ing length and orthogonality. These yield a definition for the inner (or scalar) product $\mathfrak{x}\mathfrak{y}$ of two vectors $\mathfrak{x}$ and $\mathfrak{y}$. (American readers may at first be confused by the author's use of a dot when a vector is multiplied by a scalar but *not* when two vectors are multiplied together!) The other topics in this part include congruent transformations, complex geometry, and quadrics. In Part III, projective n-space is derived from affine $(n+1)$-space by identifying the points of the former with the classes of parallel vectors in the latter. After discussing cross-ratio, collineations and correlations, polarities, and the projective theory of quadrics, the author shows how the affine and metrical geometries can be derived from projective geometry. It is unfortunate that he failed to take full advantage of his division of the book into three parts. Content (i.e., area, volume, etc.) is considered in Part II, and barycentric coordinates in Part III, whereas both these subjects properly belong to affine geometry. The text and the 76 figures are clear and accurate. The book ends with a short bibliography, a full index, and a useful list of symbols.

H. S. M. COXETER

Rational fluid mechanics, 1687–1765. By C. Truesdell. Editor's introduction to vol. II 12 of Euler's works. Offprint from Leonhardi Euleri Commentationes Mechanicae ad Theoriam corporum fluidorum pertinentes (Euleri Opera Omnia, Series II, vol. 12.) Zürich, Füssli, 1954. 125 pp.

The volumes of the great Euler edition have in recent years been provided with excellent introductions, which help to clarify the astonishing achievements of this eighteenth century mathematician. Professor Truesdell has written one of these introductions; it is the preface to the first volume of Euler's contributions to the theory of fluid motion, and an interesting piece of work it is. We find in his essay not only an extensive account of Euler's main papers on hydrodynamics, but also a report on the achievements of those authors who, from Archimedes on, prepared the way to Euler's theory. We thus have in this introduction an accurate and comprehensive account of an important field of early science, a field never before so carefully investigated; a valuable contribution to the history of science in general and to the appreciation of Euler's hydrodynamical work in particular.

We owe to Euler what we may call "classical" hydrodynamics, the theory underlying the science of our present textbooks, which he established so thoroughly that all authors before him can safely be classified as "prehistoric." However, in contrast to Euler's mechanics of points and rigid bodies, his hydrodynamics has not come to us in the form of a textbook, but only in a series of papers. The principal four papers, republished in this tome II 12, and analyzed by Professor Truesdell in his introduction, were written between 1752 and 1755. The oldest of them, the "Principia motus fluidorum," was the last to see the light; we find it in the Commentaries of the St. Petersburg Academy published in 1761. The three other essays, the "Principes généraux de l'état d'équilibre des fluides," with two sequels, form a consecutive set in vol. II of the Memoirs of the Berlin Academy of 1755, published in 1757. Students of Euler's work will be encouraged when they see that these Berlin papers are written in French. This also helped Euler's contemporaries and students, so that his hydrodynamics became best known through these articles, with their 145 pages long enough to form a book by itself. The St. Petersburg paper is in Latin, but this need not discourage too many souls; Euler was no Cicero and his Latin easily gives up its secrets to any one of good will with some high school knowledge of the language and a dictionary.

The St. Petersburg paper has often been quoted because it contains the continuity equation and the dynamical equations for ideal compressible fluids. It also contains the "Laplace" equation $\Delta V=0$ and the "Killing" equation for locally and instantaneously rigid motion. The theory in full maturity can be found in the three Berlin papers; the editor calls their appearance a turning point in the history of physics. Indeed, it is here that a definite break is made with the principle that mechanics is necessarily a theory of little material particles: Euler's hydrodynamics is a field theory of continuous matter, and as such the prototype of all later aether, electric, magnetic and caloric theories. It also led Euler to the composition of that other masterpiece, his theory of rigid bodies, with its famous "Euler equations." The four papers on fluid bodies have also other important traits; for instance, they offer for the first time an exact separation of the kinematical and dynamical equations of the theory of continua. Another feature is the lucid formulation of the ancient field of hydrostatics.

Some more of Euler's papers, mostly later ones, are analyzed in this book, by which we can see how Euler dealt with such concepts as resistance, convection currents, and the influence of temperature. Of special interest is also Professor Truesdell's analysis of the works of Euler's predecessors and contemporaries, among whom the Bernoullis and D'Alembert stand out. The reader has a chance to get an idea of what Daniel Bernoulli's famous "Hydrodynamica" of 1738 was about

—the book in which we find the kinetic derivation of Boyle's law. Here entertainment is waiting, because of the edifying spectacle of Daniel's father John, the Groningen professor, publishing a special book to tear his son's claims down; a book with some of the choicest bombast ever written by a loving father. However, as Professor Truesdell points out, the book's moral aspects should not blind us to its actual merits: John Bernoulli's "Hydraulics" of 1743 had one thing his son's "Hydrodynamica" lacked: method. In the words of the editor: "For the first time, fluid mechanics appeared in its proper station in the great system of classical mechanics." Euler recognized this fact in a nice Latin note to John Bernoulli, which is printed on p. 1 of tome II 12, and which the editor partly reproduces in translation.

This is enough to show that Professor Truesdell has given us a special treat; it is enhanced by the generous way in which his essay is published—the way all volumes of this magnificent edition are published. Let no student of the history of mathematics believe that he need not study this book because it contains only mechanics, that is, physics. With Euler, as with all great mathematicians before the middle of the last century, applied and theoretical science are so interrelated that separation is impossible. For instance, those who only deal with what officially is called mathematics, may well miss the fact that "Laplace's equation" $\Delta V=0$ appears first in Euler's hydrodynamics, and that "Lagrangian multipliers" are equally found in Euler's work on fluids. The "Cauchy-Riemann" equations also occur first in a paper on hydrodynamics, this time written by D'Alembert, and duly quoted by Professor Truesdell.

At the end of the book is a chapter on the hydrodynamical theory in Euler and Lagrange's researches on the theory of sound. It is in a paper by Euler of 1759 (printed in tome III 1) that we first find the wave equation as the expression of aerial propagation in one dimension. Euler's starting point was his hydrodynamical theory, and Professor Truesdell remarks: "It is the first example of a precise *analogy* in mathematical physics." In a subsequent paper of 1759 Euler returns to the wave equation, now in three dimensions, and tackles it for cylindrical and spherical waves. It is near this point that we are warned against the use of the terms "Lagrangian" versus "Eulerian" when we make the "material" or "spatial" choice of coordinates in describing the motion of a continuum, a discrimination which goes back to Dirichlet. However, Euler has claim to both methods, as already Riemann explained to Hankel. This is just another example of how little Euler was—and still is—known.

Essays like this one by Professor Truesdell can help to overcome this lack of information. It is not just a question of historical piety and correct assignment of priority. Euler actually is good reading, and we must consider Professor Truesdell's introduction as an invitation to read him, like Christopher Morley's introduction to Shakespeare. After all, as Jacobi already said: "Today it is quite impossible to swallow a single line by D'Alembert, while we still can read most of Euler's works with delight."

D. J. STRUIK

Algèbre locale. By P. Samuel. (Mémorial des Sciences Mathématiques, no. 123.) Paris, Gauthier-Villars, 1953. 76 pp. 950 fr.

The object of this monograph is a presentation of the theory of local rings and their generalizations, the semilocal rings and the M-adic rings. Those aspects of the theory are dealt with which are valid in rings of arbitrary dimension. Thus the special properties of one-dimensional rings such as rings of p-adic integers or of power series in one variable are not included.

All rings considered are commutative and have an identity element. *Local rings* were introduced some fifteen years ago by Krull; they are Noetherian rings with a single maximal ideal. More generally, a *semilocal ring*, in the sense of Chevalley, is a Noetherian ring A with only a finite number of maximal ideals. If M is their intersection, then $\bigcap_{n=1}^{\infty} M^n=(0)$, and the sequence of ideals $\{M^n\}$ defines a Hausdorff topology on A. More generally, a Noetherian topological ring in which the topology is Hausdorff and is defined by the powers M^n of some ideal M is called *M-adic*. A *Zariski ring* is an M-adic ring in which every ideal is a closed set. M-adic rings and Zariski rings were introduced by Zariski (who, however, called the latter type *generalized semilocal*). The semilocal rings of Chevalley are Zariski rings, as are all complete M-adic rings. The more elementary properties of these rings are considered in Chapter I. Here are discussed their completions, homomorphisms, quotient rings, direct decompositions, and finite extensions.

In Chapter II we are concerned with a semilocal ring A and a defining ideal V of A—that is, an ideal V in A such that $M^t \subset V \subset M$, t being some integer and M the product of the maximal ideals of A. It is then proved that the length of A/V^n as an A-module is a polynomial of $P_V(n)$ for n sufficiently large. The degree d of this polynomial is independent of V and is, in fact, the minimum number of generators in any defining ideal. It is called the dimension of A and thus coincides with the notion of dimension of a local ring in the