

by Loève “decomposable processes.” This is a subject for which authors like to invent at least one new nomenclature. Loève makes a distinction between “random function” (a family of random variables) and “stochastic process” (never formally defined but, roughly, a class of random functions with common conditional distributions). This reader found the distinction, going back to ideas of Lévy, somewhat confusing, but perhaps a clearer discussion would make the distinction helpful. Dynkin has a careful definition along these lines in the Markov case. Chapter XII includes detailed discussions of the strong Markov property, sample function continuity, and semi-group analysis based on the work of Feller and Dynkin. This part of the subject is still under rapid development, and many readers will find Loève’s treatment helpful as an introduction to material otherwise available only in papers scattered through the periodical literature. The relation to potential theory is not discussed.

This reader feels that Loève’s attempt to be so complete in a book of normal length would have been more successful if about 100 more pages had been allotted, and devoted to discussion and examples, but the book is an excellent pioneering text which will have an enormous influence.

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Vorlesungen über Orthogonalreihen. By F. G. Tricomi. Grundlehren der mathematischen Wissenschaften, Band LXXVI. Springer-Verlag, Berlin-Göttingen-Heidelberg, 1955. 8+264 pp. DM 37.60.

This book, like many of its author’s other well-known books, originated in courses of lectures given at the University of Turin. It is an enlarged and considerably revised version of a preliminary (mimeographed) Italian edition (*Serie Ortogonali di Funzioni*, Gheroni, Torino, 1948) which is now out of print. The very competent translation is the work of Dr. F. Kasch, of Göttingen.

The author’s aim is to provide a lucid, comparatively elementary, and highly readable introduction to orthogonal expansions, and in particular to trigonometric series and orthogonal polynomials. In this he succeeds admirably, demanding from the reader little more than a thorough knowledge of advanced calculus (some knowledge of the elementary theory of Lebesgue integrals, and perhaps a little more on infinite series than is contained in some advanced calculus courses). It is not part of the author’s plan to replace Zygmund on trigonometrical series, or Szegö on orthogonal polynomials, to aim at encyclopedic completeness or at penetrating to the most modern parts of the theory; and he valiantly resists the temptation to enter

into discussions which, in the framework of this volume, must at best remain sketchy.

If the preceding remarks stress the didactic orientation of the volume under review, they are not intended to convey the impression of a run-of-the-mill textbook. Far from it. As would be expected by the readers of the author's numerous books, individual touches abound, and some original results due to the author himself are incorporated in the presentation, especially in the second half of the book. Here we get a characterization of the classical orthogonal polynomials by their generalized Rodriguez formulas, and many original results on the asymptotic behaviour and other analytic properties of the classical orthogonal polynomials.

The work consists of three parts dealing, respectively, with general orthogonal functions on a real interval (Chapter I), trigonometric series (Chapters II and III), and orthogonal polynomials (Chapters IV to VI).

In Chapter I, orthogonal systems of functions are introduced. Both convergence in mean and pointwise convergence of orthogonal expansions are considered, and the Fischer-Riesz theorem is discussed. Several conditions for completeness of orthogonal systems are given, and the completeness of the trigonometric system as well as that of the system of powers on a finite interval is proved.

In Chapter II, first the uniform convergence of the Fourier series of an absolutely continuous periodic function with a square integrable derivative is established. Then Riemann's localization theorem is proved, and the conditions of Dirichlet, Dini, and Lipschitz for local convergence follow.

Chapter III is devoted to a more detailed investigation of the convergence of trigonometric and Fourier series. The topics taken up in this chapter are: absolute and uniform convergence, integration and differentiation of trigonometric series, Cesàro summability and Fejér's theorem, Abel summability, Riemann summability and generalized derivatives, the Cantor-Lebesgue theorem, and sets of uniqueness for trigonometric series. This chapter contains also a brief introduction to Fourier integrals.

Chapter IV begins with a construction of orthogonal polynomials for an arbitrary weight function on a finite or infinite interval, and goes on to the basic results regarding zeros, recurrence relations, the Christoffel-Darboux formula. The classical orthogonal polynomials are characterized as those suitably normalized polynomials for which a generalized Rodriguez formula

$$p_n(x) = \frac{1}{w(x)} \frac{d^n}{dx^n} [w(x)X^n]$$

holds, where the function w and the polynomial X are independent of n . The differential equations satisfied by the classical orthogonal polynomials follow.

In Chapter V the properties of the classical orthogonal polynomials on finite intervals are developed. There is some introductory work on the gamma function, on hypergeometric functions, and on confluent hypergeometric functions. Then follow the properties of Jacobi, ultraspherical, Chebychev, and Legendre polynomials. In connection with the latter, Legendre functions are discussed also for general values of the parameters. The detailed study, both in this chapter and in Chapter VI, of the analytic behaviour of the classical orthogonal polynomials, in particular of their zeros and their asymptotic behaviour, may be especially noted here.

In Chapter VI, the classical orthogonal polynomials for unbounded intervals, that is the Laguerre and Hermite polynomials are presented, the presentation including a proof of the completeness of these polynomial systems. The book concludes with a discussion of the convergence properties of expansions in classical orthogonal polynomials.

Throughout the work examples enliven the presentation, and at the end there is a very useful table of the various constants associated with the classical orthogonal polynomials as well as a bibliography. All in all a worthy addition to the series of books written by the author—and also to the “yellow series.”

A. ERDÉLYI

Special relativity. By W. Rindler. Oliver and Boyd and Interscience Publishers, Inc., 1960. 10+186 pp. \$2.25.

This small book gives an excellent, clear, and concise account of special relativity. There is a sound balance between physical ideas, analytical formulae, and space-time geometry. In a short treatment the author cannot satisfy all tastes and must make a selection of topics. The reviewer would have preferred a greater emphasis on space-time geometry, but this is a matter of personal predilection. He is in complete agreement with the choice of topics. These are perhaps best indicated by enumerating the chapter headings: 1. the special principle of relativity; 2. relativistic kinematics; 3. relativistic optics; 4. space-time; 5. relativistic mechanics of mass points; 6. relativistic