# SOLUTION OF THE PLATEAU PROBLEM FOR $m$-DIMENSIONAL SURFACES OF VARYING TOPOLOGICAL TYPE

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We use a definition due to J. F. Adams:

DEFINITION. *Let $G$ be a compact Abelian group. Let $S$ be a closed set in $N$-dimensional Euclidean space and $A$ a closed subset of $S$. Let $m$ be a non-negative integer. Then there is defined the Čech homology group $H_m(S, A; G)$; if $A$ is empty this is written $H_m(S; G)$. Let $K$ be the kernel of the inclusion homomorphism $i_*: H_{m-1}(A; G) \to H_{m-1}(S; G)$. Let $L$ be any subgroup of $H_{m-1}(A; G)$. Then we say that $S$ is a surface of class $\mathcal{S}^G$ with boundary $\supset L$ if $K \supset L$. Moreover if $S$ is a surface in the above sense but there are no closed proper subsets of $S$ containing $A$ which are surfaces with boundary $\supset L$ then $S$ is said to be a proper surface.* It will be proved that every surface contains a proper surface.

We note in passing that when $A$ is an $m-1$ sphere, dim $(S) \leqq m$, and $G$ is the group of reals mod 1 this is equivalent to saying "$S$ is a surface with boundary $A$ iff $A$ is not a retract of $S$."

We take area to be the Hausdorff spherical measure $\Lambda^m S$.

MAIN THEOREM. *The minimum area of surfaces of class $\mathcal{S}^G$ with boundary $\supset L$ is attained and if $S$ is a proper surface of minimum area then $S$ will be locally Euclidean at all nonboundary points at which the lower density does not exceed one, that is at almost all nonboundary points.*

Moreover, when $m=2$, $N=3$ and $G$ is the group of integers mod 2 the minimal surface is locally Euclidean at *all* nonboundary points. If further the boundary $A$ is polygonal the surface is a manifold. The proofs (which are very long) run as follows:

Compactness is easy.

Lower semicontinuity in a suitable subsequence is proved by methods reminiscent of A. S. Besicovitch's work here. Local Euclideaness is proved via the following:

THEOREM. *If $S_0$ is a bounded set of points in $E_N$, and $P$ is a point of $S_0$ such that to each $R < R_0$ and each $X \in S_0 S(P, R_0)$ there corresponds a m-plane ${}_R\sum_X$ through $X$ such that*

(A) $$S_0 S(X, R) \subset ({}_R\textstyle\sum_X, \epsilon R) S(X, R)$$

*and*

(B) $$_R\sum_X S(X, R) \subset (S_0, \epsilon R) S(X, R)$$

*and* $\sum$ *is an m-plane through P such that*

(C) $$(\sum, \epsilon R_0) \supset S_0.$$

*Then if* $\epsilon \leqq 2^{-2000N^2}$ *there will exist a topological m-disk* $\bar{S}$ *such that*

$$S_0 S\left(P, \frac{1}{16} R_0\right) \subset \bar{S} \subset S_0 S(P, R_0).$$

Where $S(x, r)$ is a solid ball of centre $x$ and radius $r$ while $(y, \delta)$ is the set of points lying within $\delta$ of the set $y$.

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# A CHARACTERIZATION OF THE ALGEBRA OF ALL CONTINUOUS FUNCTIONS ON A COMPACT HAUSDORFF SPACE

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This note is a complement to [1]. We consider a commutative, semi-simple and self-adjoint Banach algebra $B$ and assume that $B$ has a unit element and is regular. By $\mathfrak{M}$ we denote the space of maximal ideals of $B$ and, applying the Gelfand representation, we consider $B$ as an algebra of continuous functions defined on $\mathfrak{M}$. It is obvious that if $B$ is $C(\mathfrak{M})$ (the algebra of all the continuous functions on $\mathfrak{M}$) the idempotents in any quotient algebra of $B$ are always bounded. We prove here that this property characterizes $C(\mathfrak{M})$ and give an application of this result.

LEMMA 1. *Suppose that there exist constants* $K$ *and* $K_1$, $K_1<1$ *such that to any real,* (*resp. non-negative*) *function* $f \in C(\mathfrak{M})$ *there exists an element* $f_1 \in B$ *such that* $\|f_1\| \leqq K \operatorname{Sup}_{M\in\mathfrak{M}} |f(M)|$, $f-f_1$ *is real* (*non-negative*) *and*

$$\operatorname{Sup}_{M\in\mathfrak{M}} |f(M) - f_1(M)| < K_1 \operatorname{Sup}_{M\in\mathfrak{M}} |f(M)|;$$

*then* $B = C(\mathfrak{M})$ *and for any* $f \in B \|f\| \leqq 4K(1-K_1)^{-1} \operatorname{Sup}_{M\in\mathfrak{M}} |f(M)|$.

PROOF. Define by induction $f_n = (f - \sum^{n-1} f_i)_1$; then $f = \sum_1^\infty f_n$.