dimensional orbit of dimension k. He assumes that the dimension of the fixed point set F is exactly $n-k-1$, and looks at what happens. For example, if $M^n=S^n$, the n-sphere, then F is an $(n-k-1)$-cohomology manifold over the integers with the integral cohomology of S^{n-k-1}. Furthermore the orbit space $S^n|G$ is a cohomology $(n-k)$-cell.

The final chapter, 16, by Borel, discusses the spectral sequence of a map. The reviewer concludes by expressing his hope that this book will stimulate more interest and activity in transformation groups.

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Integral functions. By M. L. Cartwright. (Cambridge Tracts in Mathematics and Mathematical Physics, no. 44). Cambridge, at the University Press, 1956. 8+135 pp. $3.50.

For its short length this book contains a large amount of material. This is made possible in part by the choice of subject matter, but more importantly, by the beautifully compact proofs which the author has been able to construct in many instances. One may assume that the choice of material has been motivated to some extent by the author's preference for those subjects to which she herself has contributed most successfully. The result is a book largely complementary to that of Boas, which deals primarily with functions of exponential type [*Entire functions*, New York, 1954; this Bulletin 62 (1956) 57–62].

The main theme of the present book is the study of analytic functions which are of finite order in an angle. Whereas the results in Boas's book on this point may be interpreted as results on functions of mean type, the present author achieves complete generality by the use of proximate orders. The second important topic not covered by Boas is the distribution of the values of an analytic function, notably the theories of exceptional values and of lines of Julia.

Like Boas's, the present book should be well within the reach of any student who has taken a standard year course in complex analysis.

A description of the chapters in the book follows.

I. Preliminary results. II. Integral functions of finite order: Hadamard's factorization theorem, Borel and Picard exceptional values, asymptotic values. III. The Phragmén-Lindelöf principle. Various formulations are given of the principle that a not-too-large analytic function in a half-plane which is bounded by M on the frontier is bounded by M throughout. The indicator function

$$h(\theta) = \limsup \log |f(re^{i\theta})| / r^{\rho}$$

is discussed in great detail, also in the case where r^{ρ} is replaced by a function $V(r) \sim r^{\rho(r)}$ involving a proximate order.

IV. The proximate order of an integral function. It is shown that for every entire function f of finite positive order ρ there exists a Lindelöf proximate order, that is, a real, continuous, piecewise differentiable function $\rho(r)$ such that $\rho(r) \to \rho$, $\rho'(r)r \log r \to 0$, and $\limsup \log M(r)/r^{\rho(r)} = 1$. The proximate order is used to give a short proof of the Wiman-Valiron theorem which compares the minimum and the maximum modulus of an entire function of order $\leqq 1$. V. Integral functions, type results. Various relations are obtained between the distribution of the zeros and the general indicator function $h(\theta)$. A number of results are given which estimate $\log |f(re^{i\theta})|$ in terms of $V(r) \sim r^{\rho(r)}$ and (for integral order ρ) $\sum z_n^{-\rho}(|z_n| \leqq r)$. VI. Some results for angles: results relating zeros, minimum modulus and maximum modulus in the case of functions of finite positive order in an angle.

VII. Lines of Julia. Let f be regular in an angle. A line of Julia is a direction near which f takes all values with one possible exception, in some precise sense. Among the results proved are the following: if f has an isolated singularity at infinity it has at least one line of Julia; large angles in which f is large relative to a proximate order always contain a line of Julia; a direction which separates an angle in which f is large from one in which f is small is a line of Julia. It follows that entire functions of order $>1/2$ have at least two lines of Julia. The proofs are based on a precise form of Schottky's theorem which was given by Hayman. VIII. Singularities of power series and $h(\theta)$. Pólya studied entire functions $f(z) = \sum a_n z^n/n!$ of exponential type with the aid of their transforms $\phi(z) = \sum a_n/z^{n+1}$ and $F(z) = \sum a_n z^n$. The author discusses the indicator diagram of f in detail, and studies its relation to the singularities of ϕ, the Borel summability of the series $\sum a_n z^n$, and the lines of Julia of f.

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