ators considered before have analogues in connection with those systems.

Equations of mixed type and elliptic equations with coefficients that have singularities or are not analytic functions are considered at the end of the book.

The report includes a rather complete list of references to literature, and original papers are quoted throughout the text. Some of the proofs have been omitted in particular in cases where the transition to original papers is not difficult. Integral operators have applications in hydrodynamics and other fields, but these are not considered here.

ERWIN KREYSZIG

Theory of algebraic numbers. By E. Artin. Notes by Gerhard Würges from lectures held at the Mathematisches Institut, Göttingen, Germany in the winter semester, 1956–57. Translated and distributed by George Striker, Schildweg 12, Göttingen. 172 pp. $2.50.

Professor Artin is well known not only as a master of modern algebra, a subject to which he has made important contributions, but also as a master of elegant and original exposition. It is needless to say that the present volume based on his Göttingen lectures has all the characteristic features associated with his name though it is not quite clear what part he has taken in its production. Since there is now a tendency to drop the word modern, a better title perhaps for this enjoyable little book would be "Lectures on Algebra with Applications to Algebraic Numbers." It develops the subject as axiomatically as possible and makes clear the assumptions and axioms upon which the treatment depends. The knowledgeable reader will note that the ideas introduced will be available for the study of related topics and also applicable to other disciplines. The book will be highly valued and appreciated by those who are curious to see how the subject is developed from the apparently remote initial concepts and definitions, and how these lead to the establishment of the essential results in the theory and how they are brought into the picture. They are sure to find great pleasure in reading this book. They should not, however, be beguiled into thinking there is nothing more to be done in algebraic number theory than axiomatization. There are vast domains not touched upon here. The size of the book makes it obvious that its scope will be limited, and it must be borne in mind that the book had its origin in a one-semester course of lectures and is really of an introductory character.

Next a word of warning. There are other readers for whom Artin, it may be fairly said, did not intend to cater in his lectures, namely, those who are desirous of acquiring as rapidly as possible a working knowledge of algebraic number theory sufficient for the applications they have in mind. They may be interested in Diophantine equations and it must not be overlooked that the subject of the book had its real origin in Kummer's work on Fermat's last theorem. There are also many questions in analytical number theory arising from the algebraic theory, questions not only about the order of magnitude of various arithmetical functions, but also about the properties of analytic functions associated with them, e.g., the multiple theta functions, the Dedekind zeta function, and the various L and zeta functions introduced by Hecke and Artin. These readers, though it may do them good to read Artin, may be too impatient to do so, and would be well advised to turn to those writers who develop the subject in the more usual classical ways and deal with more concrete concepts and problems.

Further, the book will not be easy reading for those who are taking up the subject for the first time or who are not too familiar with algebraic concepts. The book takes for granted that the reader has an adequate knowledge of these and of the simple properties of the entities considered, for example, of rings and ideals, of fields and their closures under various processes, of separable fields, perfect fields, and algebraically closed fields, of groups and of the topological notions associated with sequences. It takes some time to acquire a knowledge of all these, but there has been an increasing tendency to do so earlier and earlier as part of a standard course in mathematics. The reader would have been helped if some indication had been given of the properties assumed to hold for various entities. Thus he will see in due course that as far as ideals are concerned, all that is required is that an ideal $\mathfrak{a}$ is a set of elements in a ring v closed under subtraction and under multiplication by the elements of v. An introductory chapter giving definitions and properties of concepts not defined in the book would have been useful.

Undoubtedly an axiomatic or abstract treatment of a subject can lead to simplicity and the unification of various concepts and has resulted in important advances in mathematics. Abstract concepts and proofs based on them are occasionally not easy to grasp, and some mathematical maturity is required to cope with them. It may require some effort to retain in mind the meanings of various symbols and what they stand for. A conglomeration of symbols may lead to considerable perplexity. They occasionally seem rather intangible and lacking in substance or in reality until one has had repeated contact with them. One may not see what they really mean unless he can associate them with more concrete notions with which he has previously become familiar. Without some such background, demonstrations may be very difficult to comprehend.

In this modest little book of 172 pages, the author gives an account of some of the fundamental and worth while results in the theory of algebraic numbers. The notion of mapping is the starting point followed by valuation rings, places and valuations. The motif in the theory can be compared to a related problem in the theory of functions of a complex variable z. Given a function $f(z)$ defined when z is in a region R by some process e.g., an infinite series or integral, how can the meaning of $f(z)$ be extended or how can $f(z)$ be continued i.e., does there exist a function $F(z)$ defined in a region containing R such that $F(z)$ and $f(z)$ have the same values at points of R? In algebraic number theory, the function associated with an element z of a given field k is the valuation $|z|$. The properties of valuations are discussed in some detail, but it is perhaps a little confusing to have first a definition in Chapter III which applies only to non-Archimedean valuations and then later in Chapter III, a more general definition which also applies to Archimedean valuations. Suppose now that k is a subfield of another field K. The fundamental problem is how can the valuation $|z|$ of elements z of k be continued or extended into valuations of the elements Z of K in such a way that the new valuation reduces to the old one when Z is an element of k. Various cases have to be considered depending upon the nature of k e.g., whether it is complete or not complete and on the nature of the valuation in k e.g., whether it is Archimedean or non-Archimedean, then it may or may not be discrete. Similar problems arise in extending the definitions of other entities associated with k e.g., there is the residue field associated with a valuation and also the "*different*."

During most of the first ten chapters, Artin's motto seems to be excelsior or ever forward. He proceeds steadily and one might almost say relentlessly with his logical development. His presentation is closely knit and economical to an extreme and he does not diverge from his chosen path. There is little room for motivation and perhaps a preface to the book would have given an opportunity for this. His treatment is quite an education in itself. But how very different the subject seems to be to one who has been brought up in the older classical tradition and who is familiar with the books by Dirchlet-Dedekind, Kronecker, Hilbert, Weber, Bachmann, Landau, Hecke, Weyl and the report by Fürtwangler-Hasse in the German Encyclopedia. In these books and even in Hasse's Zahlentheorie with its modern treatment, there is no mention of ordered groups, Zorn's lemma, normed fields and other concepts which play so important a part in Artin's book. This does not mean that the classical treatises can be discarded or dispensed with as being old fashioned or out of date. There is a great deal still to be learned from them. Holzer's recent Zahlentheorie shows that even now, there is room for expositions based upon the older ideas.

Before Chapter VIII, very little has really been said about ideals. The definition has been assumed and some very simple properties are proved e.g., if an ideal $\mathfrak{a}$ is a maximal ideal in a ring v, i.e. is $\mathfrak{a}$ contained in no other ideal than v, then $\mathfrak{a}$ is a prime ideal, i.e. if the product $\beta\gamma$ of two numbers in v is contained in $\mathfrak{a}$, then either β or γ is contained in $\mathfrak{a}$. There is as yet no suggestion that ideals arose in an effort to circumvent the difficulty arising in number theory investigations due to the fact that in general, the integers in an algebraic number field do not factorize uniquely in terms of those numbers which one would naturally regard as primes. Chapter VIII is devoted to the exposition of ideal theory based on valuation theory and the relation of ideals to divisors, a concept very useful in other disciplines. The whole theory is shown to follow from two axioms. The statement that properties hold for nearly all valuations would be clearer if it were stated that it holds except for a finite number of valuations.

Professor Artin can be relied upon to introduce unexpected novelty in his exposition. Thus in dealing with ideals, he applies results on the solution of linear Diophantine equations when the coefficients are elements of a field k and when the variables are elements of a ring v of k. He considers both local and global solvability introducing a principle which has many important applications in other parts of number theory.

In the last few chapters, Artin comes down from the heights, and devotes himself to the more concrete parts of the subject. There is an account of Minkowski's theorem on linear forms, and applications to the finiteness of the class number, and to the units of a field. Several cubic fields are considered in detail and there are given the integer basis, the factorization of small primes, the class number and the units.

The usual discussion on the units of a field includes a number of lemmas. Artin dispenses with these and replaces them by a more general result—an additive subgroup Λ of an n-dimensional vector space V over the field of real numbers is a lattice if and only if every bounded domain in V contains only a finite number of elements of Λ. This result is typical of mathematical progress in that special isolated results are included in a general one of much more importance and interest. Some readers however, may find the abstract presentation not so satisfying perhaps as the lemmas dispensed with. There is a feeling as if a well known piece of prose were translated into some foreign language. One would have thought that there was not much opportunity for new results on cubic units. It comes as a surprise when he shows that if ϵ is the fundamental unit of a cubic field with negative discriminant, D, then $4\epsilon^3+24>|D|$, a result of some use in calculating a fundamental unit.

There is no need to recommend the purchase of this excellent book to those familiar with Artin's name as they will buy it as a matter of course. It can be recommended very strongly to those who are not familiar with his work, but have the preliminary requisite knowledge, and are interested in the various aspects and facets of number theory. They will find the book an intellectual treat of the first order.

Professor Artin would confer a boon on those interested in number theory if this book was followed by another volume dealing in his own inimitable way with other aspects of algebraic number theory not dealt with in his Princeton and New York lectures on algebraic numbers and algebraic functions.

L. J. MORDELL

Representation theory of the symmetric group. By G. de B. Robinson. University of Toronto Press, Toronto, 1961. 8+204 pp. $6.00.

This monograph deals with a specialized and highly technical part of group theory, namely the calculation of the modular irreducible representations of the symmetric group. The most important part of the book (Chapters 7 and 8) contains many results, so far unpublished, due to the author and his collaborators. The earlier part of the book consists of preparatory material, classical results on group theory and representation theory in general and on the representations of the symmetric group in particular.

The pace here is fairly rapid, many details of proofs being omitted, but full references being given to original sources or more detailed discussions. Although one cannot, of course, object to sketchy treatment of classical material in a specialized monograph, there are nevertheless one or two points which seem to have been left rather obscure. For example the definition of an induced representation is rather difficult to follow. Also there appears to be some confusion in §12.1; namely the field of characteristic p which is used does not seem to be Σ itself, but rather the residue class field modulo a certain prime.

The most attractive feature of the book is the wealth of examples with which the author illustrates each stage of his argument. Indeed