ing $r \to 1$" before the third display this passage to the limit does not seem to occur until the fourth display. Also, on a slightly different level, on page 122 in the asymptotic relation (6.6) the author has replaced the term $\log \beta_n$ by $-(1-\beta_n)$, yet when he applies the result on page 130 he has just to reverse this process. Finally on page 139 there is no discussion of the determination of $\arg f(z)/z$ in the inequality (6.21) and a similar condition obtains in several places afterwards.

As far as the choice of contents is concerned, in view of the obvious limitations, the reviewer regrets only that the author was not able to include some of his more recent results on asymptotic bounds for the coefficients. Also in chapter four he tacitly restricts the discussion of circular symmetrization to domains not containing the point at infinity which excludes the possibility of some applications to meromorphic functions.

The author makes no pretensions of biographical completeness. However there are several references he might well have added. One is to Faber's paper of 1920 in which a symmetrization method was first applied to the study of univalent functions. Another is to R. M. Robinson's paper of 1942 in which Löwner's method was extensively exploited.

JAMES A. JENKINS

An introduction to combinatorial analysis. By John Riordan. New York, Wiley, 1958. 10+244 pp. $8.50.

This is the first book on combinatorial analysis in forty-three years; the last was MacMahon's two-volume treatise in 1915–1916. It is, therefore, a very welcome arrival on the mathematical scene. Much of the material appears in book form for the first time. The emphasis throughout is on the methods of finding the number of ways in which a certain operation can be performed. Unsolved combinatorial problems are in abundance. For example it is not even known for $n=8$ how many distinct latin squares of order n there are, i.e., the number of distinct square matrices of order n containing the numbers from 1 to n in each row and column. Some mathematicians feel that combinatorial analysis is not a branch of mathematics but rather a collection of clever but unrelated tricks. This book successfully refutes that viewpoint.

The subject is, without doubt, one of the hardest in which to write an effective exposition. The reason for this is the fact that so much of the material occurs in an isolated fashion in so many different applications both to pure and applied mathematics and to other fields. Combinatorial analysis is a subject in which many of the fundamental results are frequently rediscovered by people in different fields, from first principles. The author has succeeded in organizing the subject matter in a coherent manner. The book will be useful both for a one semester course in combinatorial analysis and for handling practical problems involving enumeration of possibilities. It contains not only a review of classical material but also an exposition of recent developments, several of which are due to the author. The methods are of increasing importance, e.g., in statistics and communication theory. We now describe the contents.

Chapter 1 gives a concise review of standard formulas involving permutation and combinations.

Chapter 2 is devoted to the development of ordinary and exponential generating functions, and the relationship between them. As illustrations and applications of the methods, there appear solutions of linear recurrences, moment generating functions, and the derivatives of composite functions. Stirling numbers of the first and second kind are introduced and reappear frequently in the remainder of the book.

Chapter 3 describes the principle of inclusion and exclusion, together with some variations. This is applied to solve the "Problème des rencontres," which asks for the number of permutations of n distinct objects such that no object is in its own position. An illustration of a setting for this problem is given by the author: "suppose the pages of a manuscript are scattered, by the wind, a small child, or other demonic activity, and reassembled in a hurry."

Chapter 4 presents a clear exposition of the work of Touchard on the cycles of permutations. These include the number of even and odd permutations, and those with a given number of cycles and without unit cycles. The generating function which is most useful for these purposes is the cycle index of the symmetric group. This is a typical example of combinatorial results which have often been independently discovered.

In Chapter 5, we have an exhaustive description of the distribution of objects into cells. The number of ways of putting like or unlike objects into like or unlike cells is developed. Generating functions are again effectively used.

Chapter 6 is the longest chapter in the book. It includes the study of two essentially different subjects, namely, enumeration problems for partitions and for graphs. Very appropriately, the author writes, "the real development starts, like so much else in combinatoric, with Euler (1674)." The standard theorems of Euler on partitions are developed, e.g., "The partitions with unequal parts are clearly equinumerous with those with all parts odd." Partitions and conjugate partitions are studied. Compositions, or ordered partitions, are also mentioned briefly.

The second part of Chapter 6 studies some enumeration problems for graphs. Following Cayley, a formula is first obtained for the number of rooted trees with n points. A more convenient form of this result, due to Pólya, is also given. Pólya's powerful and elegant enumeration theorem is then stated for the first time in any book, but the author chooses to state it in a form which is one step before the conclusion as given by Pólya. He then derives the generating function for (unrooted) trees in terms of that for rooted trees, obtaining Otter's equation. In the most complete such tabulation known to the reviewer, the number of trees and rooted trees with n points for $n=1, 2, \cdots, 26$ is included. Further enumeration results which appear include series-parallel networks, linear graphs, and connected graphs with one cycle. This enumeration of series-parallel networks using Pólya's Theorem is a worthy improvement over previous formulations. The author properly includes some of his own investigations on the number of labeled colored and chromatic trees.

The last two chapters study permutations with restricted position. The problems are formulated in terms of configurations of nonattacking rooks on rectangular chessboards. Chapter 7 takes up the problem of the rooks and properties of rook polynomials. Chapter 8 continues the discussion with the study of the ménage problem and Latin rectangles. Some of the interesting methods are due to Riordan and Kaplansky. "Each of these topics has a growing end and the treatment of the text and its continuation in the problems merely serve to define the open regions; a striking example is that of Latin rectangles where the enumeration for more than three lines has scarcely been begun."

In Chapter 7, the author states as an open question to find the number of distinct types of problems concerning permutations on n objects with restricted position: "The exact number of distinct problems, for any n, is not known, but some progress in this direction will appear in this chapter." In another context, this question has been answered by the reviewer (*On the number of bicolored graphs*, Pacific J. Math. vol. 8 (1958) pp. 743–755) while enumerating bicolored graphs with the same number of points of each color.

In the opinion of the reviewer, Chapter 6 should have been split into two. This split is justifiable on both the grounds of length and subject matter. It would involve little additional work, because they are practically already split. For example, the first 13 problems deal with partitions and the last 24 with networks; similarly, the first six sections discuss partitions and the last six sections study networks.

There are 197 problems in the book. These serve to present many additional results and also to provide exercises for the reader to use the combinatorial techniques developed in the text. The problems are in general well constructed. The extensive references also enhance the value of the book. The book, dedicated to E. T. Bell, refers to polynomials named after Appell, Bell, Chebyshev and Legendre, as well as to numbers attributed to Bernoulli, Euler, Fibonacci, Lah, and Stirling. In view of the technical and often subtle nature of the subject matter, it is not always easy reading. However, the persevering reader will be rewarded, especially if he works out some problems. The book makes obsolete much of the previous written material on combinatorial analysis, and in general the reviewer would recommend that if one is searching for combinatorial information, he should consult this book before looking elsewhere.

The author has succeeded in gathering together a wealth of combinatorial results and methods. Bell Telephone Laboratories is to be congratulated on having provided him with the environment and facilities to make the preparation of such a book possible.

FRANK HARARY

Equazioni A derivate parziali. By F. G. Tricomi. Rome, Edizioni Cremonese, 1957. 12+392 pp. L. 5500.

This book gives an interesting introduction to the theory of partial differential equations. It is divided into four chapters which deal with the theory of characteristics (Chapter 1), hyperbolic equations (Chapter 2), elliptic equations (Chapter 3) and equations of the parabolic and mixed type (Chapter 4). In roughly four hundred pages of text the reader is provided with a good and up to date introduction to these topics. It is true that other aspects of these topics might have been considered, but for this level the author is entitled to his tastes. This text provides a healthy balance between the mathematical methods required for the solution of certain partial differential equations and the underlying theory. Numerous problems from classical theoretical physics are discussed and there is an interesting set of problems at the end of each chapter. The written style is superb.

A. E. HEINS