BOOK REVIEWS

Homology theory on algebraic varieties. By Andrew H. Wallace, New York, Pergamon Press, 1958. 8+114 pp. $5.50.

Thirty-five years have passed since the appearance of Solomon Lefschetz's *L'analysis situs et la géométrie analytique*, Gauthier-Villars, Paris, 1924, a work of fundamental importance in the topological aspects of algebraic geometry. Intervening advances, notably the new concepts and methods which now play so great a role in homology theory, but which did not exist a generation ago, have made it important to review this work and to reestablish the results rigorously in contemporary terms. The author has rendered a valuable service to mathematical research workers in accomplishing this purpose.

In general, Mr. Wallace has followed a policy of preceding the somewhat intricate details of his proofs by intuitive geometric arguments. This device affords a useful guide, without which the motivation of many steps in the reasoning would be obscure on a first reading.

The work centers about three principal results in the earlier book by Lefschetz, all of which pertain to a nonsingular algebraic variety, V, of dimension r defined over the complex numbers, where V is immersed in L^n, the complex projective space of dimension n. These are complex dimensions, $2r$ and $2n$ being the real dimensions. Let V_0 be a non-singular hyperplane section of V; that is, the intersection of V with a linear space in L^n of dimension $n-1$, where V_0 has no singular points. The first main theorem is that all cycles of dimension $s<r$ on V are homologous to cycles on V_0. In terms of the relative homology of V modulo V_0, this means that $H_q(V, V_0)=0$ $(q<r)$.

The second important result relates to $H_r(V, V_0)$. The section V_0 is an element of a pencil π of hyperplane sections, the axis of π being of dimension $(n-2)$, such that only a finite number of the sections have singular points and each such has just one singular point. The theorem yields a set of generators for $H_r(V, V_0)$, one associated with each of the singular sections.

The third principal theorem is concerned with the Poincaré formula, which describes the variation of cycles of V_0 as V_0 varies in a pencil of sections.

Besides the three theorems of Lefschetz which motivate the book, there are numerous results and methods of intrinsic interest and importance.

The general viewpoint of the author's treatment is geometric as compared with the viewpoint of the transcendental theory of algebraic varieties.

STEWART S. CAIRNS

Trigonometric series. By A. Zygmund. 2d. ed., vols. I and II. New York, Cambridge University Press, 1959. 12+383 pp. and 7+354 pp. $15.00 each or $27.50 set.

In his course at the University of Cambridge, Professor Littlewood used to call the first edition of Zygmund's book "the Bible." This second edition, coming almost twenty-five years after the first one, will undoubtedly deserve this name even more, not only because it takes into account the work done in the field during this period, but also because the author, profiting from new experience and constant reflection on his past work, has introduced many topics which had been left aside in the first edition.

The book is dedicated to the memories of two polish mathematicians, A. Rajchman and J. Marcinkiewicz, who met both with a tragic end during the last world war: Rajchman was executed by the Nazis, while Marcinkiewicz died under circumstances not yet fully explained. The first one Zygmund calls "his teacher," the second one "his pupil," but both have considerably influenced the mathematical thought of Zygmund, who had an equal respect to the genius of these two mathematicians of the celebrated Polish school.

As the author states in the Preface, he has deliberately left aside all recent extensions of the theory to abstract fields and the second edition is, as the first one, devoted to the classical theory.

Chapter I contains the essential notions of analysis which shall be used throughout the book, such as: orthogonality, completeness and closure, Fourier-Stieltjes series, fundamental inequalities, convex functions, rearrangements and maximal theorems of Hardy and Littlewood. This is an improvement over the first edition, in which these fundamental notions were scattered through the book. It is only to be regretted, in the opinion of the reviewer, that the important theorems about abstract spaces and linear operations have been postponed until Chapter IV; they, like the other general theorems of analysis, belonged quite naturally to Chapter I, inasmuch as this chapter deals with convergence in the spaces L^r, with sets of first and second category, and with Baire's theorem.

Chapter II deals with the most elementary parts of the theory of