the periodic solution with R replaced by R_0 and for t ranging over a single period has the error estimate

$$O(\epsilon) + O\left(\frac{\epsilon^2}{\delta r}\right) + O\left(\left|\frac{(\delta r)^3}{\epsilon}\right|^{1/2}\right),$$

which is only meaningful if $\epsilon \to 0$ with $\delta r \to 0$ in such a way that $\epsilon/(|\delta r|)^{1/2} \to 0$ and $|\epsilon|/|\delta r|^3 \to +\infty$, to be replaceable by

$$O(\delta r)$$

as $\delta r \to 0$ for fixed $\epsilon \neq 0$.

F. H. BROWNELL

Theorie der Limitierungsverfahren. By K. Zeller. Ergebnisse der Mathematik und ihrer Grenzgebiete, New Series, vol. 15. Berlin, Gottingen, Heidelberg, Springer, 1958. 8+242 pp. DM 36.80.

This book is intended as a survey of the literature of the topic. To this end the bibliography contains over 2000 items! However the author has achieved far more than a survey, for there is running through the pages a hard core of self-contained material lying at the heart of the subject. A consistent notation and terminology are used and the contents of the bibliography are exposed by means of them. Thus it is possible to learn from the book itself without consulting the references, although at times proofs are compressed to a density rivalling that of a white dwarf. In spite of this the author has taken great pains to isolate the fundamental ideas (e.g. gleitende Buckel, Mittelwertsatz) behind the works of the giants of classical analysis such as G. H. Hardy. In addition much space is given to the important and extensive contributions of functional analysis (described somewhat condescendingly by Hardy as soft analysis).

After an interesting sketch of pre-twentieth century ideas—laced with metaphysics—on divergence, there is a section on definitions of limit. The book treats mainly those given by matrices. Multiple sequences, nonmatrix methods: strong and absolute summability, integral transforms, are dismissed with references. Section 5 lists the main problems.

In Chapter 2 the discussion proper begins. Descriptions of Banach and F spaces and their duals are given. For example proofs of the Baire category and Hahn-Banach theorems are given as well as three types of proof of the uniform boundedness principle. The FK space then appears; this highly useful object is an F sequence space with continuous coordinates. Banach algebras and Fourier transforms are then defined and a proof of Wiener's approximation theorem occupies just over a page.

Chapter 3 begins the study of summability. The author chooses to reverse the historical order and begin with structure theorems for the Wirkfeld (=convergence domain=set of sequences transformed into convergent sequences), and Anwendungsbereich (=existence domain =set of sequences whose transforms exist) of a matrix. These results draw richly upon functional analysis. It is in the area of this chapter that the author's own greatest contributions have been made.

These domains are FK spaces with known dual. In some, (perfect), c is dense; or, (AK), $\{\delta^K\}$ is a basis; or, (AD), $\{\delta^K\}$ is fundamental. Such topological properties reflect themselves in summability properties such as: the matrix sums a bounded divergent sequence. As another example, a complete space is not the union of an expanding sequence of subspaces if the assumptions imply that each subspace is of first category; hence no matrix is equivalent to the "union" of a sequence of matrices of increasing strength.

In this chapter is told the interesting story of the Mazur-Orlicz articles of 1933 and 1955.

Chapters 4 and 5 deal with direct and Tauberian theorems. Here at last the famous Silverman-Toeplitz conditions appear; this late because the structure theory did not depend on the considerations: permanent (=regular), konvergenztreu (=conservative), konvergenzerzeugend (=coercive). In these chapters also much use is made of functional analysis.

The deep Tauberian results of Littlewood, Wiener, et al. are discussed with indication of proof. The discussion continues into boundary value and other problems in function theory (Phragmén-Lindelöf, Vitali).

Finally the older theory appears in the last 3 chapters: inclusion relations among Cesàro, Abel, etc. methods, Borel, Euler and so on. Except for the functional analysis settings and numerous references, details of these three chapters may be found in Hardy's "Divergent series." Most of the earlier material appears for the first time in book form.

The reviewer feels that FK spaces will join the mainstream of mathematics as a topic known to all educated mathematicians.

The author follows the laudable practice of giving a reference to a review of each item in the bibliography.

An English index would be most helpful.

The author has performed a monumental task with great dignity.

ALBERT WILANSKY