RESEARCH PROBLEMS

21. Richard Bellman: *Dynamic programming.*

Given N coins, k of which are defective, either lighter or heavier than the other $N-k$ coins which are assumed to be of equal weight, and a balance, determine the weighing procedures which minimize the number of weighings required to separate the defective coins from the ordinary coins. Consider the following two cases

a. The k defective coins are all of the same weight, heavier or lighter than the regular coins.

b. The k defective coins are all of different weight.

Also determine the weighing procedures which minimize the expected time required to determine the defectives.

This is a particular case of the general "sorting" problem where an individual element of a set may be characterized by a number of properties and we have a number of testing devices for determining these properties. (Received May 31, 1955.)

22. Richard Bellman: *Analysis.*

The derivative of the gamma function satisfies the recurrence relation

$$\Gamma'(x+2) = (2x+1)\Gamma'(x+1) + x^2\Gamma'(x),$$

for $x>0$. Can one derive from this equation a convergent continued fraction expan sion for $\Gamma'(x)/\Gamma'(x+1)$, or a related expression, which can be used either

a. To obtain a rapid method for computing $\Gamma'(1)$, the negative of Euler's constant, or

b. To obtain some results concerning the arithmetic character of Euler's constant? (Received May 31, 1955.)

23. Richard Bellman: *Number theory.*

There are a number of numerical techniques available for determining the maximum over the x_i of the linear form, $L(x)=\sum_{i=1}^{n} a_i x_i$, subject to the linear constraints $\sum_{j=1}^{n} b_{ij}x_j \leqq c_i$, $i=1, 2, \cdots, M$, whenever it exists. Can one obtain a usable algorithm for the cases where we impose additional constraints of the form

a. $x_i=0$ or 1, for $i=1, 2, \cdots, N$, or

b. x_i is zero or a positive integer? (Received May 31, 1955.)

24. Sherman Stein: *Number theory.*

Let a be a positive rational fraction with odd denominator and $u_n=(2n+1)$, $n=1, 2, \cdots$. Let b_1 be the smallest of the u_i satisfying $a-(u_i)^{-1}\geqq 0$. Having defined $b_1, b_2, \cdots, b_n$, define b_{n+1} as the smallest u_i, $u_i>b_n$, with $a-(b_1)^{-1}-\cdots-(b_{n+1})^{-1}\geqq 0$. Is the sequence $b_1, \cdots, b_n, \cdots$ finite for each a? (Received May 23, 1955.)

25. Sherman Stein: *Geometry.*

Let $J\subset R_2$ be a rectifiable Jordan curve, with the property that for each rotation R, there is a translation T, depending on R, such that $(TRJ)\cap J$ has a nonzero length. Must J contain the arc of a circle? (Received May 23, 1955.)