THE FEBRUARY MEETING IN NEW HAVEN

The five hundred thirty-second meeting of the American Mathematical Society was held at Yale University in New Haven, Connecticut, on Saturday, February 23, 1957. The meeting was attended by about 130 persons including 110 members of the Society.

By invitation of the Committee to Select Hour Speakers for Eastern Sectional Meetings Professor J. T. Tate of Harvard University delivered an address entitled *Class formations* at a general session presided over by Professor A. A. Albert. Sessions for contributed papers were held in the morning and afternoon, presided over by Professor Nathan Jacobson and Dr. E. C. Schlesinger.

Abstracts of the papers presented follow. Those having the letter "*t*" after their numbers were read by title. Where a paper has more than one author, the paper was presented by that author whose name is followed by "(p)". Mr. Bomberault was introduced by Mr. Michael Held.

ALGEBRA AND THEORY OF NUMBERS

349*t*. J. W. Andrushkiw: *A note on elimination.*

Let $f(z)=a_0z^n+\cdots+a_n$, $g(z)=b_0z^m+\cdots+b_m$, $a_0\neq 0$, $b_0\neq 0$, $n>m$, be polynomials with complex coefficients, and denote by $r_1, r_2, \cdots, r_n$, $r_i\neq r_j$, $i\neq j$, the roots of $f(z)$. If s_k are the coefficients in expansion $g(z)/f(z)=\sum_{k=0}^{\infty} s_k z^{-k-1}$, it follows that (1) $\sum s_i a_j=0$, $i+j=h$, $h=0, 1, \cdots, n-m-2; n, n+1, \cdots$, and $\sum s_i a_j = b_k$, $i+j=h$, $h=n-m-1+k$, $k=0, 1, \cdots, m$; (2) for $i, j=1, 2, \cdots, n-k$; $k=0, 1, \cdots, m$ there is $\|s_i+j-2\| = \sum_{\lambda>\mu}^{1,n-k} \prod (r_{i_\lambda}-r_{i_\mu})^2 g(r_{i_1})g(r_{i_2}) \cdots g(r_{i_{n-k}})/f'(r_{i_1})f'(r_{i_2}) \cdots f'(r_{i_{n-k}})$ $=(-1)^{C(n-m,2)}a_0^{k-n-m}A_{n+m-2k}$ where A_{n+m-2k} are the principal diagonal minors of $n+m-2k$ order of A_{n+m}. The elements of A_{n+m} are the coefficients of $f(z)$ and $g(z)$, and the summation is taken over all combinations of $n-k$ roots. If $r_1=r_2=\cdots r_s=r$, the factors $f'(r_1), \cdots, f'(r_s)$ have to be replaced by $f^{(s)}(r)$, and in the product occur only differences of distinct roots. If r also a root of $g(z)$, the factors $g(r_1), \cdots, g(r_s)$ have to be replaced by $g^{(p)}(r)$ where $g^{(p)}(z)$ is a proper derivative depending upon the multiplicity of r. The relation is still true if $n=m$; (3) $f(z)$ and $g(z)$ have exactly k common roots if and only if $A_{n+m}=A_{n+m-2}=\cdots=A_{n+m-2k+2}=0$, $A_{n+m-2k}\neq 0$. The determinant A_{n+m} is related to the resultant $R(f, g)$ by the equation A_{n+m} $=(-1)^{C(n,2)-C(n-m,2)}R(f, g)$. If $g(z)=f'(z)$, A_{n+m}/a_0 = represents the discriminant $D(f)$.
(Received December 31, 1956.)

350*t*. Maurice Auslander and Alex Rosenberg: *Dimension of prime ideals in polynomial rings.*

Let R be a commutative ring and let p be a prime ideal in R such that R_p is a regular local ring. Let P be a prime ideal in $S=R[x_1, \cdots, x_n]$ with $R\cap P=p$ and denote the transcendence degree of $S-P$ over $R-p$ by d_P. Using homological methods we then show that rank $P+d_P=$ rank $p+n$. If R is a Dedekind ring, P and Q two prime ideals in S, and I the ideal in $S \otimes_R S$ generated by $P \otimes 1$ and $1 \otimes Q$, we use this equality to show that $d_I = d_P + d_Q$. (Received January 7, 1957.)

351*t*. R. W. Bass: *The real parts of eigenvalues.*

Let A be any nonsingular matrix with eigenvalues λ_i ($i=1, \cdots, n$) and let $\Lambda \equiv \text{diag}\, [\Re\lambda_1, \cdots, \Re\lambda_n]$. Theorem: *There is a nonsingular matrix R with the property that the symmetric part of $(R'R)A$ has the same signature as Λ.* (Here $'$ denotes transpose.) When the eigenvalues are all distinct a simple construction for R is given; the theorem is shown in this case to be equivalent to a theorem of Lewis (Amer. J. Math. vol. 73 (1951) pp. 48–58). See also Abstract No. 362. (Received January 10, 1957.)

352. Chandler Davis: *All invariant convex functions of hermitian matrices.*

Any function of $n \times n$ hermitian matrices with values in a partially ordered vector space, if it is convex and unitary-invariant, gives rise in an obvious way to a convex symmetric function of n real variables (values in the same partially ordered vector space). The main purpose of the present paper is to point out the converse. The proof is simple. Several well-known results are special cases. (Received January 9, 1957.)

353. Emil Grosswald: *Some theorems concerning partitions.*

Let q be an odd prime, let $\{a\} = \{a_1, \cdots, a_m\}$ stand for a set of m least positive residues mod q. $\{a\}$ is called symmetric, if with a also $q-a$ belongs to $\{a\}$. Let $p_n(q)$ be the number of partitions of n into summands congruent (mod q) to elements of $\{a\}$; define $p_n(q, l)$ similarly, if no summand may appear more than l times; $p_n^+(q)$, $p_n^-(q)$, $p_n^+(q, l)$, $p_n^-(q, l)$ are, respectively, the number of partitions of n into quadratic residues, into quadratic nonresidues (mod q) without, or with, restriction to at most l repetitions of a summand. For these, and more general, partition numbers Petersson (Abh. D. Akad. Wiss. (2) 1954) has obtained asymptotic formulae in the case of symmetric sets $\{a\}$. In the present paper the Hardy-Rademacher method is used to obtain asymptotic formulae for these partitions, corresponding to any set $\{a\}$. Typical results are: For $q \equiv 1 \pmod 4$, $p_n^+(q)/p_n^-(q) = \epsilon^h(1 - A\alpha_5(q)n^{-1/2} + O(n^{-1}))$, $p_n^+(q, l)/p_n^-(q, l) = 1 + B\alpha_5(q)n^{-1/2} + O(n^-)$, while for $q \equiv 3 \pmod 4$, $p_n^+(q)/p_n^-(q) = Cn^{h/2}(1 - Dn^{-1/2} + O(n^{-1}))$, $p_n^+(q, l)/p_n^-(q, l) = (l+1)^h(1 + O(n^{-1}))$. Here $A, B, \cdots$ are trivial constants, $\alpha_1(q)$ is the number of representations of q as a sum of five squares, $\epsilon > 1$ is the fundamental unit of $R(q^{1/2})$ and h is the class number of $R(\pm q)^{1/2}$ for $q > 3$, $h = 1/3$ for $q = 3$. (Received January 10, 1957).

354. E. C. Posner: *Derivations in prime rings.*

A prime ring is a ring in which the zero ideal is a prime ideal. *Theorem* I. In a prime ring of characteristic not two, if d_1 and d_2 are derivations such that $d_1(d_2)$ is also a derivation, then d_1 or d_2 is zero. *Theorem* II. If d is a derivation in a prime ring such that, for all elements a in the ring, a and $d(a)$ commute, then either the ring is commutative or d is zero. The proofs proceed by manipulation of identities. (Received January 10, 1957.)

355. M. O. Rabin: *The generalized word problem for free groups with an application to Post languages.*

Let $\Pi = (x: r(x))$ be a (finite) presentation. The generalized word problem of Π is the problem of finding a method for deciding, for any given set of words $w_1, \cdots, w_n$,

u (n is arbitrary), whether u is in the subgroup U generated by $w_1, \cdots, w_n$. When $n=0$ this reduces to the word problem for groups. *Theorem*: *There exists a general and effective solution for the generalized word problem for free groups (i.e. for those presentations having no defining relations). In fact, let $l(u)$ be the length of u as a product of the (free) generators of Π and let $L(u)$ be the minimal length of u as a product of the w_i and their inverses; there exists an effectively computable function $c(k)$ such that if $u \in U$ then* $L(u) \leqq (l(u)+1)c(l(w_1)+ \cdots +l(w_n))$. As a corollary the following result is obtained. *Theorem*: *Let L be normal Post language on the letters $a_1, \cdots, a_k$, having the axiom H and productions $A_iP \to PB_i$, $i=1, \cdots, m$. If L is undecidable then the words $H, A_i, B_i, i=1, \cdots, m$, considered as elements of the free group on the a_i, satisfy a nontrivial algebraic relation.* (Received January 11, 1957.)

356t. Irving Reiner: *A new type of automorphism of the general linear group over a ring.*

Let $GL_2(R)$ be the general linear group over the ring $R=K(x)$, where K is a field. Let $(a\ b)X_m=(a\ ax^m+b)$, $m=1, 2, \cdots$. The elements X_m and those in $GL_2(K)$ generate $GL_2(R)$. Let Y_m be obtained from X_m by replacing x^m by y_m, where $1, y_1, y_2, \cdots$ form a K-basis of R. The map ϕ_y which leaves $GL_2(K)$ elementwise fixed, and maps X_m onto Y_m, induces an automorphism of $GL_2(R)$. This automorphism is not expressible in terms of previously known types of automorphisms. Automorphisms of this new type do not arise for the group $GL_2(Z)$, Z=ring of rational integers, nor for $GL_n(R)$, $n \geqq 3$. The proofs make use of properties of continued fractions. A set of generators for the group of automorphisms of $GL_2(R)$ is obtained. (Received January 4, 1957.)

357. E. V. Schenkman: *A characterization of some metacyclic groups.*

Szász (*On groups every cyclic subgroup of which is a power of the group*, Acta Mathematica Acad. Scient. Hungarian vol. 6 (1955) pp. 475–477), has recently shown that a group is cyclic if and only if it satisfies condition (A) as follows. (A) Every cyclic subgroup of the group is for some positive integer k the subgroup generated by the kth powers of the elements of the group. This idea will be extended to show that a metacyclic group whose commutator subgroup has order relatively prime to its index is characterized as a solvable group satisfying condition (B) as follows. (B) Every member of a composition series (i.e., every subinvariant subgroup) is for some positive integer k the subgroup generated by the kth powers of the elements of the group. (If G denotes the group, the subgroup will be denoted by $G(k)$.) (Received January 9, 1957.)

ANALYSIS

358. R. W. Bass: *The generalization to n dimensions of Bendixson's nonexistence criterion.*

This refers to the well known criterion for the nonexistence of limit cycles of an autonomous two dimensional system of ordinary differential equations. Let $f(x)$ be an n-vector function of class C^1 on E^n. Consider the dynamical system S defined on E^n by $\dot{x}=f(x)$ ($\dot{}=d/dt$). A trajectory of S will be called a *normal almost-periodic orbit* if it is (i) uniformly (Bohr) almost-periodic, and (ii) if its closure M has dimension $\geqq (n-1)$ at some point. THEOREM. *Let G be any open set in which* div $f(x)$ *does not change its sign and is not identically zero. Then G does not contain any normal almost-*

periodic orbit of S. This theorem cannot be improved, as can be shown by constructing compressible flows in E^3 which contain a (1 dimensional) periodic orbit. The proof of the theorem is an immediate consequence of the following nontrivial lemma. *The closure M of any normal almost-periodic orbit of S is an* $(n-1)$ *dimensional torus.* (Received January 10, 1957.)

359*t*. R. W. Bass: *Irrotationality, invariance, and instability* (I).

Let S be a dynamical system defined in E^n by $\dot{x}=f(x)$ ($\dot{} = d/dt$), where $f(x)$ is a class C^1 n-vector function. Let $F(x, t)$ denote the flow defined by S so that e.g. $F(x, 0) \equiv x$. A set R is *invariant* [positively semi-invariant] if $F(R, I)=R$ [$F(R, I^+)\subset R$]. A *rest point* is an invariant point. Define a compact set to be *rotational* if its Euler-Poincaré characteristic $E(R)=0$ and define a flow F to be *P-irrotational in R* if there exists a continuous matrix $P(x)$ such that the tensor curl $P(x)f(x)\equiv 0$ and $f(x) \cdot P(x)f(x)>0$ for all x in R, where $\cdot$ denotes the scalar product in E^n. *In all that follows we assume G to be an open set whose closure* $\bar{G}$ *is* (i) *compact, and* (ii) *contains no rest point.* Lemma. There is a positive number which is smaller than the period of any periodic orbit in $\bar{G}$. Theorem 1. *If* $\bar{G}$ *is positively semi-invariant and is a Lefschetz space* (*e.g. a compact ANR*) *then* $\bar{G}$ *is a rotational set.* (Received January 10, 1957.)

360*t*. R. W. Bass: *Irrotationality, invariance, and instability* (II).

Further define a compact set R to be *Poisson unstable relative to itself* if to every x in R there correspond finite numbers $T_-(x)<0<T_+(x)$ with the property that $F(x, T_\pm(x))$ is not in R. A closed subset K of R is a *transversal* if each orbit in R intersects K exactly once (hence K is a local cross section). Theorem 2. *The following statements are entirely equivalent*: (a) $\bar{G}$ *is Poisson unstable relative to itself*; (b) $\bar{G}$ *admits a portion of an orientable* $(n-1)$*-manifold as a transversal*; (c) *there is a lower semi-continuous function* $U(x)$ *defined on* $\bar{G}$ *whose* (*positive*) *directional derivative along any orbit exists and is positive.* Theorem 3. *If* $\bar{G}$ *is connected, triangulable, and has a vanishing first Betti number, and if the boundary of* $\bar{G}$ *is a differentiable* $(n-1)$*-manifold nowhere tangent to the flow, then statements* (a)–(c) *of Theorem* 2 *are equivalent to*: (d) *the flow is P-irrotational in* $\bar{G}$. Theorem 4. *Whether or not* $\bar{G}$ *is "simply connected" as in Theorem* 3, *statement* (d) *implies statements* (a)–(c) *of Theorem* 2. Corollary. *If* $\bar{G}$ *is a semi-invariant* (*hence rotational*) *set, then the flow in* $\bar{G}$ *cannot be P-irrotational for any P.* (Received January 10, 1957.)

361*t*. R. W. Bass: *Global asymptotic stability of equilibrium* (I).

Notations as in Abstracts No. 359 and No. 360. A rest point x_0 of S is *asymptotically stable in the large* if x_0 is stable and if for every x_0 in E^n, $F(x_0, t)\rightarrow x_0$ as $t\rightarrow +\infty$. *Theorem.* Let x_0 be the only rest point of S. Then the following statements are entirely equivalent: (i) x_0 is asymptotically stable in the large; (ii) $E^n - x_0$ admits a compact connected orientable $(n-1)$ dimensional spherelike manifold K as a transversal; x_0 lies in the conditionally compact component of the complement of K, and (at least) for $n\leq 3$ K is a sphere; (iii) the flow in E^n-x_0 is P-irrotational, x_0 is asymptotically stable in some (small) neighborhood of x_0, and every positive semi-orbit of S is bounded; (iv) the flow is P-irrotational in $E^n - x_0$ and the class C^1 function $V(x) \equiv \int_0^1 u \cdot P(\theta u)f(\theta u)d\theta$, where $u\equiv x/\|x\|$, satisfies (a) $0<V(x)$ for $x\neq 0$, and $V(0)=0$, and (b) $V(x)\rightarrow +\infty$ as $\|x\|\rightarrow\infty$; (v) there exists a C^1 function having properties (a) and (b) of (iv) and also: (c) $f(x)\cdot \text{grad } V(x)<0$ for $x\neq 0$; (vi) the differential equation $\dot{x}=f(x)$ can be re-written as $\dot{x}=-Q(x) \text{ grad } V(x)$, where $V(x)$ satisfies (a), (b), as well as: (d) grad $V(x) \neq 0$ for $x \neq 0$, and (e): grad $V(x) \cdot Q(x)$ grad $V(x) > 0$ for $x \neq 0$. (Received January 10, 1957.)

362*t*. R. W. Bass: *Alternatives to the Routh-Hurwitz criterion.*

A matrix is called stable if all of the real parts of its characteristic roots are negative. Theorem. *A is stable if and only if* det $(\mathfrak{A}) \neq 0$ *and* det $(B_i) > 0$, $(i = 1, \cdots, n)$ *where* $\mathfrak{A}$ *and* B_i *are certain symmetric* ($\mathfrak{A}' = \mathfrak{A}$, $B_i' = B_i$) *matrices composed of the elements of A.* ($\mathfrak{A}$ *and B can be found readily by inspecting A.*) This solves Bellman's Research Problem No. 24, this Bulletin vol. 60 (1954) p. 501. Theorem. *If A is stable and* $x(t)$ *satisfies* $\dot{x} = Ax$ ($\dot{} = d/dt$), *then* $\|x(t)\| \leq \gamma \|x(0)\| \exp(-\lambda t)$, *where* $\gamma = (\sigma_n/\sigma_1)^{1/2}$ *and* $\lambda = \mu_1/2\sigma_n$; σ_1, μ_1 *and* σ_n, μ_n *are respectively the least and the largest, in absolute value, of the (necessarily real, hence easily computable) characteristic roots of* B_1 *and of* $(B_1A + A'B_1)$. See also Abstract No. 169. (Received January 10, 1957.)

363. Barry Bernstein: *On analytic functionals derivable from differential equations.*

The system of equations $dz_i/d\tau = G_i[\tau; z, w(\tau)]$, where z is a complex n-vector and w is a complex m-vector assigns to each function $w(\tau)$, $0 \leq \tau \leq t$, a function $z(\tau)$ such that $z_i(0) = c_i$, a fixed initial value. Thus z is a functional of w. If the quantities $G_1, \cdots, G_n$ are analytic in the components of z and w as well as continuous in all their variables, then this functional is expressible as a series of Riemann Integrals. (Received January 9, 1957.)

364. A. M. Bomberault: *A generalized minimum principle and applications.*

An extension of the minimum principle in the calculus of variations is proposed to study interacting physical systems. Necessary conditions on the functions which satisfy the principle are obtained. These correspond to the Euler-Lagrange equations, boundary conditions and Legendre conditions of the ordinary minimum principle. Using the principle and fixed point theorems, general existence theorems for systems of nonlinear partial differential equations are proved. One theorem concerns a system of equations such that each equation is quasi-linear elliptic in one variable, with its coefficients continuous functions of the remaining variables; and each variable appears as the variable in one and only one equation. For such a system one can define a Rayleigh-Ritz method which converges in the mean to the solution. The proposed principle can be enunciated as finding the solution, using Nash's definition of best play, of the following n-person game. Each player's allowable play is a function from a given function space and his payoff is an integral functional of the plays of all the players. (January 14, 1957.)

365. Frederic Cunningham, Jr.: *Uniform norm structure in Banach spaces.* Preliminary report.

Given for each t in a compact space S a Banach space $X(t)$; a *uniform direct integral* in $X(t)$ over S shall mean a linear subspace X of $\prod_{t \in S} X(t)$ invariant under the ring R of multiplications by continuous functions, with $N_x(t) = \|x(t)\|$ upper semi-continuous on S for each $x = x(t)$ in it, and closed under the norm $\|x\| = \|N_x\|_\infty$. Then R satisfies the Kakutani relation: $\|(f \vee g)x\| = \|fx\| \vee \|gx\|$ for all $f \geq 0$, $g \geq 0$ in R and x in X. A function ring of operators with this property will be called an *M-ring*. *Theorem: Any Banach space X has an essentially unique representation as a uniform direct* *integral over an appropriate compact Hausdorff space S, such that every M-ring on X is a closed subring of $R=C(S)$.* This refines a similar canonical decomposition of X into L^∞-subspaces, a notion dual to the L^1-decompositions reported on earlier (Bull. Amer. Math. Soc. Abstract 60-2-217). (Received January 10, 1957.)

366t. I. S. Gál: *On subbases for uniform structures.*

The object of this note is to give a satisfactory definition of a subbase $\mathcal{S}$ for a uniform structure $\mathcal{U}$ and to give an example where such subbases can be used to show that a given topology is a uniform topology. The notion of a subbase occurs in the present literature (See: J. Kelley, *General topology*, p. 178) but the definition is not satisfactory because not all subbases satisfy the axioms given for a subbase. Definition: *A family $\mathcal{S}$ of nonvoid sets $S\subseteq X\times X$ is a subbase for a uniform structure if it satisfies the following axioms*: (i) *If $S\in\mathcal{S}$ then $I\subseteq S$.* (ii) *If $S\in\mathcal{S}$ then there exist sets $T_1, \cdots, T_m$ such that $(T_1\cap \cdots \cap T_m)\circ(T_1\cap\cdots\cap T_m)\subseteq S\cap S^{-1}$. The uniform structure generated by $\mathcal{S}$ is determined by the base $\mathcal{B}$ consisting of all finite intersections $V=S_1\cap\cdots\cap S_n$ where $S_1, \cdots, S_n\in\mathcal{S}$.* It is easy to see that there are no other subbases for uniform structures than the ones which satisfy (i) and (ii). The proof of the sufficiency is straightforward. Using the present definition it can be proved directly without any reference to complete regularity and the Haar-König theorem that the interval topology of a linearly ordered set is a uniform topology. (Received December 27, 1956.)

367. Fritz John: *Nonadmissible data for partial differential equations with constant coefficients.*

Let $P(\xi_1, \cdots, \xi_n, \tau)$ denote a polynomial of degree m with constant coefficients. The characteristic form $Q(\xi_1, \cdots, \xi_n, \tau)$ consists of the terms of P of degree m. We assume that $Q(0, \cdots, 0, 1)\neq 0$, and that P is "strictly nonhyperbolic" in the sense that the equation $Q(\xi_1, \cdots, \xi_n, \tau)=0$ has a nonreal root τ for some real vector $(\xi_1, \cdots, \xi_n)$. The following theorem is proved: If the polynomial P is irreducible, every solution $u(x_1, \cdots, x_n, t)$ of $P(\partial/\partial x_1, \cdots, \partial/\partial x_n, \partial/\partial t)u=0$, whose Cauchy data on $t=0$ have compact support, vanishes identically. (Received December 17, 1956.)

368t. Dorothy Maharam: *On a theorem of von Neumann.*

Let S be any measure space with a σ-finite complete measure; S need not be separable. Then, from each class x of measurable sets modulo null sets of S, a member X of x may be chosen so that the correspondence $x\leftrightarrow X$ is a finite Boolean isomorphism (i.e., $x\vee y\leftrightarrow X\cup Y$, $-x\leftrightarrow S-X$, and therefore $0\leftrightarrow\emptyset$). This theorem is due to von Neumann (for the separable case see J. für Math. vol. 165 (1931) pp. 109–115); his proof for the nonseparable case has not been published, but the present proof is believed to be different. As a corollary to the method of proof, every open subset of a product of (not necessarily countably many) unit intervals, with the usual topology and measure, is measurable. (Received January 10, 1957.)

369. J. C. Mairhuber, I. J. Schoenberg (p) and R. Williamson: *On variation diminishing transformations for the circle.*

Important special examples of transformations as described in the title were given by G. Polya in his paper *Qualitatives über Wärmeausgleich*, Z. für Angew. Math. und

Mech. vol. 13 (1933) pp. 125–128. Applications of some of these were made by G. Polya and N. Wiener, *On the oscillation of the derivative of a periodic function*, Trans. Amer. Math. Soc. vol. 52 (1942) pp. 249–256. In the present paper Polya's ideas are shown to lead to a wide class of well described periodic kernels furnishing convolution transformations having the variation diminishing property. In analogy with the corresponding problem for the real axis (see I. J. Schoenberg, *On Polya frequency functions*. II: *Variation diminishing integral operators of the convolution type*, Acta Szeged. vol. 12 (1950) pp. 97–106), one would expect the class of kernels thus derived to contain all variation diminishing kernels. Using essentially new ideas the authors are able to construct infinitely many new kernels which are shown not to be included in Polya's class. (Received January 14, 1957.)

370*t*. R. R. Phelps: *Convex sets and nearest points*. II.

If S is a subset of a normed linear space E and $z\in S$ we define $S_z=\{x: \|x-z\| =\inf_{y\in S}\|x-y\|\}$, the (closed) set of all points having z as a nearest point in S. Using a result of Motzkin (Atti Acad. Naz. Lincei, Rend 6 vol. 21 (1935) pp. 773–779) it is easily shown that E is an inner product space if and only if S_z is convex for any $S\subset E$ and $z\in S$. A subset T of E is called a *nearest-point set* if there exists $S\subset E$ and $z\in S$ such that $T=S_z$. It is shown that if E is a complete inner product space then every closed convex subset of E is a nearest-point set. The converse is true if the dimension of E is at least three. If $S\subset E$ a *nearest-point map for* S is a function f which assigns to each point of E a nearest point in S. It is shown that the property "a nearest-point map shrinks distances whenever it exists for a convex set" characterizes inner product spaces of three or more dimensions. The convexity of a closed set in E^n is characterized by the fact that its nearest-point map f shrinks distances. (Received December 10, 1956.)

371*t*. M. O. Reade: *On quasi-conformal maps in three space*. Preliminary report.

Let $y=y(x)$ be a differentiable quasi-conformal map defined for $\|x\|<1$ such that $\iiint_{\|x\|<1}J(y/x)dx<\infty$. Then the following results are obtained. 1. Almost all radii, in the sense of two dimensional measure on $\|x\|=1$, have rectifiable images. 2. There is a set E of capacity zero, on $\|x\|=1$, such that if $\|x_0\|=1$, $x_0\notin E$, then almost all of the radii of sphere $\|x-x_0\|=1/2$ that lie in $\|x\|<1$ have rectifiable images. A method due to Tsuji [Tohoku Mathematical Journal vol. 2 (1950) pp. 113–125; Journal of the Japanese Mathematical Society vol. 5 (1953) pp. 307–320] is used. (Received December 17, 1956.)

372*t*. M. O. Reade: *A radius of univalence for* $\int_0^z e^{-\zeta^2}d\zeta$. Preliminary report.

It is shown that $\int_0^z e^{-\zeta^2}d\zeta$ is univalent for $|z|<R$, where R is the largest positive root of the equation $(4R^4-1)^{1/2}-\text{ArcTan}\,(4R^4-1)^{1/2}=\pi$. $R=2.1$, approximately, which improves upon a result due to Rogozin, $R=\pi/2$ [Rostov. Gos. Univ. Uc. Zap. Fiz-Mat. Fak. volume 32 (1955) pp. 135–137], and Nehari's $R=(((\pi^2+1)^{1/2}-1)/2)^{1/2}$. [Bull. Amer. Math. Soc. vol. 55 (1949) pp. 545–551]. A result due to Umezawa is used [Tohoku Math. J. vol. 7 (1955) pp. 212–228] to obtain our radius R. (Received December 17, 1956.)

373t. M. O. Reade: *On Umezawa's criteria for univalence.*

Recent results due to Umezawa [Tohoku Math. J. vol. 7 (1955) pp. 212–228] are examined. Some of Umezawa's proofs are simplified, and some of his results are generalized. A typical result is the following one, first announced by the author at the International Congress of Mathematicians, Amsterdam, 1954. If $f(z)=1/z+a_0+a_1z+\cdots$ is analytic, with nonvanishing derivative, for $0<|z|<1$, and if $\int_C d \arg df(z)<\pi$ holds for all arcs C on $|z|=r$, for all $0<r<1$, then $f(z)$ is univalent for $0<|z|<1$; moreover, $f(z)$ maps each circle $|z|=r$ onto a close-to-convex curve. The methods used to prove the results contained in this note are extensions of one due to Kaplan [Michigan Mathematical Journal vol. 1 (1952) pp. 169–185]. (Received December 24, 1956.)

374t. Martin Schechter: *On estimating partial differential operators.*

Let G be a bounded n-dimensional domain with closure $\bar{G}$ and boundary B of class C^m. Denote by $C_{mr}(G)$ the set of complex valued functions of class C^m in $\bar{G}$ having derivatives of order $<r$ vanish on B $(0\leq r<m)$. Let A be a linear mth order partial differential operator with complex valued coefficients continuous in $\bar{G}$. Consider the statement (I). There is a constant K such that $\|D^m u\|^2\leq K(\|Au\|^2+\|u\|^2)$ for all $u\in C_{mr}(G)$, where $D^m u$ denotes the generic mth order derivative of u and $\|u\|$ its $L_2(G)$ norm. For $n>2$, (I) holds if and only if A is elliptic in $\bar{G}$ and $m\leq 2r$. For $n=2$, one must consider the vector roots of the characteristic polynomial of A. Properly defined, there are, at B, m roots with imaginary parts perpendicular to B. If p is the number of these roots with imaginary parts directed inward and $q=\max(p, m-p)$, then (I) holds if and only if A is elliptic in $\bar{G}$ and $q\leq r$. (For $n>2$, this is equivalent to the first statement.) The proofs employ a method due to Gårding and Aronszajn. (Received January 7, 1957.)

375t. I. J. Schoenberg and George Polya: *The de la Vallée Poussin summation method is variation diminishing.*

The de la Vallée Poussin means $V_n(x)$ of a function $f(x)$, of period 2π, are obtained by convoluting $f(x)$ with the periodic kernel $(2\cos x/2)^{2n}/\binom{2n}{n}$. It is now shown that, if $f(x)$ is real, the number of real zeros, in a period, of the nth order trigonometric polynomial $V_n(x)$ never exceeds the number of variations of signs of $f(x)$ in a period. As an application of this new property it is shown that the de la Vallée Poussin means furnish elegant necessary and sufficient conditions for a power series $z+a_2z^2+\cdots$ to map the unit circle onto a schlicht convex domain; also criteria for such a power series to map the unit circle onto a star-shaped domain. Another consequence is as follows: If $f(x)$ is the function of support of a plane convex domain D, then also $V_n(x)$ is the function of support of a convex domain D_n. The D_n are all isoperimetric, of nonincreasing areas as n increases and, of course, converge to D as n tends to infinity. (Received January 14, 1957.)

376t. V. L. Shapiro: *A best possible result in the uniqueness of Laplace series.*

Let S designate the surface of the unit sphere in three dimensional Euclidean space and let x denote a point on S. Furthermore for every surface spherical harmonic of order n, $Y_n(x)$, defined on S let the value $(\int_S |Y_n(x)|^2 dS(x))^{1/2}$ be designated by $\|Y_n\|$ where $dS(x)$ is the two-dimensional area element on S. Then in this paper the following theorem is proved: Given a series $\sum_{n=0}^{\infty} Y_n(x)$ of surface spherical harmonics and a fixed point x_1 on S. Suppose that (i) $\|Y_n\| = o(n^{1/2})$; (ii) $\lim_{r\to 1} \sum_{n=0}^{\infty} Y_n(x)r^n = 0$ for x in $S - x_1$. Then the given series is identically zero (i.e. $Y_n(x) = 0$ for all n). This result is in a certain sense best possible for condition (i) cannot be changed to read $\|Y_n\| = O(n^{1/2})$. For letting (x, x_1) designate the scalar product it is well-known that the series $\sum_{n=0}^{\infty} (2n+1)P_n[(x, x_1)]$ is Abel summable to zero for $x \neq x_1$, and a computation shows that $\|(2n+1)P_n[(x, x_1)]\| = O(n^{1/2})$ but not $o(n^{1/2})$. (Received January 4, 1957.)

377. Norbert Wiener and E. J. Akutowicz (p): *The definition and ergodic properties of the stochastic adjoint of a unitary transformation.*

The concepts and results of the present paper depend upon a combination of elementary Hilbert space notions with the theory of the Brownian motion. To begin with, integrals of the form $\int_{-\infty}^{\infty} f(t)dX(t, \alpha)$, where $f \in L^2(-\infty, \infty)$ and $X(t, \alpha)$ is the complex Brownian motion process, are well-defined complex Gaussian random variables, the covariance of two such quantities being $\int_{-\infty}^{\infty} f_1(t)\bar{f}_2(t)dt$. With any unitary transformation U in L^2 there is associated a measure-preserving point transformation $T \equiv T(U)$ defined almost everywhere on the space of the α's, and such that $\int_{-\infty}^{\infty} Uf(t)dX(t, \alpha) = \int_{-\infty}^{\infty} f(t)dX(t, T\alpha)$ almost everywhere. The definition of T is such that from a one-parameter group U^λ, $-\infty < \lambda < \infty$, of unitary transformations there results a well-defined one-parameter group of measure-preserving point transformations T^λ. The ergodic properties of T^λ are determined as follows. *T^λ is weakly mixing if and only if the point spectrum of U^λ is absent. If the spectrum of U^λ is absolutely continuous, then T^λ is strongly mixing.* Using results of Wiener and Wintner, Amer. J. Math. vol. 60 (1938), it is shown that the last statement cannot be reversed. If the point spectrum is present, then ergodicity necessarily fails. These results have application in quantum mechanics. (Received January 9, 1957.)

378*t*. F. M. Wright: *On weighted mean Stieltjes sigma integrals of order p.*

Let $[a, b]$ be a finite closed interval of the real x axis, and let f, g be real-valued functions on $[a, b]$. Let p be any integer $\geqq 2$, and let $r = (r_1, r_2, \cdots, r_p)$ be any p real numbers satisfying $r_1 + r_2 + \cdots + r_p = 1$. If $P = (a = x_0 < x_1 < \cdots < x_n = b)$ is any partition of $[a, b]$, we consider sums $S(P) = \sum_{i=1}^{n} [\sum_{j=1}^{p} r_j \cdot f(x_{i,j})] \cdot [g(x_i) - g(x_{i-1})]$, where $x_{i-1} = x_{i,1} < x_{i,2} < \cdots < x_{i,p} = x_i$ for $i = 1, 2, \cdots, n$. If $S(P)$ has a finite limit in the sense of successive subdivisions, it is said that f has a mean Stieltjes sigma integral of order p and weight r with respect to g on $[a, b]$. It is shown that this integral exists in case f has only discontinuities of the first kind and g is monotone nondecreasing, and a formula is obtained for the integral in this case involving the Lebesgue-Stieltjes integral $(LS)\int_a^b f(x)dg(x)$ and the weights r_1, r_p. For $p = 3$ and such an f, g, the integral either assumes the value $(LS)\int_a^b f(x)dg(x)$ for all (r_1, r_2, r_3) or else assumes an arbitrary value for a suitable (r_1, r_2, r_3). The usual theorem on integration by parts for the Lebesgue-Stieltjes integral follows. (Received January 11, 1957.)

GEOMETRY

379*t*. M. O. Reade: *On certain conformal maps in three space.*

In this note the following theorem is proved. If $y = y(x)$ is a differentiable homeomorphism of $\|x\| < 1$ onto $\|y\| < 1$, with nonvanishing Jacobian, and if $y(0) = 0$, then

$y(x)$ is a rotation. The method is an elementary one depending upon properties of subharmonic functions. (Received December 17, 1956.)

380*t*. Stephen Smale: *A classification of immersions of the 2-sphere.*

An immersion of one C^1 manifold in another is a regular map (a C^1 map whose Jacobian is of maximum rank at every point) of the first into the second. A homotopy of an immersion is called regular if at each stage it is regular and if the induced homotopy of the tangent bundle is continuous. If M is a C^1 manifold, $F_2(M)$ denotes the bundle of ordered 2-frames of M. Let N be a C^2 manifold of dimension greater than two and let $x_0 \in F_2(S^2)$ and $y_0 \in F_2(N)$. It will be assumed hereafter that all immersions of S^2 in N are C^2 in some neighborhood of the base point of x_0 and their induced maps send x_0 into y_0. If f, g: $S^2 \rightarrow N$ are given immersions, an invariant $\Omega(f, g) \in \pi_2(F_2(N); y_0)$ is defined. The following theorems are proved. Two immersions f, g: $S^2 \rightarrow N$ are regularly homotopic if and only if $\Omega(f, g) = 0$. If $\Omega_0 \in \pi_2(F_2(N), y_0)$ and an immersion f: $S^2 \rightarrow N$ are given there is an immersion g: $S^2 \rightarrow N$ such that $\Omega(f, g) = \Omega_0$. For example any two C^2 immersions of S^2 in E^3 are regularly homotopic. Given even $\gamma \in H_2(S^2)$, then there is an immersion of S^2 in E^4 such that the characteristic class of the normal bundle is γ. Any two such C^2 immersions are regularly homotopic. (Received January 2, 1957.)

Logic and Foundations

381*t*. J. W. Addison and S. C. Kleene: *A note on function quantification.*

The question raised in Bull. Amer. Math. Soc. vol. 61 (1955) p. 211, whether for $k > 0$ each predicate expressible in both the $k+1$-function-quantifier forms is hyperarithmetical in predicates expressible in the k-function-quantifier forms, is answered in the negative. The predicates of the finite $\mathfrak{L}$-hierarchy $\mathfrak{L}_2, \mathfrak{L}_3, \mathfrak{L}_4, \cdots$ (after $\mathfrak{L}_0, \mathfrak{L}_1$) and its extension into the transfinite up through the constructive third number class lie properly between $\mathfrak{N}_1$ and $\mathfrak{N}_2$ in hyperdegree. (Received January 2, 1957.)

382*t*. Angelo Margaris: *A problem of Rosser and Turquette in many-valued logic.*

An affirmative solution is given for the following problem posed by Rosser and Turquette [*Many-valued logics*, Amsterdam, 1952, p. 110]. "Given an S and T such that $1 \leqq S < T < M$, can one define a many-valued logic so that if P is a statement formula with the truth-value function p, then the following conditions are satisfied? (1) If p is such that we always have $p \leqq S$, then $\vdash P$. (2) If p is ever such that $p > T$, then not $\vdash P$. (3) If neither condition 1 nor condition 2 is satisfied, then we have $\vdash P$ in some cases and not $\vdash P$ in other cases." A solution is given for the case $S = 1$, $T = 2$, $M = 3$, and is then extended to the general case. A further extension solves the problem on the level of the restricted predicate calculus. The key device is the use of two valuations, one of which is the classical one. (Received December 11, 1956.)

Statistics and Probability

383*t*. K. S. Miller and R. I. Bernstein: *Generalized Rayleigh processes.*

Let $\mathfrak{X}_1, \mathfrak{X}_2, \cdots, \mathfrak{X}_N$ be N independent stationary Gaussian processes with means

a_i respectively and with correlation function $\psi(\xi)$. A generalized Rayleigh process $\mathfrak{R}$ is defined as $\mathfrak{R}^2 = \sum_{n=1}^{n} \mathfrak{X}_n^2$. Properties of $\mathfrak{R}$ including second and third order distributions are investigated. Typical results are: (i) The joint probability density of $\mathfrak{R}$: $p(R_1, R_2) = (R_1R_2)^{N/2}[\psi_0^{(N+2)/2}(1-\lambda^2)\Gamma(N/2)(2\lambda)^{(N-2)/2}]^{-1/2} I_{N/2-1}(\lambda R_1 R_2 \psi_0^{-1}(1-\lambda^2)^{-1}) \cdot \exp[-(R_1^2+R_2^2)/2\psi_0(1-\lambda^2)]$ where $I_{N/2-1}$ is a Bessel function, $\lambda = \psi(\xi)/\psi_0$ is the normalized autocorrelation function and $a_1 = a_2 = \cdots = a_N = 0$. (ii) The correlation function of $\mathfrak{R}$: $C(x) = [2\psi_0(1-\lambda^2)^{\alpha+1}][\Gamma(\alpha)]^{-1} D^{\alpha-1}[E(\lambda)/(1-\lambda^2)^2 - K(\lambda)/2(1-\lambda^2)]$ where $N = 2\alpha$, $D^{\alpha-1}$ is the $(\alpha-1)$st derivative with respect to λ^2 and E and K are complete elliptic integrals. Applications to radar and other problems are considered. A brief survey is made of similar formulas which occur in the literature. (Received January 2, 1957.)

384t. Valdemars Punga: *On solution of Dirichlet problem by Monte Carlo method in regions covered by a nonuniform network.*

Consider the two-dimensional $\nabla^2\psi = 0$, ψ prescribed at the boundary of the region. The stochastic model for the above stated Dirichlet problem is usually formulated in the following way: subdividing the region of integration into square network of mesh size h, a random walk along the mesh lines starts at an interior nodal point (x, y) and ends upon reaching any boundary point. It is assumed that it is equally likely to proceed from (x, y) to any one of the four neighboring points, i.e. the probability of each of the four directions is 1/4. If $p(x, y)$ is the probability starting from (x, y) to reach a boundary point B, then (1) $p(x, y) = p(x+h, y)/4 + p(x, y+h)/4 + p(x-h, y)/4 + p(x, y-h)/4$, $p(B) = 1$, $p = 0$ at all other points of the boundary. Now, if the distances from the point (x, y) to the four neighboring points are not all equal (what usually happens near the curved boundary), (1) is not adequate for definition of $p(x, y)$. It is shown in this paper that, in case the distances from (x, y) to $(x+h, y)$, $(x, y+h)$, $(x-h, y)$ and $(x, y-h)$ are h_1, h_2, h_3, and h_4, respectively, (1) can be replaced by (2) $p(x, y) = A_1 p(x+h, y) + A_2 p(x, y+h) + A_3 p(x-h, y) + A_4 p(x, y-h)$, where $A_1 = h_2h_3h_4/(h_1h_3+h_2h_4)(h_1+h_3)$, $A_2 = h_1h_3h_4/(h_1h_3+h_2h_4)(h_2+h_4)$, $A_3 = h_1h_2h_4/(h_1h_3+h_2h_4)(h_3+h_1)$, $A_4 = h_1h_2h_3/(h_1h_3+h_2h_4)(h_4+h_2)$. Since $p(x, y)$ is a Green function of Dirichlet problem, the solution of $\nabla^2\psi = 0$ follows in usual pattern. Note that $\sum_{i=1}^{4} A_i = 1$, so that $A-s$ can be interpreted as the probabilities of proceedings to the neighboring points. (Received January 11, 1957.)

TOPOLOGY

385t. R. W. Bass: *Generalizations to n dimensions of the combined theorems of Poincaré-Bendixson, Poincaré-Denjoy, and Haas* (I).

Notations as in Abstract No. 359, except that E^n is replaced by an n-manifold D^n. Other definitions as in Nemickii, Amer. Math. Soc. Translations, no. 103. Define a dynamical system S to be *ultimately dissipative* if its center is compact. define S to be *smooth* if every Poisson stable orbit is uniformly Lyapunov stable relative to (at least) itself, and to be *extremely smooth* if in addition every quasi-minimal set of dimension $\leqq (n-2)$ is locally connected. S is defined to be *very regular* if every compact quasi-minimal set of S is actually minimal. S is defined to be *nice* if every compact quasi-minimal set containing no rest point is a manifold. Theorem 1. *The positive limit set of any bounded asymptotic semi-orbit is nowhere dense.* Theorem 2. *If S is very regular and ultimately dissipative then the closure of any nonasymptotic orbit is either* (i) *nowhere dense, or* (ii) *equal to* D'', *in which case S has no rest points and* D^n

is a rotational manifold. Corollary 1. *If S is very regular and ultimately dissipative then either the positive limit set of any given orbit is nowhere dense, or the orbit is dense on* D^n *and* D^n *is a rotational manifold.* (Received January 10, 1957.)

386*t*. R. W. Bass: *Generalizations to n dimensions of the combined theorems of Poincaré-Bendixson, Poincaré-Denjoy, and Haas* (II).

Theorem 3. *If S is ultimately dissipative and nice, and if the presence of a rest point in the positive limit set of an orbit implies that there are no other points in the limit set, then there are only two kinds of orbits in S*: (a) *asymptotic orbits, and* (b) *Poisson stable orbits dense on rotational manifolds. Moreover, each bounded positive semi-orbit of type* (a) *contains at least one of type* (b) *in its positive limit set.* The dimensional restrictions in the following theorem are nontrivial. Theorem 4. *Let* $D^n = E^n$. *If S is ultimately dissipative and smooth, then there are only three kinds of orbits of S*: (a) *asymptotic orbits, and* (b) *almost-periodic orbits ergodic on* (b_1) *k-dimensional tori,* $k \in \{0, 1, \cdots, n-1\}$; *or* ($b_2$) *k-dimensional solenoids,* $k \in \{0, 1, \cdots, n-2\}$. *Moreover, each bounded positive semi-orbit of type* (a) *contains at least one orbit of type* (b) *in its positive limit set.* Theorem 5. *If, in addition to the hypotheses of Theorem* 4, *S is extremely smooth, then the orbits of type* (b_2) *cannot occur.* One can show by examples that Corollary 1 and Theorems 4 and 5 cannot be improved. If S is defined by a C^1 vector field in the plane, or by a C^2 vector field on a compact orientable 2-manifold, then S is very regular, nice, and extremely smooth. Since the only rotational orientable 2-manifold is the 2-torus, Corollary 1 and Theorems 3 and 5 are each generalizations of the (combined) theorems of Poincaré-Bendixson, Poincaré-Denjoy, and Haas. (Received January 10, 1957.)

387*t*. R. H. Bing: *Pseudo-arc is only nondegenerate homogeneous chainable compact continuum.*

It is shown that if M is a homogeneous snake-like continuum, then M contains an end point p in the sense that for each positive number ϵ there is an ϵ-chain covering M one of whose end links contains p. This insures that M is hereditarily indecomposable. It is known [R. H. Bing, *Concerning hereditarily indecomposable continua*, Pacific J. of Math. vol. 1 (1951) pp. 43–51] that any two nondegenerate hereditarily indecomposable chainable compact continua are topologically equivalent and that each is a pseudo-arc. (Received January 7, 1957.)

388*t*. M. R. Demers and Herbert Federer: *The norm of a cohomology class, with applications to the theory of area.*

With each integral Čech cohomology class $\lambda \in H^k(X, A)$, the authors associate the norm $|\lambda|$, a nonnegative integer or ∞. In case X is a k dimensional finitely triangulable space, $|\lambda|$ measures the minimal multiplicity with which a continuous map, representing λ, of (X, A) into a relative k-cell (S, T) must cover S. Using this norm, the authors define the multiplicity $M(f, y)$ with which a map f of X into Euclidean k-space assumes the value y. It is proved that the *Lebesgue area* of f equals $\int M(f, y)dy$, an integral with respect to *Lebesgue measure* over k-space. Furthermore, the Lebesgue area of a map f of X into n-space, where $n > k$, does not exceed the sum of the Lebesgue areas of the $\binom{n}{k}$ principal projections of f into k-space, provided the image of f has $k+1$ dimensional Hausdorff measure zero. Previously such results had been established only for the special case in which X can be embedded in a k dimensional manifold. (Received December 21, 1956.)

389t. M. R. Demers and Herbert Federer: *On 2 dimensional Lebesgue area.*

Suppose X is a finitely triangulable 2 dimensional space and f is a *light* continuous map of X into n-space. It is proved that the Lebesgue area of f does not exceed the sum of the Lebesgue areas of the $\binom{n}{2}$ principal projections of f into 2 space. In obtaining this result, known previously only for the case in which X can be embedded in a 2-manifold, the concepts described in the preceding abstract are used, as well as certain information concerning the topological structure of the sections of f by $n-1$ dimensional planes, together with a dimension theoretic modification theorem. (Received December 21, 1956.)

390. I. S. Gál: *On a generalized notion of compactness.*

One can introduce a generalization of the notion of compactness which permits a more systematic treatment of compact uniform spaces: *A topological space X is $\langle m, n\rangle$-compact if from every open covering $\{0_i\}$ $(i \in I)$ of X whose cardinality* card I *is at most n one can select a subcovering $\{0_{i_j}\}$ $(j \in J)$ of X whose cardinality* card J is at most m. A $\langle 1, n\rangle$-compact space is by definition an n-compact space and $\langle 1, \infty\rangle$-compactness reduces to compactness in the usual sense. An $\langle \alpha_0, \infty\rangle$-compact space is a Lindelöf space. X is $\langle m, n\rangle$-compact if and only if every family $\{C_i\}$ of closed sets $C_i \subseteq X$ having the m-intersection property also has the n-intersection property. In the following theorems let X be a uniform space and let u be a cardinal such that there is a base for a uniform structure of X whose cardinality is at most u. I. *If X is $\langle m, u\rangle$-compact for some $m<u$ then X is $\langle m, \infty\rangle$-compact.* II. *If X is $\langle m, n\rangle$-compact for some m and n satisfying $m<n$ and $u<n$ then X is $\langle m, \infty\rangle$-compact.* III. *X is completely $\langle m, \infty\rangle$-compact for some $m \geqq u$ i.e. every subspace of X is $\langle m, \infty\rangle$-compact if and only if every subset of cardinality greater than m has an accumulation point in X and thus also in the set itself.* (Received January 8, 1957.)

391t. S. L. Gulden: *A set of invariants for relative singular homotopy type.*

The above mentioned invariants are essentially obstructions in a suitable set of type complexes. The resulting classification is given in a purely abstract manner without recourse to topological spaces. In order to do this an abstract homotopy theory for complete semi-simplicial complexes is introduced, the concept of a relative minimal complex is defined and an abstract analogue for the space of paths is employed. The classification yields as a special case the Postnikov invariants for absolute singular homotopy type. (Received January 10, 1957.)

392t. W. S. Mahavier: *A theorem on spirals in the plane.*

It follows from the results of J. T. Mohat (Bull. Amer. Math. Soc. Abstract 60-6-772) that if R is a rectangular disc in the Euclidean plane such that each side of R is either vertical or horizontal, then there is a reversibly continuous transformation of R into itself such that if I is a vertical interval whose endpoints belong to the boundary of R and which lies, except for its endpoints, wholly in the interior of R, then the image of I under this transformation is a spiral with only one whirl point. In the present paper it is shown that there is a reversibly continuous transformation of R into itself such that if I is either a vertical or a horizontal interval whose endpoints belong to the boundary of R and which lies, except for its endpoints, wholly in the interior of R, then the image of I under this transformation is a spiral with only one whirl point. (Received January 7, 1957.)

393t. G. D. Mostow: *Star bounded coverings.*

A covering C of a space E is called *star-finite* if each set of C meets only a finite number of others; C is called *star-bounded* if there is a finite number b such that each set of C meets no more than b others. *Theorem.* Any open covering of a finite dimensional separable metric space E admits a star bounded refinement. As a direct consequence, one obtains the following: *Corollary.* Let E be a finite dimensional metric space. Then any fiber bundle over E admits a finite number of local cross-sections whose projections cover E. (Received January 10, 1957.)

394t. G. D. Mostow: *On the fundamental group of a homogeneous space.*

P. A. Smith has proved that the first Betti number of a Lie group G cannot exceed the dimension of G. Inasmuch as the fundamental group of a Lie group is abelian, the following result is a generalization of Smith's. *Theorem.* If the fundamental group F of a homogeneous space M (i.e. a Lie group operates transitively on M) is solvable, then F is finitely generated and its rank cannot exceed the dimension of M. In particular, the first Betti number of M cannot exceed the dimension of M if F is solvable. On the other hand if F is not solvable, examples show that the first Betti number of M may be infinite even with dim $M=3$. It follows directly from this theorem that a discrete solvable subgroup of a connected Lie group is finitely generated. More generally, a solvable component group of a closed subgroup is finitely generated. (Received January 10, 1957.)

395t. G. D. Mostow: *Equivariant embeddings in euclidean space.*

Let G be a compact Lie group of transformations on a completely regular space E. A *pseudo-section* to the orbit at p is a closed subset K satisfying (a) $gK\subset K$ for $g\in G_p$, the isotropy subgroup of p; (b) $gK\cap K$ is empty for $g\notin G_p$; (c) GK is a neighborhood of p. *Theorem.* There is a pseudo-section to each orbit of G at each point of E. This theorem has been proved under certain topological restrictions on E by Montgomery and Yang using other methods. The orbits through points p and q are called equivalent if there is an $h\in G$ such that $gp\leftrightarrow ghq$ is a well-defined one-to-one correspondence, i.e. $G_p=G_{hq}=hG_qh^{-1}$. *Theorem.* If E is a finite dimensional separable metric space and G has but a finite number of inequivalent orbits in E, then there exists an *equivariant embedding* of (E, G) into euclidean space, i.e. a homeomorphism ϕ of E into finite dimensional euclidean space and a unitary representation θ of G such that (1) $\phi(gp)=\theta(g)\phi(p)$ and (2) $\theta(G)$ keeps only the origin fixed if G has no fixed points in E. (Received January 10, 1957.)

396t. G. D. Mostow: *On a conjecture of Montgomery.*

It has been conjectured by Montgomery that a compact Lie group operating on a compact manifold admits at most a finite number of inequivalent orbits (cf. *Equivariant embeddings* abstract for definitions). E. E. Floyd has proved the conjecture is correct if G is a toral group (i.e. compact connected abelian) operating on an orientable manifold. Here the theorem is extended to arbitrary compact Lie groups operating on any compact manifold. The result is "best possible" in the sense that if either the group or the space is not compact, the result may fail. Upon adding a point at infinity to euclidean space, it is seen that Montgomery's conjecture holds for a compact Lie group operating in euclidean space E. Consequently (E, G) admits an equivariant embedding in euclidean space. (Received January 10, 1957.)

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