BOOK REVIEWS

Polynomial expansions of analytic functions. By Ralph P. Boas, Jr. and R. Creighton Buck. Ergebnisse der Mathematik und ihrer Grenzgebiete, New series, No. 19. Berlin, Springer, 1958, 8+77 pp. Paperbound, $4.75.

This is an excellent little book written with a commendable attention to detail. The authors take pains to indicate frequently what is going on behind the scenes and they include numerous pertinent and useful illustrations of the theory. The book contains much new material in addition to an organized treatment of known phenomena.

The work is motivated by a desire to examine in some detail the fact that certain polynomial sets yield nontrivial representations of zero. Boas and Buck find that, for a specified polynomial set, (a) the nonexistence of such representations of zero, or (b) the existence and enumeration of distinct representations of zero can be deduced from properties of generating functions of certain forms. A very few misprints were noted, none of them likely to disturb the reader.

Chapter I (about 20 pages) contains motivation of the study and some underlying function theoretic results. A set of pseudo Laguerre polynomials (superscript dependent upon the subscript) is used in an illustration. A fairly general class of generating relations previously studied by Boas and Buck is introduced and the corresponding polynomials are given the name generalized Appell polynomials. This class of polynomials is to play a major role in the succeeding chapters. On page 18 two problems, which have long confronted workers in the study of generating relations for polynomial sets, are pointed out to the reader.

Chapter II (about 27 pages) is a study of the representation of entire functions by series of generalized Appell polynomials. There are determinations of the existence of convergent, Mittag-Leffler summable, or Borel summable expansions in such series. Numerous useful illustrations are given. These include polynomials associated with names Bernoulli, Euler, pseudo Laguerre, reversed proper Laguerre (superscript independent of the subscript), Hermite, and a rearranged form of Legendre polynomials. Additional application of much interest is made to the Sheffer polynomials (Sheffer's type zero classification) which include proper Laguerre polynomials, those of Angelescu, Mittag-Leffler and Newton, the actuarial polynomials and some others associated with interpolation problems. Boas and

Buck then treat Sister Celine's polynomials including Bateman's Z_n and others as special cases. The authors also treat what they term Bessel polynomials, which are generalizations of one of Rainville's generalizations of what Krall and Frink called generalized Bessel polynomials. A few other polynomial sets are also considered in this chapter.

Chapter III (about 19 pages) is a study of the representation of functions regular at the origin. Again the Boas and Buck generalized Appell polynomials play a major role. Applications are made to Jacobi, Gegenbauer, Humbert, Lerch, Faber, and Sheffer polynomials among others.

Chapter IV (about 5 pages) gives applications to uniqueness theorems and certain functional equations. The book closes with an extensive bibliography and an index.

This book should be of interest to students of function theory. It certainly belongs in the library of everyone working with special functions.

EARL D. RAINVILLE

Eigenfunction expansions associated with second order differential equations, Part 2. E. C. Titchmarsh. Oxford University Press, 1958. 11+404 pp. $11.20.

One of the major difficulties facing an author of a book on eigenfunction expansions associated with partial differential equations is the decision as to which topics to discuss and at what level of generality to place the discussion. How general are the differential equations and domains to be considered? Should "expansion" mean "expansion in L^2" or something less or more general? Are the methods employed to be those of strictly classical analysis or those of abstract operator theory or a mixture of the two?

Professor Titchmarsh had faced similar, but perhaps simpler, versions of these questions when in 1946 he wrote Part 1 dealing with the ordinary differential equation $\{d^2/dx^2+\lambda-q(x)\}\psi(x)=0$. In the present volume, he stands by the decisions he made then and his object here is to treat the partial differential equation $\{\Delta+\lambda-q(x)\}\psi(x)=0$ on the entire x-space in a similar fashion, that is, by methods of classical analysis. The decision to deal only with the fundamental case of constant coefficients is probably a wise one. The complete avoidance of standard results and even the language of operator theory in Hilbert space leads a reader (particularly, an inexperienced reader, who may know the elements of operator theory) to lose a great deal of perspective.