There are a few minor misprints, but on the whole the book has been very well printed and proofread—this latter not always being the case today. The book is a valuable addition to the literature on line geometry, and a good translation into English makes it available to many more readers.

ALICE T. SCHAFER

Drei Perlen der Zahlentheorie. By A. J. Chintschin. Trans. from the 2d (1948) Russian ed. by W. v. Klemm. Berlin, Akademie-Verlag, 1951. 61 pp. 6.50 DM.

Three pearls of number theory. By A. Y. Khinchin. Trans. from the 2d (1948) Russian ed. by F. Bagemihl, H. Komm, and W. Seidel. Rochester, Graylock, 1952. 64 pp. $2.00.

The author, one of the leading Russian mathematicians, attempts to present three important recent results in such a way that they can be understood without much knowledge of number theory. He tries to create admirers of number theory by showing that elementary number theory is not yet a finished field since highly interesting new results were obtained by ingenious methods during the last few years, and further progress can be expected.

The author has been extremely successful in writing an excellent book for trained mathematicians. However, it is stated in the German edition that the book can be read by students of the upper grades of high schools and amateurs of mathematics and in the American edition that it can be understood by beginning college students. It is the reviewer's opinion that this is impossible. No such reader could study it with success. Even if he could understand some pages, he would not recognize the beauty of the results and their proofs.

The simplest part of the book is certainly the first chapter. The reviewer has proved its results in his classes at the University of Berlin and at the University of North Carolina, and he knows from this experience that it is not easy to present these theorems even to students who had taken a course in number theory.

In the first chapter the author proves the following theorem of van der Waerden published under the title *Beweis einer Baudetschen Vermutung*, Nieuw Archief voor Wiskunde (2) vol. 15 (1927) pp. 212–216. Let k and l be arbitrary integers. There exists a constant $W = W(k, l)$ such that for any distribution of the numbers $1, 2, \cdots, W$ into k classes at least one of the classes contains an arithmetic progression of l terms.

Khinchin states the history of this theorem as follows. When he arrived in Goettingen in the summer of 1928, the topic of the day was this result of van der Waerden proved in Goettingen a few days before. One of the young mathematicians there had come upon this problem. Everyone believed it was a simple problem, but all attempts by the mathematicians of Goettingen and many foreign visitors there remained unsuccessful. After several weeks of strenous efforts the problem was solved by van der Waerden who was in Goettingen at that time.

This history is not quite correct. While van der Waerden attributes the problem to Baudet, it is much older and was an unsolved problem for many years. It is due to I. Schur. In 1906, E. Jacobsthal had published his results on sequences of 3 and 4 quadratic residues. Schur conjectured then that there exist sequences of l quadratic residues for every l and all sufficiently large primes, and he conjectured for the proof of this result the proposition attributed to Baudet for $k=2$ in a little stronger form, namely that the difference of the sequence belongs to the same class as the sequence.

van der Waerden did not find his proof only a few days before Khinchin arrived in Goettingen in the summer of 1928; he had read it as a paper already in September 1927 at the meeting of the Deutsche Mathematiker-Vereinigung. The paper was published in 1927. The reviewer was present when von Neumann, just returned from the meeting of the D.M.V., came to see Schur to inform him that his conjecture was proved. Schur was highly excited, but on the other hand disappointed that only the weaker form of the conjecture was proved which did not give the theorem on the sequences of residues. But a few days later, the reviewer succeeded in proving Schur's conjecture for the quadratic residues, the corresponding theorems for the quadratic non-residues, the kth power residues, and later the theorem for the $k-1$ classes of kth power non-residues while Schur proved the stronger form of van der Waerden's theorem (Sitzungsber. Preussische Akademie d. Wiss. Phys.-Math. Kl. (1928) pp. 9–16 and (1931) pp. 329–341). For the proof of each of these results the theorem of van der Waerden is used. Although these results are the only known applications of the theorem of van der Waerden they are not mentioned in the book. One may wonder why the theorem of van der Waerden is called a theorem of number theory without these applications since actually it is a theorem of combinatorial analysis.

In the second chapter the theorem of Mann is proved. In his important investigations on Goldbach's theorem, L. Schnirelmann defined the sum of two sets $A=(a_i)$ and $B=(b_j)$ of integers as the set containing the elements of A, those of B, and all numbers of the form a_i+b_j. Let $A(n)$ be the number of elements $a_i \leqq n$. We define the Schnirelmann density $d(A)$ as the lower bound of the quotients $A(n)/n$. Schnirelmann proved $d(A+B)=1$ for $d(A)+d(B)\geqq 1$, and Landau conjectured in 1931 that

$$d(A + B) \geqq d(A) + d(B) \text{ if } d(A) + d(B) < 1.$$

This problem was studied in a number of papers, but for some years it was proved only for special cases, and other estimates were obtained (see the report of H. Rohrbach, Jber. Deutschen Math. Verein. vol. 48 (1938) pp. 199–236). These papers were so interesting since everything had to be obtained only from the definition of Schnirelmann density. In 1941, the reviewer gave a course on additive number theory whose only aim was to develop everything then known about the problem. At the end of the semester one of the students, H. B. Mann, succeeded in proving the conjecture. His proof is very ingenious, but difficult. One year later, a simpler proof was given by Artin and Scherk. The latter proof is given in the book.

The third chapter of the book is the most valuable. It gives an elementary proof of Waring's problem, which for hundreds of years was one of the most famous unsolved problems. The proofs of Hilbert and of Hardy-Littlewood are very difficult and use complicated analytic methods. In 1942, the young Russian mathematician Linnik published an elementary proof of this theorem. But this proof was not accessible to those who cannot read Russian. While Linnik's paper is only 6 pages long, Khinchin needs 28 pages for the presentation to facilitate the understanding. It is of great importance that this proof is now available in English and German.

ALFRED BRAUER

Existence theorems for ordinary differential equations. By F. J. Murray and K. S. Miller. New York University Press, 1954. 10+154 pp. $5.00.

The following quotation from the Introduction of this book indicates the scope and level of treatment, as well as the aims of the authors: "We assume a knowledge of basic real variable theory (and for certain specialized results only, of elementary functions of a complex variable) and establish the fundamental existence theorems for ordinary differential equations which are the culmination of the nineteenth-century development. We do not consider the elementary methods for solving certain special differential equations nor the more advanced specialized topics. By restricting ourselves in this fashion