SLENDER GROUPS

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Let P be the direct product of countably many copies of the integers Z, i.e., the group of all sequences $x=(x_1, x_2, \cdots)$ of integers with term-wise addition; and, for each natural number n, let δ^n be the element in P whose nth coordinate is 1 and whose other coordinates are 0. Łoś calls a torsion-free abelian group A *slender* if every homomorphism of P into A sends all but a finite number of the δ^n into 0. The concept first appeared in [**3**]. E. Sąsiada [**6**] has shown that all reduced countable groups are slender. In this note I give a new description of the slender groups and apply it to show that certain classes of groups are slender. All groups in this paper are abelian.

A group is slender if and only if every homomorphic image of P in it is slender. It is therefore desirable to know the structure of the homomorphic images of P.

THEOREM 1. *A homomorphic image of P is the direct sum of a divisible group, a cotorsion group, and a group which is the direct product of at most countably many copies of Z.*

A group A is a *cotorsion* group if it is reduced and is a direct summand of every group E containing it such that E/A is torsion-free. These groups were introduced by Harrison [**4**]. A special case of Theorem 1 (namely the structure of P/S where S is the direct sum) was proved by S. Balcerzyk [**2**].

A torsion-free cotorsion group contains a copy of the p-adic integers for some prime p. For each prime p the p-adic integers are not slender: the homomorphism $x \rightarrow \sum_{i=1}^{\infty} x_i p^i$ sends δ^i into p^i. Theorem 1 and the remark preceding it then give

THEOREM 2. *A torsion-free group is slender if and only if it is reduced, contains no copy of the p-adic integers for any prime p, and contains no copy of P.*

A group is called $\aleph_1$-free if every at most countable subgroup is free.

COROLLARY 3. *An $\aleph_1$-free group is slender if and only if it contains no copy of P.*

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A group A is a *B-group* if $\text{Ext}(A, T) = 0$ for every torsion group T and a *W-group* if $\text{Ext}(A, Z) = 0$. The names for these classes of groups are due to J. J. Rotman. All B-groups and W-groups are $\aleph_1$-free. Baer showed in [1] that P is not a B-group. It is also true that P is not a W-group. Since every subgroup of a B-group (W-group) is a B-group (W-group) we have

THEOREM 4. *Every B-group and every W-group is slender.*

This theorem was first proved (with an additional condition on the B-groups) by Rotman [5].

The above scheme can be applied to various other classes of groups, for example the torsion-free groups such that $\text{Ext}(A, Z)$ is countable. The property is hereditary, every such group is $\aleph_1$-free, and P is not one of them. The structure of $\text{Ext}(P, Z)$ is completely known. Let Q be the additive group of rational numbers and c the cardinal of the continuum.

THEOREM 5. $\text{Ext}(P, Z)$ *is the direct sum of* 2^c *copies of* Q *and* 2^c *copies of* Q/Z.

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