Cluster sets. By Kiyoshi Noshiro. Ergebnisse der Mathematik und ihrer Grenzgebiete, Neue Folge, Heft 28. Springer, Berlin, 1960. 7+135 pp. DM 36.00.

This monograph is the first systematic treatment of the boundary behavior of analytic functions and of some other related classes of functions. Since the theory, initiated some fifty years ago in the work of Iversen and Gross, has shown considerable development, a comprehensive account has been highly desirable. The author has accomplished this task in exemplary fashion.

The first chapter is devoted to definitions and some preliminary discussion. The central notion here is that of cluster set: Let D be an arbitrary domain of the complex plane, z_0 a point of the boundary of D, and $f(z)$ a single-valued, meromorphic function in D. The set of all complex numbers α for which there exists a sequence $\{z_n\}$ of points in D, such that $z_n \rightarrow z_0$ and $f(z_n) \rightarrow \alpha$, is called the cluster set of $f(z)$ at the point z_0. Some variants of this definition are introduced, as well as definitions of the notions of range of values and asymptotic set. This is followed by the statements, mostly without proof, of classical theorems, associated with the names of Iversen, Gross, Fatou, F. and M. Riesz, Lindelöf, and Koebe.

The second chapter takes up the behavior of meromorphic functions with a compact set of essential singularities of capacity zero. Here, reference to Bagemihl's paper, which appears in the bibliography under number [3], would have been appropriate. Next, various extensions of the theorems of Iversen and Gross are discussed, and the chapter is concluded with a group of theorems due to Hervé.

The longest chapter in the book is the third. It begins with functions of class (U); that is, bounded holomorphic functions in the unit disk the modulus of whose radial limits at almost all points of the circumference is equal to 1. This section is particularly noteworthy in that earlier results whose proofs depended heavily on special techniques are now derived and extended in a uniform manner by means of elegant topological methods applied to the Riemann surfaces of the inverse functions. The author himself has made important contributions to this section.

The next section of the chapter deals with a group of theorems due to Cartwright and Collingwood on the relations, both in the small and in the large, between cluster sets, ranges of value, and asymptotic sets. The reviewer noticed here that in mentioning continua which are not global cluster sets of any meromorphic function in the unit disk, no credit is given to Potyagaĭlo, who obtained an example of such a continuum in a paper, in Russian, entitled *On the set of boundary values of meromorphic functions,* which appeared in 1952 in the Doklady of the Soviet Academy.

Another section is devoted to the notion of Baire category, as applied to the theory of cluster sets, which in recent years was developed by Bagemihl and the reviewer and by Collingwood. Among other results appearing here are the well-known theorems of Plessner and of Lusin and Privalov, as well as the "two-chord" theorem of Meier. It is not clear to the reviewer why the latter is referred to as a sharpening of Plessner's theorem.

The final section of Chapter III studies the boundary behavior of functions of bounded type, investigated recently by Lehto, and of normal functions. In connection with functions of bounded type, reference should have been made to Gehring's interesting paper which appeared in the Quarterly Journal of Mathematics in 1958. The class of normal functions was first introduced by the author in an undeservedly little known paper in 1939, in which he also derived some important properties of this interesting class of functions. Subsequently, in 1957 Lehto and Virtanen independently rediscovered these functions and made significant contributions to their theory.

Two omissions in Chapter III should be noted. The Uniqueness Theorem 7 in §3 is very closely related to Corollary 3 of the first of Bagemihl's papers listed in the bibliography, but no mention is made of this. Also, no mention is made of the fact that Theorem 15 in §3 was proved independently by Bagemihl and the reviewer in the paper listed as [5] in the bibliography.

The last chapter goes into extensions of the theory of cluster sets to single-valued analytic functions on open Riemann surfaces, and a short appendix touches on extensions to pseudo-analytic functions. The book ends with an excellent bibliography.

The author is to be congratulated for his concise, but very readable, survey of a highly technical and extensive subject. It can be warmly recommended to any one who wishes to become acquainted with this branch of function theory.

W. SEIDEL

Funktionalgleichungen der Theorie der geometrischen Objekte. J. Aczél and S. Gołab. Panstowe Wydawnictwo Naukowe, Warsaw, 1960. 172 pp. DM 36.00.

In the 1930's a rather interesting correspondence took place between the Dutch differential geometers J. A. Schouten and J. Haantjes on one side, and their Polish colleague A. Wundheiler on