THE APRIL MEETING IN CHICAGO

The five hundred and thirty-fourth meeting of the American Mathematical Society was held at the University of Chicago on Friday and Saturday, April 19–20, 1957. Sessions began at 10:00 A.M. on Friday and the Meeting concluded with a session on analysis and two special sessions for late papers at 10:00 A.M. on Saturday. There was a total of 238 registrations. Among these were 205 members of the Society.

The Committee to Select Hour Speakers for Western Sectional Meetings had invited Professor René Thom of the University of Chicago and the University of Strasbourg to address the Society. Professor Thom spoke on the topic *Topology of differentiable function spaces*. Professor E. H. Spanier presided at the session.

The Council of the Society met at 5 P.M. on Friday, April 19.

The Secretary announced the election of the following two hundred seventy-three persons to ordinary membership in the Society:

Mr. S. O. Albert, United States Army;
Mr. W. O. Alexander, Jr., University of Corpus Christi;
Mr. J. H. Ames, Bankers Life Insurance Company of Nebraska;
Mr. T. L. Anderson, Federal Life Insurance Company, Chicago, Illinois;
Mr. A. L. Armstead, National Advisory Committee on Aeronautics, Cleveland, Ohio;
Mr. G. L. Armstrong, Convair, San Diego, California;
Miss Thelma I. Arnette, Oak Ridge National Laboratory;
Mr. J. R. Ashcraft, Remington Rand UNIVAC Corp., St. Paul, Minnesota;
Mr. Samuel Asofsky, National Jewish Welfare Board, New York, New York;
Mrs. Joy H. Atkin, Bell Telephone Laboratories, Inc., Murray Hill, N. J.;
Mr. R. W. Bains, United Gas Research Laboratory, Shreveport, Louisiana;
Dr. J. G. Baker, Harvard College Observatory;
Mr. Edward Barber, Johns Hopkins University;
Mr. C. W. Barnes, University of North Carolina;
Mr. Alexander Basil, Airborne Instruments Laboratory, Mineola, N. Y.;
Dr. Frances B. Bauer, Reeves Instrument Corp., New York, N. Y.;
Mr. L. D. Baumert, Douglas Aircraft Company, Long Beach, California;
Dr. V. E. Benes, Bell Telephone Laboratories, Inc., Murray Hill, New Jersey;
Mr. M. C. Benson, Radio Corporation of America, Camden, New Jersey;
Professor W. H. Benson, Dickinson College;
Mr. R. D. Berlin, General Electric Co., Syracuse, N. Y.;
M. A. Bershad, U. S. Bureau of the Census, Washington, D. C.;
Miss Barbara B. Betts, D. C. Heath and Company, Boston, Mass.;
Mr. F. T. Birtel, USN Nuclear Power School, New London, Conn.;
Mr. D. L. Blackhurst, Mellon National Bank & Trust Co., Pittsburgh, Pennsylvania;
Professor C. R. Blyth, University of Illinois;
Dr. B. P. Bogert, Bell Telephone Laboratories, Inc., Murray Hill, N. J.;
Dr. D. M. Boodman, Operations Evaluation Group, Massachusetts Institute of Technology, Navy Dept., Washington, D. C.;

Mr. J. A. Bond, Texas Technological College;

Mrs. Janez Y. Bordeaux, Ramo-Wooldridge Corporation, Los Angeles, California;

Mr. T. A. Bordeaux, G. O. Noville and Associates, Santa Monica, California;

Mr. H. A. Bott, Minneapolis-Honeywell Regulator Co., Morton Grove, Ill.;

Mr. R. L. Boyell, Ramo-Wooldridge Corporation, Los Angeles, California;

Mr. S. R. Boyle, Hughes Aircraft Company, Culver City, California;

Professor K. H. Bracewell, Hamline University;

Mr. T. F. Bridgland, Jr., University of Florida;

Mr. R. O. Brooks, Northeastern University;

Mr. Andrew Browder, Massachusetts Institute of Technology;

Mr. W. H. Burgin, Jr., Dartmouth College;

Mr. J. F. Burke, Rochester, New York;

Professor Emeritus G. P. Bush, American University;

Mr. T. W. Chellis, North American Aviation, Bellflower, California;

Miss Ermine A. Christian, Naval Ordnance Laboratory, Silver Spring, Maryland;

Mr. Chia-kun Chu, Institute of Mathematical Sciences, New York University;

Mr. C. J. Cillay, Alamo, Texas;

Mr. M. L. Clinnick, University of California Radiation Laboratory, Livermore, California;

Mr. R. G. Clow, University of California, Berkeley;

Professor R. W. Clower, State College of Washington;

Mr. J. E. Coachman, Prudential Insurance Company of America, Newark, New Jersey;

Col. H. M. Cochran, U. S. A., Retired, Atherton, California;

Mr. W. J. Cody, Jr., University of Oklahoma;

Mr. E. L. Cohen, Harvard University;

Mr. R. C. Collmer, Lincoln Laboratory, Massachusetts Institute of Technology, Lexington, Massachusetts;

Dr. J. M. Cook, Argonne National Laboratory, Lemont, Illinois;

Miss Doris F. Corley, Los Alamos Scientific Laboratory, University of California;

Mr. W. A. Cowan, University of Adelaide;

Mr. C. W. Cox, American Bosch Arma Corp., Garden City, N. Y.;

Mr. J. R. Cox, Robert W. Lowry, Inc., Harrisburg, Pennsylvania;

Mr. G. E. Crane, Westinghouse Electric Corp., Pittsburgh, Pennsylvania;

Mr. W. C. Crehl, Holloman Air Development Center, Holloman Air Force Base, New Mexico;

Mr. D. F. Criley, International Business Machines Corp., Glendale, California;

Mr. W. S. Currie, The Texas Company, Bellaire, Texas;

Mr. Vassili Daiev, Sea Cliff, New York;

Dr. R. H. Davis, Bell Telephone Laboratories, Inc., New York, New York;

Mr. J. D. Dickinson, Chance Vaught Aircraft, Incorporated, Dallas, Texas;

Mr. R. E. Dowd, Grumman Aircraft Engineering Corp., Bethpage, New York;

Professor B. S. Dreben, Harvard University;

Mr. C. D. Dunn, Convair, Fort Worth, Texas;

Mr. E. S. Eby, University of Illinois;

Miss Ruth B. Eddy, D. C. Heath and Company, Boston, Massachusetts;

Mr. E. A. Enrione, University of Miami, Miami, Florida;

Mr. H. M. Farkas, Brown University;

Mr. M. H. Farrant, Coates, Herfurth & England, San Francisco, California;

Mr. C. T. Fike, University of North Carolina;

Dr. G. B. Findley, U. S. Navy Mine Defense Laboratory, Panama City, Florida;
Mr. R. J. Fiore, Bethlehem Steel Company, Lackawanna, New York;
Professor F. B. Fitch, Yale University;
Mr. H. E. Flesner, North American Aviation, Inc., Downey, California;
Mr. J. F. Foley, Baltimore Gas and Electric Company, Baltimore, Md.;
Mr. J. M. Frank, Convair, Pomona, California;
1st Lt. D. A. Franks, United States Army;
Mr. J. R. Franks, U. S. Naval Air Missile Test Center, Point Mugu, California;
Mr. F. J. Gagliuso, Paul Revere Life Insurance Co., Worcester, Massachusetts;
Mr. K. S. Gale, Armour Research Foundation, Chicago, Illinois;
Miss Rheba E. Galloway, Arlington County Public Schools, Arlington, Virginia;
Dr. J. L. Gammel, Los Alamos Scientific Laboratory, Los Alamos, New Mexico;
Professor S. E. Ganis, Ohio Wesleyan University;
Dr. J. J. Gilvarry, Allis-Chalmers Co., Milwaukee, Wisconsin;
Mr. W. G. Goeller, New York Telephone Co., New York, New York;
Mr. Barry Gordon, Equitable Life Assurance Society, New York, New York;
Dr. M. J. Gottlieb, Market Facts, Inc., Chicago, Illinois;
Mr. H. W. Gould, University of Virginia;
Mr. W. H. Gregory, Sacramento Junior College;
Mr. E. D. P. Gross, Jr., The Underwood Corp., New York, New York;
Mr. Nathaniel Grossman, California Institute of Technology;
Dr. A. A. Groth, A. S. Hansen Consulting Actuaries, Chicago, Illinois;
Mr. J. S. Guastavino, Remington Rand Div., Sperry Rand Corp., New York, New
    York;
Mr. C. A. Haase, Nelson and Warren, Kansas City, Missouri;
Reverend M. A. Hanhauser, Siena College;
Mr. F. L. Hardy, Emory University;
Professor M. L. Harris, Philander Smith College;
Professor C. H. Heinke, Capital University;
Mr. J. J. Henrick, Convair, Ft. Worth, Texas;
Mr. Bert Henry, NACA, Lewis Flight Propulsion Laboratory, Cleveland, Ohio;
Professor A. F. Herbst, La Verne College;
Mr. P. R. Hildebrandt, RAND Corporation, Lexington, Massachusetts;
Mr. J. V. Holberton, U. S. Naval Bureau of Ships, Washington, D. C.;
Mrs. Dorothy B. Horning, Standard Oil Company, Cleveland, Ohio;
Mr. R. R. Hudson, International Business Machines Corp., Oklahoma City, Okla-
    homa;
Dr. J. L. Hult, RAND Corporation, Santa Monica, California;
Mr. R. M. Humphrey, Presto Recording Corporation, Paramus, New Jersey;
Miss Martha A. Jennison, U. S. Naval Proving Ground, Dahlgren, Virginia;
Dr. M. V. Johns, Jr., Stanford University;
Mr. M. S. Johnson, Melpar, Incorporated, Falls Church, Virginia;
Dr. E. E. Jones, Minneapolis-Honeywell Research Center, Hopkins, Minn.;
Mr. W. D. Kahn, U. S. Army Map Service, Washington, D. C.;
Dr. Jerome Karle, Naval Research Laboratory, Washington, D. C.;
Mr. Robert Katz, Tufts University;
Mr. Ellwood Kauffman, Electronic Associates, Inc., Princeton, New Jersey;
Mr. B. S. Kawar, Remington Rand UNIVAC, St. Paul, Minnesota;
Mr. R. C. Keating, Arthur Stedry Hansen Consulting Actuaries, Lake Bluff, Illinois;
Lt. Col. L. N. King, U. S. Marine Corps, Washington, D. C.;

Mr. E. L. Kirkpatrick Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland;

Mr. T. H. Kirkpatrick, Paul Revere Life Insurance Company, Worcester, Massachusetts;

Mr. B. S. Kleinman, Columbia University;

Mrs. Mary E. Knight, U. S. Naval Aviation Supply Office, Philadelphia, Pennsylvania;

Dr. E. C. Koenig, Allis-Chalmers Manufacturing Company, Milwaukee, Wisconsin;

Mr. H. W. Kreiner, Operations Evaluation Group, Massachusetts Institute of Technology, Alexandria, Virginia;

Mr. A. M. Kunis, Mount Vernon Life Insurance Company of New York, Mount Vernon, New York;

Mr. Ali Kyrala, Lessells and Associates, Inc., Boston, Massachusetts;

Mr. J. B. Lackey, United States Army;

Assistant Professor Emeritus Nellie T. Landblom, Colorado Agricultural and Mechanical College;

Mr. Harold Levenstein, The W. L. Maxson Corp., New York, New York;

Mr. D. M. Levy, S. Levy and Sons, Suffolk, Virginia;

Mr. J. E. Levy, Washington Engineering Services Company, Bethesda, Maryland;

Mr. Victor Lewicke, U. S. Army Map Service, Washington, D. C.;

Mr. T. L. Libby, Wright Air Development Center, Wright-Patterson Air Force Base, Ohio;

Mr. O. A. de Lima, Roger Smith Hotels Corporation, New York, New York;

Mr. G. E. Lindquist, United Air Lines, Chicago, Illinois;

Mr. Leo Liolios, Bastian-Blessing Company, Chicago, Illinois;

Professor J. M. Long, College of William and Mary, Norfolk, Virginia;

Mr. W. M. Lowney, University of Notre Dame;

Lt. R. E. Lynch, U. S. Air Force, Wright Patterson Air Force Base, Dayton, Ohio;

Mr. J. F. MacLean, Bankers Life Insurance Company of Nebraska, Lincoln, Nebraska;

Captain A. T. Magnell, United States Navy;

Professor J. G. Marica, Humboldt State College;

Dr. Sidney Marks, General Electric Company, Richland, Washington;

Dr. E. L. Marshall, Lafayette Life Insurance Company, Lafayette, Indiana;

Mr. N. F. G. Martin, Iowa State College;

Mr. Johann Martinek, Reed Research Incorporated, Washington, D. C.;

Mr. R. M. Mason, Naval Research Laboratory, Washington, D. C.;

Mr. W. C. Mason, Lincoln Laboratory, Massachusetts Institute of Technology, Lexington, Massachusetts;

Mr. M. D. McCord, University of Tennessee;

Mr. J. S. McCoy, Metropolitan Life Insurance Company, New York, New York;

Mr. R. E. McCrocklin, Terre Haute, Indiana;

Mr. D. C. McCune, Jones and Laughlin Steel Corporation, Pittsburgh, Pennsylvania;

Mr. R. B. McNab, Railroad Retirement Board, Chicago, Illinois;

Mr. J. C. McPherson, International Business Machines Corporation, New York, New York;

Dr. V. V. McRae, Johns Hopkins University;

Mr. E. A. Means, Means Laboratories, Incorporated, Wichita, Kansas;

Mr. Harold Mechanic, Nuclear Development Corporation, White Plains, New York;

Mr. R. L. Meyer, Jr., UARCO, Incorporated, Chicago, Illinois;

Mr. J. R. Miller, Ramseyer & Miller, Incorporated, New York, New York;

Mr. R. A. Moore, Northrop Aircraft, Incorporated, Hawthorne, California;

Mr. R. E. Moore, Missile Systems Div., Lockheed Aircraft Corp., Palo Alto, California;

Mr. C. S. Morell, American Machine & Foundry Company, Boston, Massachusetts;

Mr. H. D. Morgan, Eugene M. Klein and Associates, Cleveland, Ohio;

Mr. W. S. Morris, Talmage and Company, New York, New York;

Professor D. E. Moser, University of Massachusetts;

Mr. R. A. Mugele, Shell Development Company, Emeryville, California;

Mr. J. R. Muller, Sperry Rand Corporation, Long Island, New York;

Mr. A. A. Mullin, Massachusetts Institute of Technology;

Mr. A. E. Murray, Bausch and Lomb Optical Company, Rochester, New York;

Professor Peter Musen, University of Cincinnati;

Mr. G. F. Nardin, Jr., Armco Steel Corporation, Middletown, Ohio;

Mr. J. O. Neilson, RAND Corporation, Lexington, Massachusetts;

Professor R. J. Nelson, Case Institute of Technology;

Dr. Anil Nerode, University of Chicago;

Mr. P. E. Ney, Columbia University;

Professor E. D. Nichols, Florida State University;

Mr. L. F. Nichols, Picatinny Arsenal, Dover, New Jersey;

Mr. R. J. O'Keefe, Arthur D. Little, Inc., Cambridge, Massachusetts;

Dr. R. D. O'Neal, Bendix Aviation Corporation, Ann Arbor, Michigan;

Dr. R. K. Osborn, Oak Ridge National Laboratory;

Mr. W. L. Ostrowski, American Chemical Society, Washington, D. C.;

Mr. R. W. Paul, Jr., University of California, Radiation Laboratory, Livermore, California;

Mr. C. R. Paulson, ERCO Division, ACF Industries, Incorporated, Riverdale, Maryland;

Mr. Sidney Penner, Illinois Institute of Technology;

Mr. F. S. Perryman, Royal Insurance Company, Ltd., New York, New York;

Mr. B. I. Pesin, Los Angeles City Schools, North Hollywood, California;

Mr. C. F. Pinzka, Xavier University;

Mr. M. F. Pollack, Stockton, California;

Mr. K. F. Powell, International Business Machines Corp., Cleveland, Ohio;

Professor Carlo Pucci, University of Maryland;

Professor E. K. Rabe, University of Cincinnati Observatory;

Mr. George Rabinowitz, New York University;

Dr. J. H. Ramser, Atlantic Refining Company, Philadelphia, Pennsylvania;

Dr. R. B. Randels, Corning Glass Works, Corning, New York;

Professor Philburn Ratoosh, Ohio State University;

Mr. G. A. Reilly, DATA Processing, San Antonio, Texas;

Mrs. Dorothea J. Rhea, Bryn Mawr College;

Pvt. R. A. Rieger, United States Army, Fort Gordon, Georgia;

Dr. G. M. Roe, General Electric Research Laboratory, Schenectady, New York;

Mr. Azriel Rosenfeld, Ford Instrument Company, New York, New York;

Mr. Frank Rosett, Vickers, Incorporated, Detroit, Michigan;

Mr. D. J. Ross, Operations Research, Incorporated, Silver Spring, Maryland;

Mr. David Rothman, Rocketdyne, North American Aviation, Canoga Park, California;

Mr. R. E. Rowley, General Electric Company, Richland, Washington;

Reverend B. M. Russell, Gannon College;

Mr. A. A. Sagle, University of Washington;

Mr. Daihachiro Sato, University of California, Los Angeles;

Mr. Murray Sawits, Teleregister Corporation, Stamford, Connecticut;

Lt. Col. C. J. Schauers, 6th Army Signal Section, San Francisco, California;

Mr. I. S. Schechtman, Philco Corporation, Philadelphia, Pennsylvania;

Mr. E. D. Schell, Remington Rand UNIVAC Corporation, New York, New York;

Mr. H. E. Schnur, Texas Company, Houston, Texas;

Mr. R. L. Schwaller, Marquette University;

Mr. A. A. Schwartz, U. S. Railroad Retirement Board, New York, New York;

Mr. S. J. Scott, Vitro Corporation of America, Eglin Air Force Base, Florida;

Mr. J. F. Scott-Thomas, University of Toronto;

Mr. A. N. Seares, Remington Rand, New York, New York;

Professor C. F. Sebesta, Duquesne University;

Mr. L. A. Segel, Massachusetts Institute of Technology;

Mr. H. Manvel Semarne, Douglas Aircraft Company, Inc., Santa Monica, California;

Mr. Robert Serrell, Radio Corporation of America, RCA Laboratories, Princeton, New Jersey;

Mr. Harry Sherman, Reeves Instrument Corp., New York, New York;

Mr. T. R. Sherrow, Sinclair Oil and Gas Company, Tulsa, Oklahoma;

Dr. R. L. Shuey, General Electric Company, Schenectady, New York;

Mr. R. W. Sielaff, Northern Trust Company, Chicago, Illinois;

Mr. R. L. Sisson, Canning, Sisson and Associates, Los Angeles, California;

Mr. T. D. Sloan, New York Life Insurance Company, New York, New York;

Mr. H. L. Slonecker, Jr., North American Aviation, Incorporated, Columbus, Ohio;

Mr. C. A. Smith, City Schools, St. Louis, Missouri;

Miss Jean F. Smolak, E. R. Squibb and Sons, New Brunswick, New Jersey;

Dr. A. P. Speiser, International Business Machines Research Laboratory, Adliswi-Zurich, Switzerland;

Mr. R. F. Steinhart, International Business Machines Corporation, Newark, New Jersey;

Miss Fern G. Stenwick, David Taylor Model Basin, Carderock, Maryland;

Mr. W. F. Stephan, International Business Machines Corporation, Kingston, New York;

Mr. Erick Stern, ARMA Corporation, Garden City, New York;

Mr. A. I. Sternhell, Metropolitan Life Insurance Co., New York, New York;

Dr. S. A. Stone, Bradford Durfee Technical Institute;

Mr. A. J. Strecok, Argonne National Laboratory, Lemont, Illinois;

Mr. D. A. Swick, U. S. Naval Research Laboratory, Washington, D. C.;

Professor W. B. Swift, University of Wisconsin;

Mr. Irvin Tarnove, Ramo-Wooldridge Corporation, Los Angeles, California;

Mr. J. R. Teller, Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland;

Miss Eileen J. Theisen, North American Aviation, Incorporated, Canoga Park, California;

Mr. A. T. Thomas, Brown-Forman Distillers Corporation, Louisville, Kentucky;

Mr. J. D. Thomas, University of Oklahoma;

Mr. R. E. Thomas, North American Aviation, Inc., Columbus, Ohio;

Mr. D. G. Thoroman, International Business Machines Corporation, Dayton, Ohio;

Mr. J. K. Thurber, New York University;

Mr. D. B. J. Tomiuk, Catholic University of America;

Professor Cosimo Torre, Syracuse University;

Mr. A. L. Tritter, Lincoln Laboratory, Massachusetts Institute of Technology;

Mr. R. E. Tuck, Hughes Aircraft Company, Los Angeles, California;

Dr. Jaroslav Tuzar, Salerno-Megowen Biscuit Company, Chicago, Illinois;

Mr. S. A. Tyler, Argonne National Laboratory, Lemont, Illinois;

Mr. J. B. Tysver, University of Michigan;

Miss Theresa D. Vasques, Convair, San Diego, California;

Mr. J. B. Vieaux, Agricultural and Mechanical College of Texas;

Mr. R. M. Vogt, University of Illinois;

Mr. F. J. Wall, Sperry Rand Corporation, St. Paul, Minnesota;

Mr. R. C. Warth, Bendix Aviation Corporation, Eatontown, New Jersey;

Mr. Frederick Way, III, Case Institute of Technology;

Mr. M. R. Weinstein, Massachusetts Institute of Technology;

Mr. Burton Wendroff, Los Alamos Scientific Laboratory, University of California;

Mr. O. J. White, Ford Motor Company, Dearborn, Michigan;

Professor F. L. Wolf, Carleton College;

Mr. A. W. Yonda, Temple University;

Mr. P.A. Zaphyr, Westinghouse Electric Corporation, East Pittsburgh, Pennsylvania;

Mr. E. C. Zeiger, Guardian Life Insurance Company of America, New York, New York;

Mr. L. W. A. Zolldan, Jr., U. S. Naval Ordnance Laboratory, White Oak, Maryland.

It was reported that the following forty-six persons had been elected to membership on nomination of institutional members as indicated:

University of California, Los Angeles: Mr. R. J. Mercer, Mr. F. C. Reed, Mr. E. M. Scheuer.

University of Chicago: Mr. R. H. Szczarba.

Illinois Institute of Technology: Mr. R. C. Bzoch.

Johns Hopkins University: Mr. P. M. Anselone, Mr. Paul Berliner, Mr. J. H. De Boer, Mr. J. P. Falvey, Mrs. Nancy J. Hagelgans, Professor J. R. Hammond, Mr. J. T. Hundley, Mr. D. L. McQuillan, Mr. Richard Molloy, Professor N. O. Niles, Miss Marilyn J. Pettit, Mr. R. C. Sacksteder, Mr. B. C. Stebbings, Mr. C. E. Thompson, Mr. D. R. Tilley.

University of Miami: Mr. N. W. Hill, Jr., Miss Mabel A. Pauley.

Michigan State University: Mr. D. G. Johnson, Mr. J. R. Smart, Mr. W. W. Turner.

New York University: Mrs. Tilla S. Klotz.

Princeton University: Mr. A. M. Adelberg, Mr. M. L. Balinski, Mr. P. X. Gallagher, Mr. M. D. George, Mr. S. C. H. Gitler, Mr. R. A. Kurtz, Mr. James McKenna, Mr. B. C. Mazur, Mr. C. St. J. A. Nash-Williams, Mr. Kenichi Shiraiwa, Mr. R. M. Smullyan, Mr. J. R. Stallings, Mr. J. D. Stasheff, Mr. Nobuo Yoneda, Mr. R. O. Winder, Mr. Martin Zerner.

Stanford University: Mr. W. J. Buckingham, Mr. J. R. Hatcher, Dr. Eva M. Wirth.

Washington University: Mr. R. L. Pratt.

The Secretary announced that the following had been admitted to the Society in accordance with reciprocity agreements with various mathematical organizations: Deutsche Mathematiker-Vereinigung:

Professor W. M. Meyer-Koenig; London Mathematical Society: Professor E. T. Davies; University of Southampton, England, Professor G. B. Preston: Schweizerische Mathematische Gesellschaft: Dr. Walter Gautschi; Unione Mathematica Italiana: Mr. Tristano Manacarda.

Montana State University was elected to Institutional Membership and the Sun Oil Company and International Business Machines Corporation were elected to Corporate Membership in the Society.

The Secretary announced that by mail votes Professors W. S. Massey and A. H. Taub had been elected to the Executive Committee; a Summer Institute in 1958 on the topic, *Surface area and related topics* had been approved, the Invitations Committee for this Institute to be comprised of Tibor Rado, Chairman, L. C. Young, L. Cesari, Herbert Federer, and J. W. T. Youngs; the President was authorized to appoint two delegates of the Society to the constitutional convention of a proposed Institute of Mathematics. The delegates appointed were Professors A. E. Meder, Jr., and Saunders MacLane.

The following Presidential appointments were reported: As an arrangements Committee for the April Meeting in New York: S. C. Lowell, Chairman, J. W. John, P. D. Lax, R. D. Schafer, H. E. Wahlert, G. B. Whitham. As an Arrangements Committee for the Summer Meeting at Pennsylvania State University: Evan Johnson, Jr., Chairman, J. B. Bartoo, Orrin Frink, W. O. Gordon, H. L. Krall, H. M. Gehman, R. D. Schafer. As an Arrangements Committee for the Annual Meeting in Cincinnati: I. A. Barnett, Chairman, J. L. Baker, Arno Jaeger, H. T. LaBorde, H. D. Lipsich, C. I. Lubin, G. M. Merriman, W. E. Restemeyer, H. M. Gehman, J. W. T. Youngs. As a Committee to Nominate Candidates for the Executive Committee: E. G. Begle, J. W. Green, Saunders MacLane. To the Committee on Places of Meetings of the Association and Society: R. D. Schafer. To the Committee to Organize Summer Institutes: A. M. Gleason, Ralph Phillips. To the Committee on Applied Mathematics: R. S. Burington, R. C. Bellman. As Tellers for the 1956 election: David Gale, W. G. Lister, J. F. Randolph.

The President had appointed the following as delegates of the Society: Professor Mary V. Sunseri, at the Centennial Year Convocation of San Jose State College; Professor B. H. Bissinger, at the inauguration of F. de Wolfe Bolman, Jr. as President of Franklin and Marshall College; Professor J. H. Roberts at the inauguration of President William C. Friday of the University of North Carolina; Professor W. H. Gottschalk, at the Centennial Convention of the

National Educational Association; Professor K. C. Schraut, at the one-hundredth Anniversary of the founding of Wilberforce University; Professor W. L. Hart, at the inauguration of the Very Reverend James P. Shannon at St. Thomas College, St. Paul, Minnesota; Professor Harriet Griffin, at the inauguration of Sister Vincent Therese Tuohy as President of St. Joseph's College in Brooklyn; Professor W. C. McDaniel, at the installation of the president at Southeast Missouri State College.

The Secretary reported that the following persons have accepted invitations to deliver hour addresses: A. L. Whitman, Berkeley, California, April 20; René Thom, University of Chicago, April 19–20; Mahlon M. Day, State College of Washington, June 15; Harold Levine, University of California, Los Angeles, November 15–16; A. P. Calderon and M. Rosenlicht, at Pennsylvania State University, August 26–30.

The Council set meetings at the University of California, Los Angeles, November 15–16, 1957; at Stanford University April 18, 1958; at Chicago April 18–19, 1958; and the Summer Meeting at Massachusetts Institute of Technology August 25–30, 1958.

The Council discharged the following committees: Committee on the relationship between the Headquarters and the Mathematical Reviews; Committee to Look into the Relations between the Transactions and Proceedings; Committee to Survey the Society's Program of Periodical Publication. The President was authorized to appoint a committee to select the Gibbs Lecturer for 1958 and 1959 and a committee to award the Bôcher Prize for 1958.

The Council passed the following resolution:

Whereas, on January 1, 1951, Professor Edward G. Begle assumed the office of Secretary of the American Mathematical Society, and Whereas at that time the duties of the Secretary had attained an unprecedented importance due to the growth in Society membership and other factors, and Whereas Professor Begle performed the duties of Secretary faithfully and devotedly thereafter for a six-year period, during a time when many crises, financial, administrative, and other, were faced by the Society, and Whereas the Society is deeply indebted to Professor Begle for the unswerving loyalty and excellent judgment which he displayed during his term of office, Be it therefore resolved that the Council hereby express its profound gratitude to Professor Begle for his services to American mathematics through his work as Secretary of the American Mathematical Society, and Be it further resolved that copies of this resolution be sent to Professor Begle, spread upon the minutes of the Council, and appropriately published by the Society.

The Council approved an amendment to the by-laws setting the minimum annual dues of corporate members at One Thousand Dollars. The Council also approved a number of amendments to the by-laws having to do with the election of officers and members of the Council and the dates of Annual Meetings and meetings of the Board of Trustees. These amendments are made necessary by the fact that the next Annual Meeting will be in January, instead of at Christmas.

The recommendation of the Applied Mathematics Committee that a symposium on Applied Mathematics be held in the spring of 1958 in conjunction with a meeting of the Society was approved. The subject of this symposium will be *Combinatorial designs and analysis.*

The Council voted to approve the recommendation of the Colloquium Editorial Committee that Volume 8, *Non-Riemannian geometry* by L. P. Eisenhart be reprinted. It also voted to recommend to the Board of Trustees that in view of the large backlog in the Transactions, as many extra pages as they think feasible be published in the last volume of 1957.

On Saturday at 9:00 A.M. there was a meeting of representatives of the American Mathematical Society, Association for Symbolic Logic, Institute of Mathematical Statistics, Mathematical Association of America, National Council of Teachers of Mathematics and the Society for Industrial and Applied Mathematics.

The ladies of the Department of Mathematics entertained the Society at a tea on Friday afternoon.

In all there were six sessions for the presentation of contributed papers. Presiding Officers were Professors Ewing, MacLane, Roberts, Mickle, Herstein, and Randolph.

Abstracts of the contributed papers follow. Those with "*t*" after the abstract number were presented by title. In the case of joint papers, the name of the author who read the paper is followed by (p). Dr. Heinz Renggle was introduced by Professor B. J. Pettis.

ALGEBRA AND THEORY OF NUMBERS

500*t*. C. W. Curtis: *Modules whose annihilators are direct summands.*

Let $B = \Delta(G, \rho)$ be a ring which is a crossed product of a finite group G and a division ring Δ with factor set ρ. In particular B may be the group algebra of a finite group with coefficients in a field. Then B has a basis over Δ consisting of elements $b_1, b_s, \cdots$ in (1-1) correspondence with the elements of G such that $b_s b_t = b_{st}\rho_{s,t}$ and $\xi b_s = b_s \xi^{\bar{s}}$ for all $s, t \in G$, $\xi \in \Delta$, where the mappings $\xi \to \xi^{\bar{s}}$ are automorphisms of Δ. Let M be a right B-module, and let M' be the space of all Δ-linear functions on M. Let B_M be the set of all finite sums of the elements $\sum_{s \in G} \psi(x b_s) b_s^{-1}$ where $x \in M$, $\psi \in M'$. Then B_M is a two-sided ideal in B, and M is called a regular B-module if B_M contains a two sided identity element. Let $\epsilon_1 B, \cdots, \epsilon_s B$ be the block ideals in B, where the ϵ_i are

central primitive idempotents. A reduced block component of B is the direct sum of a full set of nonisomorphic indecomposable right ideals eB, $e^2=e$, belonging to a fixed block. Then the following statements are equivalent: (i) M is a regular module; (ii) $B = B_M \oplus (0:M)$, where $(0:M)$ is the annihilator of M; and (iii) $M\epsilon_i \neq 0$ implies that $M\epsilon_i$ contains the ith reduced block component of B as a direct summand. This work is an extension of the author's previous investigation (Canadian J. Math. vol. 8 (1956) pp. 271–292.) (Received February 21, 1957.)

501. W. E. Deskins: *On the radical of a modular group algebra.*

Let G be a group of order g, p a prime divisor of g, F a field of characteristic p, and $F(G)$ the group algebra of G over F. If P is a Sylow p-subgroup of G, let L be the left ideal of $F(G)$ generated by the set of all elements of the form $P_i - P_j$. Define I to be the ideal $\cap GLG^{-1}$ for all G in G. Then I is nilpotent and Lombardo-Radici have proved (Rend. Sem. Mat. Roma vol. 3 (1939) pp. 239–256) that I is the radical of $F(G)$ if P is unique or if $g=pq$, q a prime. These results are extended here to more general classes of groups; for example: Theorem. *If G is a super-solvable group then I is the radical of $F(G)$.* If G contains an invariant p-subgroup then it is easy to show that $F(G)$ is bound to its radical, so the following question is raised: If $F(G)$ is a bound algebra, does G contain an invariant p-subgroup? A partial solution is provided by the following: Theorem. *If I is the radical of $F(G)$, if P is cyclic, and if $F(G)$ is a bound algebra, then G contains an invariant p-subgroup.* (Received February 20, 1957.)

502. Philip Dwinger: *Complete homomorphisms of complete lattices.*

A homomorphism α of a lattice L (all lattices in this paper are complete) is complete, if it preserves joins and meets of arbitrary subsets. Equivalent to this is the condition, that the congruence relation θ generated by α is complete, i.e. it enjoys the substitution property for infinite joins and meets. The complete congruence relations of L form a complete lattice $C^*[L]$ which is a $\cap\cup$-sublattice of the lattice $C[L]$ of all congruence relations of L. A necessary condition that θ is complete is the condition, that the residue classes of L modulo θ are closed intervals of L. Whether this condition is sufficient is answered affirmatively by the following theorem: A congruence relation θ is complete if and only if all residue classes of L modulo θ are closed intervals. If L is relatively complemented then this condition can be weakened to the condition that the ideal $\{x \mid x \equiv O(\theta)\}$ is a principal ideal of L. In this case $C^*[L]$ is a complete Boolean algebra and isomorphic to a $\cap\cup$-sublattice of L (see also Dilworth, Ann. of Math. vol. 51 (1950) pp. 350–359). If A is a complete Boolean algebra then $C^*[A]$ is a complete $\cap\cup$-sublattice of $C[A]$ and isomorphic to A. (Received February 20, 1957.)

503. C. C. Faith: *Normal extensions in which every element with nonzero trace is a normal basis element.* II.

If K/F is a normal extension, then $u \in K$ is a *completely basic element* (CBE for brevity) *of K/F* provided u generates a normal basis of K/Δ, for each inter-field Δ, $K \geq \Delta \geq F$. The author has shown previously that K/F always possesses a CBE when F is infinite, and that an extensive class $\mathfrak{C}$ exists consisting of normal extensions with the property that every normal basis element is a CBE (Bull. Amer. Math. Soc. Abstract 62-4-403). The existence of a CBE in an arbitrary normal extension is now reduced to that of Q/F, where F is finite of characteristic p, and Q/F is cyclic of deg q^s, q a prime $\neq p$. This is achieved with the aid of (1) a general lemma which asserts if

$K = K_1X \cdots XK_t$ over F, and if u_i is a CBE of K_i/F, $i = 1, \cdots, t$, that $u = \prod_{i=1}^{t} u_i$ is a CBE of K/F, and (2) the knowledge that $P/F \in \mathfrak{C}$, when P/F has deg p^e, and F has characteristic p (Bull. Amer. Math. Soc. Abstract 63-2-162). Let N/F be an abelian extension, and let ζ be a primitive r root of unity in a field containing N, where r is the l.c.m. of the orders of the automorphisms of N/F. It is known (Abstract 62-4-403 loc. cit.) that $N/F \in \mathfrak{C}$ when $N \wedge F(\zeta) = F$. This result is now extended: If $K = P \times N$ over F (P/F defined as above), then $K/F \in \mathfrak{C}$. (Received February 20, 1957.)

504. J. W. Gaddum: *Quadratic forms and linear inequalities.*

Let the quadratic form $\sum_1^n a_{ij}x_ix_j$ be denoted by $z(x)$ and let A be the matrix of this form. A form is called conditionally definite (semi-definite) if $Z(x) > 0$ ($\geqq 0$) when $x \geqq 0$. It is shown that $Z(x)$ is conditionally definite (semi-definite) if and only if for each principle minor B of A the system $By > 0$ ($By \geqq 0$) has a solution $y \geqq 0$. The system $Ax \geqq 0$ is related to the form $Z(x)$ in a slightly more complicated way. It can be stated that $Ax \geqq 0$, $x \geqq 0$ is solvable if and only if $Z(x)$ is conditionally semi-definite and $Z(x) > 0$ if $x > 0$. (Received February 20, 1957.)

505. D. Hawkins and W. E. Briggs (p): *The lucky number theorem.*

The lucky numbers of Ulam (Math. Mag. vol. 29 (1956) pp. 117–122) resemble prime numbers in their apparent distribution among the natural numbers and are defined by the following sieve. If S_n is the sequence $t_{n,m}$, $m = 1, 2, 3, \cdots$, S_{n+1} is obtained for $n > 1$ by removing from S_n every $t_{n,m}$ for which $t_{n,n}$ divides m. $S_2 = (2, 3, 5, 7, 9, \cdots)$ and the lucky number sequence is $S = \lim_{n \to \infty} S_n = (2, 3, 7, 9, 13, 15, 21, \cdots)$. If $R(n, x)$ is the number of numbers not exceeding x in S_n, then the fundamental recurrence relation $R(n, x) = R(n-1, x) - [R(n-1, x)/s_{n-1}]$, s_k being the kth lucky number, yields the fact that $(1 - 1/s_1)(1 - 1/s_2) \cdots (1 - 1/s_n) \sim 1/\log n$ and that $s_n \sim n \log n$. S. Chowla has improved this last result to $s_n = n \log n + n(\log \log n)^2/2 + o(n(\log \log n)^2)$. (Received February 14, 1957.)

506. R. H. Oehmke: *On simple flexible algebras of degree $t > 3$.*

If A is an algebra over a base field F, the degree of A is the maximum number of pairwise orthogonal idempotents in any scalar extension of A over F. A flexible algebra is an algebra satisfying the identity $(xa)x = x(ax)$. If in addition the algebra also satisfies the identity $(x^2a)x = x^2(ax)$ it is called a noncommutative Jordan algebra. The results and methods of Albert [Trans. Amer. Math. Soc. vol. 64 (1948) pp. 552–593] can be generalized to show that a simple flexible power-associative algebra of degree $t > 2$ is a noncommutative Jordan algebra. R. D. Schafer [Proc. Amer. Math. Soc. vol. 6 (1955) p. 472] proved that a simple Jordan algebra of characteristic 0 is of degree 2, Jordan or quasi-associative. By use of a partial trace function it is possible to extend this result to include the characteristic $p \neq 2, 3$ and degree $t > 1$ case. (Received February 21, 1957.)

507t. Ingram Olkin: *On a class of integral identities with matrix argument.* Preliminary report.

Bellman [Duke Math. J. vol. 23 (1956) pp. 571–577] obtained a generalization of the Ingham-Siegel integral identity (1) $\int_{X>0} |X|^a \prod_2^p |X^{(i)}|^{-a_i} \exp - \operatorname{tr} XA \{dX\}$ $= \pi^{p(p-1)/4} \prod_1^p \Gamma(\sum_{j=i}^p a_j - (p-i)/2) |Y_i|^{-a_i}$, where $a = \sum_1^p a_j - (p+1)/2$, $X: p \times p$ is symmetric, $X^{(k)} = (x_{ij})$, $i, j = k, \cdots, p$, $Y_k = (y_{ij})$, $i, j = 1, \cdots, k$. The proof is similar

to that by Ingham. An alternative and more elementary proof based on certain matrix transformations is presented. The method permits the evaluation of a number of other integral identities of a form similar to (1). In particular, generalizations are given for (a) an integral indentity involving a correlation matrix in the argument, (b) the Beta function formula of Siegel, (c) a matric analogue of a certain Dirichlet integral. These integrals have a connection with distribution problems in multivariate analysis. (Received February 21, 1957.)

508t. D. S. Rim: *On relatively complete fields.*

It was proved by Kaplansky and Schilling (Bull. Amer. Math. Soc. vol. 48) that if K is relatively complete under rank one valuation, and if K has a unique separable extension of degree n for each integer n, then every subfield k with K/k finite algebraic is also relatively complete with respect to the induced valuation. One of the results of the paper is a generalization of this theorem to any field K which is not algebraically closed. In the case when K is algebraically closed, it is shown that the same statement is true if and only if the residue class field of k is real-closed. A simpler proof for Hensel's lemma, eliminating the usual approximation process is also given. (Received February 20, 1957.)

509. D. S. Rim (p) and George Whaples: *Axiomatic approach to cohomology theory of finite groups.*

Cohomology theory of finite groups is known to be essentially the theory of satellites of the 0-dimensional cohomology group. In this paper, without introducing satellites, the theory is axiomatized by demanding the decent behavior of induced homomorphisms and connecting homomorphisms, cohomological triviality of G-regular modules and specifying the 0-dimensional cohomology group. To accomplish a purely axiomatic approach, the cup product is constructed using only the connecting homomorphisms given in the axioms and one natural homomorphism, without introducing complexes at all. (Received February 20, 1957.)

510. M. P. Schützenberger: *D-representation of semigroups.*

A representation of a semigroup S by matrices over a subgroup of S is defined. This representation is an extension to the whole of S of a closely related representation given for a $\mathfrak{D}$-class, D, of S by D. D. Miller and A. H. Clifford (Trans. Amer. Math. Soc. vol. 82 (1956) pp. 270–280) whose results and notations are used below. Let L^i $(i \in I)$ and R_j $(j \in J)$ be the $\mathfrak{L}$- and $\mathfrak{R}$-classes of D; $H = L^1 \cap R_1$, a subgroup of S with idempotent e. For all $i \in I$ and $j \in J$, fixed arbitrary elements are chosen, satisfying: $a^1 = b_1 = e$; $a^i \in R_1 \cap L^i$; $b_j \in L^1 \cap R_j$; $a^i_* \in L^1$; $b^*_j \in R_1$; $a^i a^i_* = b^*_j b_j = e$. The $I \times J$ matrix P and, for any $s \in S$, the $I \times I$ matrix $M(s)$ and the $J \times J$ matrix $N(s)$ are defined by: $p^i_j = a^i b_j$, if $a^i b_j \in D$ and $= 0$, otherwise; $m^{i'}_i(s) = a^i s a^{i'}_*$, if $a^i s \in L^{i'}$ and $= 0$, otherwise; $n^j_{j'}(s) = b^*_{j'} s b_j$, if $sb_j \in R_{j'}$ and $= 0$, otherwise. All the entries of the matrices P, $M(s)$ and $N(s)$ belong to $H \cup 0$ and, identically, $M(s)P = PN(s)$. The mapping; $s \to M(s) \otimes (N(s))$† is shown to give a representation of S, independant (up to transformations by diagonal matrices) of the choice of $H \subset D$, of the a^i's and so forth. The homomorphisms of S induced by the restrictions of this representation and by the homomorphisms of H are discussed. (Cf. Comptes Rendus vol. 242 (1956) pp. 2907–2908.) (Received February 20, 1957.)

511*t*. M. F. Smiley: *Jordan homomorphisms and right alternative rings.*

The methods of a previous paper (*Jordan homomorphisms onto prime rings*, Trans. Amer. Math. Soc. vol. 84 (1957) pp. 426–429) are used to give a brief and characteristic-free proof that certain right-alternative rings are alternative. Our result includes those previously obtained by E. Kleinfeld (*Right alternative rings*, Proc. Amer. Math. Soc. vol. 4 (1953) pp. 939–944) and by R. L. San Soucie (*Right alternative rings of characteristic two*, Proc. Amer. Math. Soc. vol. 6 (1955) pp. 716–719). (Received February 20, 1957.)

512. F. B. Wright: *On Hölder groups.*

Let G be a topological (additive) abelian group, and let M be a semigroup in G such that (1) M is open, (2) $0 \notin M$, and (3) M is maximal with respect to (1), (2). Then the complement $b(M)$ of $M \cup -M$ is a closed subgroup of G. For each such semigroup, set $s(M) = M \cup b(M)$; then $s(M)$ is a semigroup. Let T be the intersection of all such groups $b(M)$; T is called the radical of G. (For details: a forthcoming paper in the American Journal of Mathematics.) The classical theorem of Hölder on archimedean ordered abelian groups can be translated into this terminology: G is continuously isomorphic to a subgroup of the additive reals if and only if for every such M, it is true that $b(M) = 0$ and the semigroup $s(M)$ is a maximal semigroup (in the strictest sense) in G. This suggests defining a Hölder group to be an abelian group whose radical is 0, and which has the property that for every M the semigroup $s(M)$ is a maximal semigroup in G. Then G is a Hölder group if and only if it is continuously isomorphic to a subgroup of a locally convex real linear topological space. (Received February 18, 1957.)

ANALYSIS

513. R. J. Blattner: *Lie algebras of measurable operators.* Preliminary report.

Let $\mathfrak{A}$ be a ring of operators with countably decomposable center and $\mathfrak{M}$ the algebra of measurable operators over $\mathfrak{A}$. Then the family of all *-convergent sequences in $\mathfrak{M}$ is the family of all convergent sequences for a metrizable topology on $\mathfrak{M}$, called the *-topology. $\mathfrak{M}$ is a nonlocally convex topological algebra in the *-topology. The exponential series *-converges for any normal operator, the resultant map (exp) being *-continuous. Let G be a Lie group, $\mathfrak{L}$ its Lie algebra, ρ a representation of $\mathfrak{L}$ as skew-adjoint operators in $\mathfrak{M}$, and log the logarithm map from a suitable neighborhood of the identity in G to $\mathfrak{L}$. Theorem: exp o ρ o log is a local homomorphism of G into the unitary group of $\mathfrak{A}$. This is proved by reducing to the two cases of abelian and compact semi-simple G. The first case is trivial, while the second can be reduced further to the case where $\rho(\mathfrak{L}) \subseteq \mathfrak{A}$, which can easily be handled by the Campbell-Hausdorff formula. Corollary of the proof: there is no representation of the Heisenberg commutation relations as measurable operators. (Received February 22, 1957.)

514*t*. D. G. Bourgin: *Some fixed point theorems.*

The following is a typical result: Let X be a retract of the polytope $P = P_A I_a$ where the power of A is unrestricted. Let $\{Y_i \mid i = 1, \cdots, n\}$ be n, $n \neq 1$, open sub-

sets of X whose closures $\{\bar{Y}_i\}$ are retracts and are pairwise disjunct. Denote the boundary of Y_i by $\dot{Y}_i$. Let f map X into X and $\dot{Y}_i$ into $\bar{Y}_i$. Then f has a fixed point in X. The proof of the theorem depends on certain index notions. (Cf. D. G. Bourgin; Rend. Accad. Naz. dei Lincei vol. 22 (1956) pp. 43–48. A special finite dimensional case of the theorem, with a quite different proof which does not extend, has been found by F. Bagemihl; (Fundamenta vol. 40 (1953) pp. 3–12). (Received February 19, 1957.)

515. G. U. Brauer: *Remark on a result of Utz.*

The following theorem is proved: Let $c(t)$ be a positive function with a nonnegative derivative, for large t, and let $g(t)$ be a function such that $g(0)=0$, $g'(t)\leqq 0$ for all values of t. Then if $x(t)$ is a solution of the equation $x''(t)+g(x')+c(t)x(t)=0$, then $x(t)$ is oscillatory, $\lim_{t\to\infty} x(t)=\infty$, or $\lim_{t\to\infty} x(t)=-\infty$. Utz recently proved this theorem for the case where c is a positive constant. (Received February 11, 1957.)

516. R. H. Cameron: *Differential equations involving a parametric function.*

This paper deals with the differential system $dz/dt+f[t, y(t)+z(t)]=0$, $z(0)=0$ on the interval $[0, 1]$, where $y(t)$ is a parametric function continuous on $[0, 1]$ and vanishing at $t=0$. Conditions are given on $f(t, u)$ such that the system has solutions $z(t)$ on $[0, 1]$ for almost every choice (in the sense of Wiener measure) of the parametric function y. Nevertheless it is shown that there is a function $f(t, u)$ satisfying the conditions but such that the system does not have a solution z for *every* y. (Received February 20, 1957.)

517. A. M. Chak: *A generalization of Lommel polynomials.*

This paper concerns the study of polynomials $R_{p,m,n}(x)$ obtained from the recurrence relation $J_{m+3,n+3}(x)-(3(n+n+3)/x)J_{m+2,n+2}(x)+(9(m+1)(n+1)/x^2)J_{m+1,n+1}(x)-J_{m,n}(x)=0$, where $J_{m,n}(x)$ is the generalized Bessel function studied by P. Humbert [Atti della Pont. Acc. delle Sci. Nouvi Lincei; 1930 (128–), 1934 (323–331), 1935 (154–158)]. These polynomials $R_{p,m,n}(x)$ are of degree p in $1/x$ and also of the same degree in m and n respectively. A number of recurrence relations, several term relations, etc. have been found out; also polynomials associated with $J_{1/3,2/3}(x)$ and $J_{-1/3,-2/3}(x)$ have been studied on the lines of the interesting relationship which Bessel functions of order half an odd integer have with sine and cosine functions. (Received February 18, 1957.)

518t. W. F. Darsow: *Ideals and states in C^*-algebras.*

With the appropriate modifications certain theorems of I. E. Segal in *Two-sided ideals in operator algebras* (Ann. of Math. vol. 50 (1949) pp. 856–865) extend readily to one-sided ideals. If K is a closed left ideal in a C^*-algebra A, then T is in K if and only if T^*T is in K. Let $s(K)$ be the set of all pure states of A that vanish on the self-adjoint elements in K; then T is in K if and only if $f(T^*T)=0$ for all f in $s(K)$. For a pure state f of A the *left-kernel* $k(f)$ of f is the closed left ideal of all T in A for which $f(AT)=0$; then a closed left ideal K is the intersection of the left-kernels $k(f)$ as f ranges over all of $s(K)$. (Received February 19, 1957.)

519. W. H. Fleming: *Functions with generalized gradient and generalized surfaces.*

Let $f(x)$ denote an integrable function of k real variables ($k\geqq 2$) vanishing out-

side a fixed cube K, for which the total gradient variation $I(f)$ defined by DeGiorgi [Annali di Mat. vol. 36 (1954) pp. 191–213] is finite. Each such f defines a vector measure $\Phi = (\Phi_1, \cdots, \Phi_k)$ called *exact*. A $k-1$ dimensional generalized surface L situated in K also defines a vector measure Φ, termed *closed* if L is closed [Fleming-Young, Trans. Amer. Math. Soc. vol. 76 (1954) pp. 457–484]. *Theorem.* Closed if and only if exact. The set F_N of all f for which $I(f) \leq N$ is convex and compact in the l_1 topology. *Theorem.* Every extreme point of F_N is a multiple of the characteristic function f_E of a set $E \subset K$. For $k=2$ the extreme points are completely characterized. *Mixture Theorem.* There exists a function $W(x, E)$, equal to f_E a.e. for each fixed E, such that to every f with $I(f)$ finite corresponds a signed measure ω on E-space for which $f(x) = \int [I(f_E)]^{-1} W(x, E) d\omega$ a.e. (Received February 18, 1957.)

520. W. R. Fuller: *Existence of cycles for weakly nonlinear real differential systems containing differences.*

Consider the system: (1) $\ddot{x}_1 + \sigma_1^2 x_1 = \epsilon\{\sum_{l=0}^{r} \dot{x}_1(t - \lambda_l)f_l[x_1(t - \lambda_l)]$ $+ \sum_{l=1}^{r} x_1(t-\lambda_l)g_l[\dot{x}_1(t-\lambda_l)]\} + \epsilon q_1(y, \dot{y}; \epsilon) + \epsilon h(y, \dot{y}; \epsilon); \ddot{x}_j + \alpha_j \dot{x}_j + \sigma_j^2 x_j = \epsilon q_j(y, \dot{y}; \epsilon)$, $j=2, \cdots, n; \ddot{x}_\mu + \beta_\mu x_\mu = \epsilon q_\mu(y, \dot{y}; \epsilon), \mu = n+1, \cdots, N$; where $\lambda_l, l=0, \cdots, r$ are distinct, $\lambda_0 = 0, \epsilon > 0$ is a small parameter, $y, \dot{y}$ denote the $N(r+1)$- and $n(r+1)$-vectors $[x(t-\lambda_l), l=0, \cdots, r], [\dot{x}(t-\lambda_l), l=0, \cdots, r], \sigma_j > 0, \alpha_j > 0, \beta_\mu > 0, \gamma_j^2 \equiv 4\sigma_j^2 - \alpha_j^2 > 0$, $\alpha_j + i\gamma_j \neq 2im\sigma_1$ $(m=0, \pm 1, \cdots; j=1, \cdots, n; \mu = n+1, \cdots, N)$, each function is analytic for $\epsilon < \epsilon_0, |x_k|, |\dot{x}_k| < K; f_l, g_l$ even polynomials, $q_1(-y, -\dot{y}; \epsilon) = q_1(y, \dot{y}; \epsilon)$, $h(y_1, 0, \cdots, 0, \dot{y}_1, 0, \cdots, 0; 0) = 0$. Then an algebraic equation $P(|a|^2) = 0$ is given for each of whose simple roots $|a_0|^2 = c_0^2 > 0$, a periodic solution of (1) exists: (2) $x_1 = |a|\sigma_1^{-1} \sin (\tau t + \phi) + \epsilon W_1(\tau t + \phi, \lambda_1, \cdots, \lambda_r; \epsilon); x_k = \epsilon W_k(\tau t + \phi, \lambda_1, \cdots, \lambda_r; \epsilon)$, $k=2, \cdots, N$; where W_1, W_k are periodic of period 2π in $\tau t + \phi, \phi$ is an arbitrary constant, and $\tau = \sigma_1 + O(\epsilon), |a| = c_0 + O(\epsilon)$. The series for τ and $|a|$, and conditions assuring that $P = 0$ has simple positive roots are given. For instance the Van der Pol like equation $\ddot{x}(t) + \epsilon[x^2(t) - 1]\dot{x}(t) + \epsilon[x^2(t-\lambda) - 1]\dot{x}(t-\lambda) + x = 0$ has a cycle. The method of successive approximations successively developed by L. Cesari [Atti Acad. Italia (6) vol. 11 (1940) pp. 633–692], J. K. Hale [Bull. Amer. Math. Soc. Abstracts 60-1-118, 60-1-119, 62-6-668, 62-6-669] and others, is here modified and its convergence proved for systems with differences. (Received February 18, 1957.)

521. R. R. Goldberg: *Convolution transforms of almost periodic functions.*

Let $G(t)$ be any Pólya frequency function the reciprocal of whose bilateral Laplace transform has the form $E(s) = e^{bs}\prod_{k=1}^{\infty} (1 - a_k s)e^{a_k s}$ where $\sum_{k=1}^{\infty} a_k^2 < \infty$. Let $P_n(s) = e^{b_n s}\prod_{k=1}^{n} (1 - a_k s)e^{a_k s}$ where $(b - b_n)^2 = O(\sum_{k=n+1}^{\infty} a_k^2)$ as $n \to \infty$. The following theorem is proved: Necessary and sufficient conditions that a given function $f(x) \in C^\infty$ $(-\infty < x < \infty)$ be representable in the form $f(x) = \int_{-\infty}^{\infty} G(x-t)\phi(t)dt$ for some $\phi(t)$ almost periodic in the sense of Bohr are that the functions $\{P_n(D)f(x)\}_{n=1}^{\infty}$ (where D stands for differentiation) be uniformly bounded and be uniformly dominated by some almost periodic function. This last means that there should exist an almost periodic function $\beta(x)$ such that for any $\epsilon > 0$ every translation number for $\beta(x)$ corresponding to ϵ is also a translation number for each $P_n(D)f(x)$ corresponding to ϵ. (Received February 15, 1957.)

522. J. H. B. Kemperman: *Continuous solutions of some functional equations on a topological group.*

Let X be a topological Hausdorff group satisfying the first axiom of countability,

and let Y be a topological space. Let $f(s)$ be a function on an open subset U of X to Y, which is continuous on a Borel subset $D \subset U$. We suppose either that D is of second category or that X is locally compact and D is of positive right (regular) Haar measure. Finally, let $I_1 \subset X$, $I_2 \subset X$ with I_2 open and $I_1 I_2 \subset U$. (A) Suppose that $D \subset I_2 \subset U$ and that $f(st) = G(s, f(t))$ for $s \in I_1$, $t \in I_2$, where, for each $s \in I_1$, $G(s, v)$ is a continuous function from Y to Y. Then $f(s)$ is continuous at each point t_0 with the property that for each $d \in D$ there exists a finite sequence of points $s_i \in I_1$ with $s_i s_{i-1} \cdots s_1 d \in I_2$, $(i = 1, \cdots, k-1)$ and $s_k s_{k-1} \cdots s_1 d = t_0$. This implies some known results concerning measurable functions $f(s)$ on $s > 0$, cf. E. Hille, *Functional analysis and semi-groups*, Chapter VIII. (B) Suppose that $I_1 I_2$ is a connected set containing D, further, I_1 is open, while for each $t \in I_2$ there exists an open connected set V with $t \in V \subset I_2$. Finally, let Y be a topological vector space over the field $\mathfrak{F}$ and let, for $s \in I_1$, $t \in I_2$, $f(st) = \sum_1^q g_i(s) h_i(t)$, where $g_i(s) \in \mathfrak{F}$, $h_i(t) \in Y$, $(i = 1, \cdots, q < \infty)$. Then $f(s)$ is continuous on $I_1 I_2$. (Received February 20, 1957.)

523. M. Z. Krzywoblocki: *Mathematics and the first law of the mo-dynamics.*

Max Planck opposed using the differential notation for the heat input in the first law of thermodynamics, maintaining that the differential dQ cannot be regarded as the differential of a finite quantity Q. The author shows that this reasoning is correct only with reference to the exact differential. Using the elementary notion of the functions of bounded variation and that of the Riemann-Stieltjes integral, the author shows that the heat input function Q exists in almost all the cases of physical importance. (Received February 18, 1957.)

524. Karel deLeeuw: *Analytic almost periodic functions in a sector.*

Let α be a real number satisfying $0 \leq \alpha \leq \pi/2$ and S_α the subset $\{re^{i\theta}: -\alpha \leq \theta \leq +\alpha\}$ of the complex plane. Let $B(S_\alpha)$ be the space of bounded continuous functions on S_α that are analytic at interior points, supplied with the topology of uniform convergence on S_α. For each σ in S_α and f in $B(S_\alpha)$, let $T_\sigma f$ in $B(S_\alpha)$ be defined by $T_\sigma f(\tau) = f(\sigma + \tau)$. Define $A(S_\alpha)$ to be the closed linear subspace of $B(S_\alpha)$ consisting of those f which are such that the subset $\{T_\sigma f: \sigma \in S_\alpha\}$ has compact closure. Let $\widehat{S}_\alpha$ be the subset $\{re^{i\theta}: \pi/2 + \alpha \leq \theta \leq 3\pi/2 - \alpha\}$ of the complex plane. *Theorem:* $A(S_\alpha)$ consists of precisely those functions that can be approximated uniformly on S_α by linear combinations of those exponential functions of the form $f(z) = e^{\lambda z}$ with λ in $\widehat{S}_\alpha$. The case $\alpha = \pi/2$ of this theorem follows from results of Bohr. That for $0 < \alpha < \pi/2$ is established by obtaining a canonical direct sum decomposition of $A(S_\alpha)$ into four direct summands and identifying each of these direct summands. If $\alpha = 0$, there is a similar decomposition into two direct summands. (Received February 18, 1957.)

525t. M. J. Mansfield: *Some generalizations of full normality, I.*

A collection $\mathfrak{B}$ of subsets of a set X is said to be an $\mathfrak{m}$-*star refinement* ($\mathfrak{m}$ any cardinal number ≥ 2) of a collection $\mathfrak{a}$ if (i) $\mathfrak{B}$ is a refinement of $\mathfrak{a}$ and (ii) if $\mathfrak{C} \subset \mathfrak{B}$, $2 \leq |\mathfrak{C}| \leq \mathfrak{m}$, and $\bigcap\{C: C \in \mathfrak{C}\} \neq \phi$, then there is an $A \in \mathfrak{a}$ such that $\bigcup\{C: C \in \mathfrak{C}\} \subset A$. $\mathfrak{B}$ is an *almost-$\mathfrak{m}$-star* refinement of $\mathfrak{a}$ if (i) $\mathfrak{B}$ is a refinement of $\mathfrak{a}$ and (ii) if $M \subset St(x, \mathfrak{B})$ $(= \bigcup\{B \in \mathfrak{B}: x \in B\})$ for some $x \in X$ and $2 \leq |M| \leq \mathfrak{m}$, then there is an $A \in \mathfrak{a}$ such that $M \subset A$. A topological space X is said to be $\mathfrak{m}$-*fully normal* (resp. *almost-$\mathfrak{m}$-fully normal*) if each open covering of X admits an open $\mathfrak{m}$-star (resp. almost-$\mathfrak{m}$-star) refinement. Each fully normal space is $\mathfrak{m}$-fully normal for every $\mathfrak{m} \geq 2$,

and each $\mathfrak{m}$-fully normal space is almost-$\mathfrak{m}$-fully normal. An almost-2-fully normal space is collectionwise normal (cf. H. J. Cohen, C. R. Acad. Sci. Paris vol. 234 (1952) pp. 290–292). An example, due to R. H. Bing, is given of a collectionwise normal space which is not almost-2-fully normal. The class of almost-2-fully normal spaces is shown to coincide with the class of all spaces X for which the family of all neighborhoods of the diagonal in $X \times X$ is a uniformity for X (cf. Cohen, *op. cit.*). (Received February 18, 1957.)

526. N. G. Meyers: *Asymptotic behavior of solutions of elliptic equations.*

The paper is concerned principally with the behavior of solutions of linear elliptic equations of the form $\sum_{i,k=1}^{n} a_{ik}(x)\partial^2 u/\partial x_i \partial x_k + \sum_{i=1}^{n} b_i(x)\partial u/\partial x_i + c(x)u = f(x)$, ($n > 2$), defined in a neighborhood of infinity. The main result gives the leading term in the expansion of the solution about infinity. For simplicity it is stated here only for the case $b_i(x) \equiv 0 \equiv c(x)$. Theorem: If (1) $|a_{ik}(x) - \delta_{ik}| <$ constant $|x|^{-\alpha}$ where δ_{ik} is the Kronecker delta (2) $|x|^{n+\alpha}|f(x)| <$ constant (3) the Hölder conditions $|a_{ik}(x) - a_{ik}(y)| <$ constant $d^{-\alpha}|x - y|^{\alpha}$, $|f(x) - f(y)| <$ constant $d^{-2-\alpha}|x - y|^{\alpha}$ ($d = \min |x|$, $|y|$) hold (4) $u(x) \to u^0$ as $|x| \to \infty$ then $u = u^0 + \text{constant}/|x|^{n-2} + O_2(1/|x|^{n-2+\alpha})$. Asymptotic estimates on derivatives of solutions are also obtained assuming conditions on the coefficients and their derivatives similar to those above. The method of proof exploits the Schauder estimates coupled with Green's integral representation of the solution. A similar method can also be applied to prove removability of singularities at finite points. (Received April 19, 1957.)

527. J. C. C. Nitsche: *A uniqueness theorem on minimal surfaces in cylindrical coordinates.*

Let S be a minimal surface which admits of a representation in cylindrical coordinates $x = r(z, \phi) \cdot \cos \phi$, $y = r(z, \phi) \cdot \sin \phi \cdot r(z, \phi)$ is assumed to be positive and periodic: $r(z, \phi+2\pi) = r(z, \phi)$. The author proves the following *theorem*: Let the function $r(z, \phi)$ be of class C^2 for all values of z and ϕ and suppose that the curves of intersection of S with the planes $z = $ const. are convex. Then S must be a catenoid: (*) $r(z, \phi) = [\gamma \cdot \sin (\phi+\delta) + (\cosh^2 (\alpha z+\beta) - \gamma^2 \cos (\phi+\delta)^{1/2}]/\alpha$. $\alpha, \beta, \gamma, \delta$ are real constants, $\alpha > 0$, $|\gamma| < 1$. The axis of revolution is parallel to the z-axis and passes through $x = \gamma \sin \delta$, $y = \gamma \cos \delta$. One may regard this as another example of a theorem of "Bernstein's type": The entire solutions of the corresponding differential equation under certain conditions depend on finitely many parameters only (cf. a remark by C. Loewner). The proof makes use of the following facts: The function $\zeta = \xi + i\eta$, where $\xi = z$, $\eta = \int_{0,0}^{z,\phi} W^{-1}[r_z r_\phi dz + (r^2 + r_\phi^2)d\phi]$, $W = (r^2 + r_\phi^2 + r^2 r_z^2)^{1/2}$, yields a one-to-one and isothermic mapping of the z, ϕ-plane onto the ζ-plane. $F(\zeta) = (r + ir_\phi)(W + rr_z)^{-1}e^{-i\phi}$ is an analytic function of ζ and a schlicht function of $\zeta^* = e^\zeta$ in $0 < |\zeta^*| < 1$ without zeros. Hence $F(\zeta) = \text{const} \cdot e^{-\zeta}$. Thus a differential system is obtained whose integration leads to (*). (Received February 19, 1957.)

528. Heinz Renggli: *On quasiconformal mappings.*

By $\lambda\{\gamma\}$ we denote the extremal length of a family $\{\gamma\}$ of curves γ [L. Ahlfors and A. Beurling, Acta Math. 83 (1950) p. 101]. Let f be a homeomorphism of a region D of the complex plane onto another region. Then f is said to be quasiconformal and denoted by F, if $\lambda\{\gamma'\} = 0$ if and only if $\lambda\{\gamma\} = 0$ where $\{\gamma'\}$ is the image of $\{\gamma\}$ under F. We then have: (i) the class $\{N(SB)\}$ of sets $N(SB)$ [l.c.] is quasiconformally

invariant, i.e. let F be a mapping of the complement in the extended plane of a set $N(SB)$; then the complement of the image is also a set $N(SB)$, (ii) let F map a Jordan region onto another Jordan region; then F induces a homeomorphism of the boundary, (iii) let F map a Jordan region D onto the unit-circle and let $N(SB)$ be situated on the Jordan curve bordering D; then the image of $N(SB)$ in the boundary of the unit-circle is also a set $N(SB)$. The statements are proved by a special study of families $\{\gamma\}$ with $\lambda\{\gamma\} = 0$. For (iii) reflection in the boundary of the unit-circle is used. (Received February 18, 1957.)

529t. Diran Sarafyan: *Singular points of exact differential equations.*

Concerning the singular points of $y' = f(x, y)$ the following theorem has been given in a previous paper (Bull. Amer. Math. Soc. Abstract 63-2-256): The necessary and sufficient condition under which through any point $P_0(x_0, y_0)$ of a closed simply connected region R, there exists a unique integral curve is that $\exp \int_a^{x_0}(\partial f/\partial y)(x, y_0)dx = 0$. This theorem will be applied to the exact differential equation $M(x, y)dx + N(x, y)dy = 0$ which can also be written $y' = -M/N = f(x, y)$. It will be assumed that the functions M, N and $f(x, y)$ are continuous in the region R. Now $\partial f/\partial y = (M/N^2)(\partial N/\partial y) - (1/N)(\partial N/\partial X)$. But $\partial N/\partial x = dN/dx + (M/N)(\partial N/\partial y)$. Thus by substitution one gets $\partial f/\partial y = -(1/N)(dN/dx)$. Then $\int_a^{x_0}(\partial f/\partial y)dx = -\int_a^{x_0}dN/N = [\log (1/|N|)]_a^{x_0}$. The application of the above stated theorem indicates that, there are no singular points in the region R. Outside of R they are determined from $1/N(x_0, y_0) = 0$ and/or $1/M(x_0, y_0) = 0$. (Received February 20, 1957.)

530. A. B. Simon: *Subalgebras of group algebras.* Preliminary report.

Let S be a measurable subset of a locally compact Abelian group G and $L = L(G)$ the group algebra of G. We denote by L_S the set of all functions of L whose support lies in S. In the case L_S forms a subalgebra of L, we call it a vanishing subalgebra. It is proved that L_S is a vanishing subalgebra if and only if there is a subset T whose closure is a subsemigroup of G and $L_T = L_S$. We say that a subset A of L is separating if the set of Fourier transforms of elements of A separate points of the character group. It is shown that L_S is separating if and only if S generates G. (Received February 18, 1957.)

531. Hidehiko Yamabe: *On a problem of Bellman.*

In the Bulletin of American Mathematical Society, January 1957, R. Bellman proposed a problem. Let $f(x)$ and $g(x)$ be two strictly positive functions over $[0, 1]$. Then are the eigenvalues λ of $u'' + \lambda(f + \epsilon g)u = 0$ analytic in ϵ when $R(\epsilon) \neq 0$. An affirmative answer is given for real. This is shown by showing a solution $\beta(\lambda, \epsilon; x)$ of $u'' + \lambda(f + \epsilon g)u = 0$, $u(0) = 1$, $u(1) = 0$, which is analytic in λ and ϵ and is 0 at $x = 1$ and $\beta_\lambda \neq 0$ at $x = 1$. From this the answer follows by the implicit function theory. β is the (1.2) member of the product integral of $\begin{pmatrix} 0 & 1 \\ -\lambda(f+\epsilon g) & 0 \end{pmatrix}$ which gives the solution for any initial data. (Received April 8, 1957.)

APPLIED MATHEMATICS

532. R. P. Kanwal: *On curved shock waves in three-dimensional steady rotational gas flows.*

In this paper we discuss the problem of shock waves in three-dimensional steady

rotational gas flows of fluids devoid of viscosity and heat conductivity. The main object is to obtain, by the help of the Rankind-Hugoniot relations, formulas for the determination of the derivatives of velocity components, pressure, density and entropy behind the shock surface when the flow in front is known. The detailed treatment is given to the case when the flow in front is uniform. Gauss-Weingarten formulas and various other results of the geometry of surfaces concerning principal normal curvature are found useful in the analysis. As anticipated, the flow is found in general to be rotational behind the shock surface. The explicit determination of the components of vorticity has been carried out. This leads to the formulation of a general theorem regarding the characterization of surfaces behind which the flow remains irrotational. It is found that the plane, the right circular cone, the cylinder and the developable helicoid are the only such surfaces. (Received April 15, 1957.)

533. W. R. Wasow: *On the accuracy of implicit difference approximations to the equation of heat flow.*

Let $u(x, t)$ be the isolution of the boundary value problem: $\partial u/\partial t = \partial^2 u/\partial x^2$, for $0 < t$, $0 \leqq x \leqq \pi$; $u(x, 0) = f(x)$, $u(0, t) = u(\pi, t) = 0$. Let $U(x, t)$ satisfy the difference equation $[U(x, t+k) - U(x, t)]k^{-1} = [U(x+h, t+k) - 2U(x, t) + U(x-h, t+k)]$ $\cdot (1-s)h^{-2} + [U(x+h, t) - 2U(x, t) + U(x-h, t)]sh^{-2}$ and the same subsidiary conditions as $u(x, t)$. It is proved that the truncation error $U - u$ is of the order $O(t^{-1}h^\alpha)$, where $\alpha = 1$ for $1/2 < s \leqq 1$, $\alpha = 2$ for $s = 1/2$, provided $f(x) \in C^3$, $f(0) = f(\pi) = 0$, and $0 < p \leqq h/k$ (p independent of h and k). For the convergence of U to u as $h \to 0$, $k \to 0$, without any restriction on h and k it is sufficient that $f'(x)$ is piecewise continuous, that $f(0) = f(\pi) = 0$, and $s \geqq 1/2$. For any $f(x)$ that is bounded, $U = O$ (max $|f|$ log $(1/h)$). This last fact implies that the difference equation is always reasonably stable. (Received March 18, 1957.)

GEOMETRY

534. W. R. Abel (p) and L. M. Blumenthal: *Metric arcs with a tangent property.*

This paper continues the systematic study of metric properties of metric arcs, begun in a previous article (*Metric arcs with Menger curvature*, I., Bull. Amer. Math. Soc. Abstract 63-1-52). A metric arc has property T_A at a point p, provided that $r, s, t \in A$, $r < s < t$ implies lim $\sphericalangle rst = \pi$ as $r, s, t \to p$. A metric continuum K has property T_K at a point p, provided $r, s, t \in K$, lim max $\sphericalangle$ (triple r, s, t) $= \pi$ as $r, s, t \to p$. It is proved that (1) if K has property T_k at each of its points, then K is a rectifiable arc or a rectifiable simple closed curve, (2) properties T_A, T_K are equivalent for $K = A$, and hence a metric arc with property T_A at each point is rectifiable. The latter statement remains valid if lim $\sphericalangle rst < \pi/2 + $ arcsin $(1/\pi)$, as $r, s, t \to p$. These results are intimately related to theorems announced without proof by C. V. Pauc (Rend. R. Accad. Naz. Lincei. vol. 24, ser. 6, 1936; Thése, Fac. Sci. l'Univ. Paris, Hermann et Cie., Paris, 1941). (Received February 27, 1957.)

LOGIC AND FOUNDATIONS

535t. Hugo Ribeiro: *Universal completeness.* I.

Terminology and some of the methods are as in Tarski, *Contributions to the theory of models*, Proc. Acad. Soc. Amsterdam, A, no. 5 (1954) pp. 572–588 and no. 1 (1955)

pp. 56–64. Equational completeness for abstract algebras has been defined by Kalicki and Scott, see Indag. Math. vol. 17 (1955) pp. 650–659 and bibliography therein. The following is a slightly different version of the definition given by the author in Bull. Amer. Math. Soc. vol. 62 (1956) p. 176, where the result on the class of all linearly ordered sets should have been attributed to Langford, Proc. London Math. Soc. vol. 25 (1926) pp. 115–142. A consistent set, Σ, of sentences of the formalized theory, $T(K)$, constructed for given similarity class, K, of relational systems is said to be universally complete if and only if for any universal sentence, ϕ, of $T(K)$, either ϕ follows from Σ or there is, for some n, a (universal) sentence, α (involving only the identity symbol and the other logical constants) saying that there are at most n elements and such that $\phi \leftrightarrow \alpha$ follows from Σ. Langford's results on universal completeness for several classes of linearly ordered sets follow from that for the class of all linearly ordered sets by the remark that if Σ is consistent and has a universally complete subset then Σ is universally complete. Every set of equalities which is universally complete is also equational complete. Several of the sets of equalities considered by Kalicki and Scott are universally complete. (Received February 21, 1957.)

536. Hugo Ribeiro: *Universal completeness* II. Preliminary report.

For terminology see abstract I. In the following Σ is a consistent set of sentences and K is the correspondent class of systems. (I) If Σ is universally complete, $\mathfrak{A}, \mathfrak{B} \in K$, $\mathfrak{A}$ finite and card $\mathfrak{A} \leq$ card $\mathfrak{B}$ then $\mathfrak{A}$ is isomorphically embeddable in $\mathfrak{B}$. (II) Let K satisfy the condition of (I) and the following one: if $\mathfrak{S} \in K$ and S' is finite subset of the set of $\mathfrak{S}$ then there is $\mathfrak{R} \in K$ such that $\mathfrak{R}$ is finite and the restriction of $\mathfrak{S}$ to S' is isomorphically embeddable in $\mathfrak{R}$. Then Σ is universally complete. (III) Σ is universally complete if and only if for any $\mathfrak{W} \in K$ and for any finite subset W' of the set of $\mathfrak{W}$ the restriction of $\mathfrak{W}$ to W' is isomorphically embeddable in all elements of K with the eventual exception of finite elements $\mathfrak{W}$ such that card $\mathfrak{W} <$ card $\mathfrak{W}$ and which, up to isomorphism, are finitely many. The following are examples of classes defined by universally complete sets of sentences: the class of all Boolean algebras, the class of all cyclic groups whose order is power of a fixed prime and smaller than a fixed number, the class of all finite projective geometries over the same field. Other classes, K, are discussed in connection with I, II and III. (Received February 21, 1957.)

STATISTICS AND PROBABILITY

537t. J. H. B. Kemperman: *An asymptotic expansion concerning the Smirnov test.*

Let $2^n p_n(a, b, d)$ denote the number of sets $(\epsilon_1, \cdots, \epsilon_n)$ with $\epsilon_i = \pm 1$, $-a < \epsilon_1 + \cdots + \epsilon_i < b$, $(i = 1, \cdots, n)$, $\epsilon_1 + \cdots + \epsilon_n = d$; here, a, b, n denote positive integers, d is an integer with $-a < d < b$, $n - d$ even. Then (*) $p_n(a, b, d)$ $= 4/c \sum_1^K \sin k\pi a/c \sin k\pi(a+d)/c \cos^n k\pi/c$, where $c = a+b$, $K = [(c-1)/2]$. From (*), we obtain the following asymptotic expansion: (1) Let $H_{2r}(x) = D^{2r} \exp(-x^2)$ and $g_r = 2^{-r} \sum_{k=-\infty}^{\infty} [H_{2r}((d+2kc)/(2n)^{1/2}) - H_{2r}((2a+d+2kc)/(2n)^{1/2})]$; (2) Let the constants $A_{ij} \geq 0$ be defined by the expansion $\exp(u/2 + u/z \log \cos z^{1/2})$ $= \sum_0^\infty \sum_0^\infty A_{ij} z^i (-u)^j$, (holding for $|z| < \pi^2/4$); (3) Let $c_i = (-1)^i (2/\pi)^{1/2} \sum_{j=0}^i A_{ij} g_{i+j}$; (4) Let R_m be defined by $p_n(a, b, d) = \sum_{i=0}^{m-1} c_i n^{-i-1/2} + R_m n^{-m-1/2}$; (5) Assertion: for each integer $m \geq 0$, both c_m and R_m are bounded functions of n, in fact, uniformly bounded with respect to a, b, d. Note that $2^{2n} p_{2n}(a, b, 0)/C_{2n}^n = \Pr(-a/n < F_2(x) - F_1(x) < b/n$ for all $x)$, where $F_1(x), F_2(x)$ denote the empirical distributions of two independent samples of size n from a continuous population. (Received February 20, 1957.)

TOPOLOGY

538t. D. G. Bourgin: *Real functions on spaces with homeomorphisms of even period.*

Let P be a unicoherent locally connected compact Hausdorff space with a finite Borel measure. Let R_1 and R_2 be two measure preserving homeomorphisms of periods $2m_1$ and $2m_2$ and suppose R_1^{m} and R_2^{m} are fixed point free. Let f be a continuous real valued function on P which satisfies $f(rR_i^{2l}) = f(r)$, $l = 1, 2, \cdots$. Then, for some $\bar{r}$, $f(\bar{r}) = f(\bar{r}R_1) = f(\bar{r}R_2)$. A special case of this theorem with $m_1 = m_2 = 2$ and P the projective 3 space has been given by the writer previously (Rend. di Mat. e delle sue applicazioni 15, pp. 177–189, 1956; Theorem 3). The proof is similar to that of the special case and pivots on introducing an identification space Y, admitting a fixed point free involution R' for which P is a covering space, and observing that the inverse projection of an R_1' (or R_2') symmetric closed connected separating set in Y is an R_1 (or R_2) symmetric connected separating set in P. (Received February 14, 1957.)

539t. A. H. Clifford: *Connected ordered topological semigroups with idempotent endpoints.* I.

By a *thread* we shall mean a system $S(\circ, <)$ such that: (1) $\circ$ is an associative binary operation in S; (2) $<$ is a total (i.e. linear) order relation in S; (3) the mapping $(x, y) \rightarrow x \circ y$ of $S \times S$ into S is continuous in the order topology; (4) S is connected; (5) S has a least element f and a greatest element e, and these are idempotent. The systematic study of threads was initiated by W. M. Faucett (Proc. Amer. Math. Soc. vol. 6 (1955) pp. 741–747 and 748–756). The present paper continues the study to the point of obtaining a complete determination of all threads with zero. (All threads without zero will be described in Part II.) A thread S is called *standard* if its endpoints are the zero and identity elements of S. P. S. Mostert and A. L. Shields (Ann. of Math., to appear) determined all these when S is a real interval; the general case is similar. If $S = [f, e]$ is any thread, the intervals $[f, 0]$ and $[0, e]$ are standard subthreads. If e is the identity element, S is completely determined by the homomorphism $x \rightarrow xf$ of $[0, e]$ onto $[f, 0]$, a result found independently by H. Cohen and L. I. Wade (Trans. Amer. Math. Soc., to appear). Every commutative thread with zero is a trivial extension of a thread of this type. Every noncommutative thread is a trivial extension of a thread like Faucett's Example 3, p. 747, with the real unit interval replaced by any standard thread. (Redeived February 25, 1957.)

540. H. H. Corson: *On the relatively compact subsets of a space.*

A subset A of a topological space X is called relatively countably compact if every sequence in A has a cluster point in X. It is proved that if a completely regular topological space has property (1) every open covering has a point finite refinement, or (2) its product with one of its compactifications is normal then (3) every relatively countably compact subset is relatively compact. It also is shown that if a space X has (3) then every closed continuous image of X has (3), and that X has (3), if it is the union of a countable number of closed subsets having (3). An example is given of a normal topological group which does not have (3), and is therefore not paracompact. (Received February 18, 1957.)

541. Edward Halpern: *A primitive hyperalgebra.*

A *hyperalgebra* over A (commutative ring with unit) consists of an A-module H, associative product $\phi: H \otimes H \rightarrow H$ with unit 1, and associative coproduct $\eta: H \rightarrow H \otimes H$

with $\eta(1) = 1 \otimes 1$ which is multiplicative. An *augmentation* is an algebra augmentation $\alpha: H \to A$ satisfying $(i \otimes \alpha + \alpha \otimes i)\eta(x) = x \otimes 1 + 1 \otimes x$, where i is the identity and $x \in H^+$, the kernel of α. An element $x \in H^+$ is *primitive* if $\Delta(x) = \eta(x) - x \otimes 1 - 1 \otimes x = 0$; H is primitive if generated (under ϕ) by primitive elements (and 1). Define subalgebras $\pi^{(k)}$, $k \geq 0$, inductively as follows: $\pi^{(0)}$ is generated by the primitive elements (and 1), $\pi^{(k+1)}$ is generated by elements such that $\Delta(x) \in \pi^{(k)} \otimes \pi^{(k)}$. The hyperalgebra is of *class k* if $H = \pi^{(k)}$, $H \neq \pi^{(k-1)}$. In a graded hyperalgebra, $H = \sum_{p \geq 0} H^p$ (weak) and ϕ and η are homogeneous of degree zero; it is assumed $H^0 = A \cdot 1$ and the augmentation is "standard." The following theorem is proved: If H is a graded hyperalgebra of class 2 over a field of characteristic zero and ϕ is anticommutative then the hyperalgebra $H/\pi^{(0)}$ (with naturally induced product and coproduct) is primitive. (Received February 18, 1957.)

542t. Edward Halpern: *An exact sequence.*

A spectral sequence of A-modules (E_r) is *canonical* if E_r is defined and bigraded for $r \geq 2$ with $E_r^{p,q}$ zero if p or q is negative and $E_r^{0,0} \cong A$ (naturally induced for $r > 2$) and the differential d_r has bidegree $(r, 1-r)$. If $E_\infty^{p,q} = 0$ for $p+q > 0$ and $E_\infty^{0,0} \cong A$ (naturally induced) it is *acyclic*. The following theorem is proved: If (E_r) is acyclic and canonical and $E_2^{p,q} = 0$ for $p \neq 0$, m, n where $2 \leq m \leq n$, $n - m \neq 1$, then $0 \to E_2^{0,1} \to^\alpha E_2^{m,2-m} \to^\beta E_2^{n,3-n} \to^\gamma E_2^{0,2} \to^\alpha \cdots \to^\gamma E_2^{0,q} \to^\alpha E_2^{m,q-m+1} \to^\beta E_2^{n,q-n+2} \to^\gamma E_2^{0,q+1} \to^\alpha \cdots$ is exact, where α, β, γ are given essentially by d_m, d_{n-m}, d_n respectively. The cohomology $H^*(\Omega)$ of the loop spaces over the (a) complex projective plane, (b) quaternionic projective plane, (c) Cayley plane, is computed using this sequence. The results when A is a principal ideal domain of characteristic zero are: $H^*(\Omega)$ is isomorphic to a tensor product of an exterior algebra with one generator of degree $m-1$ and a twisted polynomial algebra in "one variable" of degree $3m-2$ where $m = 2, 4, 8$ in cases (a), (b), (c) respectively. Corresponding results are obtained if the characteristic is not zero. (Received February 18, 1957.)

543. L. F. McAuley: *On complete collectionwise normality and paracompactness.*

The concept of complete collectionwise normality is defined. It bears a relation to Bing's notion of collectionwise normality [Canadian Journal of Mathematics (1951)] like that of complete normality to normality. Some results are as follows. (1) In a regular semi-metric topological space, paracompactness is equivalent to complete collectionwise normality. (2) There is a separable normal Hausdorff space which is collectionwise normal and which satisfies the first axiom of countability but which is not completely collectionwise normal. (3) In a Moore space, complete collectionwise normality is equivalent to collectionwise normality. (4) A normal, pointwise paracompact and separable semi-metric topological space is hereditarily separable, completely collectionwise normal, and paracompact but may fail to have a countable basis. As a consequence of (4), a normal, separable and pointwise paracompact Moore space is metrizable. (Received February 20, 1957.)

544t. M. J. Mansfield: *Some generalizations of full normality,* II.

It is shown that every linearly ordered space is $\aleph_0$-fully normal and that, for any ordinal α, the linearly ordered space $W(\omega_{\alpha+1})$ is $\aleph_\alpha$-fully normal but not almost-$\aleph_{\alpha+1}$-fully normal. It is an open question as to whether an almost-$\mathfrak{m}$-fully normal space is $\mathfrak{m}$-fully normal, even for $\mathfrak{m}$ finite. It is known that if a space X is k-fully normal for some finite k, then X is n-fully normal for every finite $n = 2, 3, \cdots$. It is not known

if the analogous statement for almost-k-fully normal spaces is true or not. An example is given of a 2-fully normal space which is not almost-$\aleph_0$-fully normal; this space, however, is not a Hausdorff space. It is an open question as to whether every 2-fully normal Hausdorff space is almost-$\aleph_0$-fully normal. The questions of topological completeness and separation of arbitrary subsets by means of open sets are investigated for $\mathfrak{m}$- and almost-$\mathfrak{m}$-fully normal spaces. In particular, it is shown that every almost-$\aleph_0$-fully normal space is countably paracompact. (Received February 18, 1957.)

545. P. S. Mostert: *On one-parameter transformation groups in the plane.*

THEOREM. Let E be the plane and R the real line acting on E as a group of transformations without fixed points. If the space E/R of orbits is Hausdorff, then E is fibred as a direct product of R and a cross sectioning line. That is, the action of R is equivalent to a group of translations. The main tools used in the proof are the local cross section theorem for local orbits and the Jordan curve theorem. (Received February 18, 1957.)

546. D. E. Sanderson: $H(E^3)$ *is uniformly locally arcwise connected.*

By specializing previous results by the author (Bull. Amer. Math. Soc. Abstract 59-6-717) to the case of Euclidean 3-space, E^3, the affirmative answer is given to the following question raised by R. H. Bing at the Summer Institute on Topology at Madison, Wisconsin in 1955: given $\epsilon > 0$, does there exist a $\delta > 0$ such that if f is a homeomorphism of E^3 onto itself moving no point as much as δ, then there is an isotopy of f to the identity moving no point as much as ϵ? Considering the author's theorem (Bull. Amer. Math. Soc. Abstract 60-4-545) that for any $\epsilon > 0$ an arbitrary member of the space $H(E^3)$ of autohomeomorphisms of E^3 is ϵ-isotopic to a piecewise linear homeomorphism, it suffices to prove Bing's conjecture for piecewise linear homeomorphisms. This is accomplished by deforming f onto the identity on an expanding infinite sequence of polyhedral 2-spheres, noting that the requirement of compactness for such a fitting process can be replaced by the requirement of a uniform structure for the 2-spheres and certain neighborhoods involved. The required deformation between the 2-spheres is carried out by an application of a theorem of Alexander (Proc. Nat. Acad. Sci. vol. 9 (1923) pp. 406–407). (Received February 21, 1957.)

547. Patrick Shanahan: *An axiomatic characterization of the reduced homology theory.*

A homology theory on an admissible category $\mathfrak{A}$ is a covariant ∂-functor defined on $\mathfrak{A}$ satisfying the seven axioms of Eilenberg and Steenrod. (For terminology see Eilenberg and Steenrod, *Foundations of algebraic topology.*) Let H be a homology theory on the admissible category $\mathfrak{A}$ consisting of all pairs of spaces and all maps of such pairs. It is shown that if $\tilde{H}$ is the reduced homology theory associated with H, then $\tilde{H}$ is a covariance ∂-functor on $\mathfrak{A}$ having seven properties analogous to the axioms of Eilenberg and Steenrod. Conversely, if H is a covariant ∂-functor defined on $\mathfrak{A}$ and enjoys the properties alluded to above, then $\tilde{H}$ is isomorphic to the reduced theory of some homology theory on $\mathfrak{A}$. The properties thus serve to axiomatize the reduced theory. They are closely allied to the seven axioms of Eilenberg and Steenrod; in fact the basic difference is in the dimension axiom. (Received February 20, 1957.)

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