The remainder of the book consists of "optional" topics, the major ones being change of variable in an integral, signed and complex measures, conjugate spaces and weak convergence, Fourier transforms, and ergodic theory.

The book is well supplied with exercises and should make quite a satisfactory text. Its brevity comes not from terseness but from judicious editing. I would guess that students will find it quite readable and that professors will object because their pet theorems are missing.

I have three specific criticisms.

(i) Zaanen defines $I(f)$—the integral of f—as $I(f^+)+I(f^-)$. He calls f integrable if this exists and summable if it is finite. I object mildly to this terminology, but my real criticism is that he has listed $\infty - \infty = 0$ as a postulate for the extended real number system. So, I do not think he means what he says about integrability.

(ii) He does not define the word function and seems to use f and $f(x)$ interchangeably. In certain transfinite proofs he refers quite properly to a "union of functions" with a parenthetic explanation that will probably be meaningless to the student who has never seen a function defined as a set of ordered pairs.

(iii) He introduces the metric space of measurable sets with the measure of the symmetric difference as a distance function but so far as I can discover he does not prove it complete or even discuss the question of completeness. Yet, on p. 183 he applies the Baire category theorem to this space deriving the usual inference, and this requires completeness.

M. E. MUNROE

Introduction to difference equations. By Samuel Goldberg. New York, Wiley, 1958. 12+260 pp. $6.75.

This extremely well-written book is "primarily intended for social scientists who wish to understand the basic ideas and techniques involved in setting up and solving difference equations." In the introduction, examples are given of problems which give rise to difference equations—a learning model and the study of the variations in national income. Throughout, numerous exercises are chosen from economics, psychology, and sociology with references to the literature of these subjects. Discussions are devoted to economic dynamics, inventory analysis, learning models, communication theory, and income variation.

On the mathematical side, the author gives an extremely lucid and careful explanation of the techniques of solving linear difference equa-

tions with constant coefficients. Illustrative examples are worked out in meticulous detail and the theory is developed from simple beginnings. Goldberg presupposes very little in the way of mathematical background, explaining as he goes along the function concept, mathematical induction, the binomial formula, the summation notation, limits, convergence and divergence, determinants, and de Moivre's Theorem. Knowledge of the calculus or infinite series is presupposed only in four starred sections which may be omitted without affecting the remainder of the text. In spite of the slight mathematical knowledge necessary to read this text, the highest standards of logical clarity are maintained in a manner which is calculated to build up the reader's mathematical maturity in a very skillful way.

The first two chapters treat respectively the calculus of finite differences, and an introduction to difference equations centering about first order linear equations. Chapter 3 deals with linear difference equations with constant coefficients. Chapter 4, Selected Topics, includes a study of equilibrium values of difference equations, and their stability, and introductions to boundary-value problems, generating functions and matrix methods. The section on generating functions (starred) assumes a modest knowledge of infinite series. It includes applications to probability theory (binomial distribution). The author devotes 38 pages to matrix methods. No previous knowledge of matrices is assumed. The interesting discussion of Markov chains is motivated by setting up difference equations for neural networks, panel surveys and Estes' statistical theory of spontaneous recovery and regression in connection with learning. After developing techniques of solution, the author makes further applications to foreign trade models and to the stability of societies with certain laws of marriage and descent.

Because of the wealth of illustrative material and the clarity of the exposition, this book should be a great boon to students of the social sciences with modest mathematical equipment. The book should also be useful to students of mathematics who will encounter many important ideas (e.g., boundary value problems, fundamental set of solutions) in a form free of the technical difficulties with which they are usually associated. The text will also help cultivate habits of clear thinking so often claimed for courses in mathematics.

<div style="text-align: right">DONALD E. RICHMOND</div>