical details to a minimum. Such statements could have been avoided without detracting from the appeal this book will unquestionably have with the audience for which it is primarily intended.

H. A. ANTOSIEWICZ

Die innere Geometrie der metrischen Räume. By Willi Rinow. Springer, Berlin, 1961. 16+520 pp. DM 83.

The intrinsic geometry of metric spaces studies their invariants under isometries. Its history dates from Gauss' Theorema Egregium, and it developed in the nineteenth century under Riemann, Helmholtz, Lie, and Klein. In this century intrinsic *differential* geometry has been pursued with great vigor, quite apart from the larger context. In fact, despite great successes in general metric geometry achieved in the past thirty years, mathematical fashion has all but ignored it as a bona fide field for research.

Recently, however, the climate has begun to change. Interest in differential geometry in the large has started many a mathematician studying intrinsic metric geometry. But such a study is made difficult by the scarcity of books on the subject. Until the present volume appeared, there were only three in the field: A. D. Aleksandrov's, *Intrinsic geometry of convex surfaces* (Moscow, 1948), L. M. Blumenthal's, *Theory and applications of distance geometry* (Oxford, 1953), and H. Busemann's, *Geometry of geodesics* (New York, 1955).[1] Since these treat restricted aspects of the whole subject, some survey which could help describe and define the field was clearly called for.

Rinow's book apparently was planned as just such a survey. However, as he states in the introduction, the lack of uniformity in assumptions and definitions among writers in the field soon made it

[1] A predecessor of Busemann's book, his monograph, *Metric methods in Finsler spaces and in the foundations of geometry* (Princeton, 1942), should also be mentioned.

apparent that a systematic development was needed. The latter is what this book is advertised to be.

Chapter I, Metric Geometry and Topology, is a detailed exposition of metric space topology. Chapter II, Continuous Mappings, continues in this vein. A not-so-standard topic treated here is the problem of defining equivalences of mappings into an abstract metric space to correspond with the notion of different parameterizations of the same arc or surface. These ideas are used, however, only for the special case of curves.

In Chapter III, The Intrinsic Metric, the study of intrinsic geometry as it is commonly understood begins. The intrinsic metric is introduced into any space whose points are joinable by rectifiable curves; it is given by the infimum of the lengths of paths joining points. This metric induces the same topology as the original if and only if the space has no "detours," that is, points close in the original metric can be joined by short arcs.

A segment between two points has length less than or equal to that of any other curve joining them. Chapter IV is devoted to their study, including an exhaustive discussion of the various hypotheses which are sufficient to ensure the existence of segments, such as Menger's convexity conditions and Busemann's prolongability axioms. Singularities of the intrinsic metric analogous to branch points and "sinks" are defined. Sections are also devoted to geodesics, cut points (here called "absolute conjugate points"), and perpendiculars.

Chapter V contains mostly standard material on fundamental groups and covering spaces.

Chapter VI, Existence Theorems for Geodesic Curves, is mainly concerned with proving that closed geodesics exist in quite general (even simply connected) compact spaces. These results, at least in the Finsler space case, are due to Lusternik and Fet, and it is their proof, based on category theory, which is generalized. This is the first detailed exposition of this important method in the calculus of variations in the large to appear in book form.

The Theory of Curvature, Chapter VII, treats the Aleksandrov theories of angles and spaces of bounded curvature. Chapter VIII, The Clifford-Klein Space-form Problem, reproduces, among other standard results, the Bieberbach Theorems on Euclidean space-forms.

Except for a section on spaces without conjugate points, Chapter IX is devoted to the geometry of spaces with non-positive curvature. Chapter X, Spheroids and Spaces of Elliptic Type, examines spaces in which all geodesics are closed and behave like the great circles in spherical or elliptic geometry, respectively.

The author has succeeded very well in examining the hypotheses needed for various theorems in intrinsic geometry. Many results and reformulations have not appeared in print before, and one wishes the author had expanded his scattered historical notes to give credit to himself where it is due. Chapters III and IV are especially valuable for clarifying the basic axioms of intrinsic geometry. The remaining chapters, however, represent a selection rather than a systematic development, albeit a selection with which this reviewer wholeheartedly agrees. For, despite Aleksandrov's and Blumenthal's results in embedding problems, and scattered beginnings of a theory of area, the most flourishing division of intrinsic geometry is the geometry of geodesics. Indeed, Chapters V, IX, and X are largely made up of material that may be found in Busemann's book of the same name. One wishes that less trouble had been taken to make the book almost self-contained in regard to topological prerequisites in order that the space could be used for descriptions of these other aspects of the field.

However, we should be grateful for the wealth of material that is here, especially for the chapter on Lusternik's theory. But it may not be out of order to ask the author to reconsider his decision on writing a survey volume. His knowledge of intrinsic geometry is certainly equal to the task, and this book reemphasizes its need.

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