# LEONARD EUGENE DICKSON
## 1874–1954

Leonard Eugene Dickson was born in Independence, Iowa on January 22, 1874. He was a brilliant undergraduate at the University of Texas receiving his B.S. degree as valedictorian of his class in 1893. He was a chemist with the Texas Biological Survey from 1892–1893. He served as a teaching fellow at the University of Texas receiving the M.A. degree in 1894. He held a fellowship at the University of Chicago from 1894 to 1896 and was awarded its first Ph.D. in Mathematics in 1896. He spent the year 1896–1897 in Leipzig and Paris, was instructor in mathematics at the University of California 1897–1899, Associate Professor at Texas 1899–1900, Assistant Professor at Chicago 1900–1907, Associate Professor 1907–1910, and Professor in 1910. He was appointed to the Eliakim Hastings Moore Distinguished Professorship in 1928, and became Professor Emeritus in 1939. He served as Visiting Professor at the University of California in 1914, 1918, and 1922.

Professor Dickson was awarded the $1,000 A.A.A.S. Prize in 1924 for his work on the arithmetics of algebras. He was awarded the Cole Prize of the American Mathematical Society in 1928 for his book *ALGEBREN UND IHRE ZAHLENTHEORIE.* He served as Editor of the Monthly 1902–1908, and the Transactions from 1911 to 1916, and he was President of the American Mathematical Society from 1916–1918. He was elected to membership in the National Academy of Sciences in 1913 and was a member of the American Philosophical Society, the American Academy of Arts and Sciences, and the London Mathematical Society. He was also a foreign member of the Academy of the Institute of France, and an honorary member of the Czechoslovakian Union of Mathematics and Physics. He was awarded the honorary Sc.D. degree by Harvard in 1936 and Princeton in 1941.

Professor Dickson died in Texas on January 17, 1954.

Dickson was one of our most prolific mathematicians. His bibliography (prepared by Mr. Richard Block, a student at the University of Chicago) contains 285 titles. Of these 18 are books, one a joint book with Miller and Blichfeldt. One of the books is his major three-volume *History of the theory of numbers* which would be a life's work by itself for a more ordinary man.

Dickson was an inspiring teacher. He supervised the doctorate dissertation of at least 55 Chicago Ph.D's. He helped his students to get started in research after the Ph.D. and his books had a world-wide influence in stimulating research.

Attention should be called to the attached bibliography. It includes Dickson's books with titles listed in capitals. It does not include Dickson's portion of the report of the Committee on Algebraic Number Theory, nor does it include Dickson's monograph on ruler and compass constructions which appeared in Monographs on Modern Mathematics.

We now pass on to a brief discussion of Dickson's research.

1. *Linear groups*. Dickson's first major research effort was a study of finite linear groups. All but seven of his first forty-three papers were on that subject and this portion of his work led to his famous first book, [**44**]. The linear groups which had been investigated by Galois, Jordan, and Serret were all groups over the field of $p$ elements. Dickson generalized their results to linear groups over an arbitrary finite field. He obtained many new systems of simple groups, and he closed his book with a still valuable summary of the known systems of simple groups.

Dickson's work on linear groups continued until 1908 and he wrote about 44 additional papers on the subject. In these later papers he studied the isomorphism of certain simple groups and questions about the existence of certain types of subgroups. He also derived a number of theorems on infinite linear groups.

2. *Finite fields and Chevalley's Theorem*. In [**44**] Dickson gave the first extensive exposition of the theory of finite fields. He applied his deep knowledge of that subject not only to linear groups but to other problems which we shall discuss later. He studied irreducibility questions over a finite field in [**113**], the Galois theory in [**114**], and forms whose values are squares in [**139**]. His knowledge of the role of the non-null form was shown in [**155**]. In [**142**] Dickson made the following statement: "For a finite field it seems to be true that every form of degree $m$ in $m+1$ variables vanishes for values not all zero in the field." This result was first proved by C. Chevalley in his paper *Démonstration d'une hypothèse de M. Artin*, Hamb. Abh. vol. 11 (1935) pp. 73–75. At least the conjecture should have been attributed to Dickson who actually proved the theorem for $m=2, 3$.

3. *Invariants*. Several of Dickson's early papers were concerned with the problems of the algebraic geometry of his time. For example, see [**4**], [**48**], [**54**]. This work led naturally to his study of algebraic invariants and his interest in finite fields to modular invariants. He wrote a basic paper on the latter subject in [**141**], and many other papers on the subject. In these papers he devoted a great deal of

space to the details of a number of special cases. His book, [**172**], on the classical theory of algebraic invariants, was published in 1914, the year after the appearance of his colloquium lectures. His amazing productivity is attested to by the fact that he also published his book, [**173**], on linear algebras in 1914.

4. *Algebras.* Dickson played a major role in research on linear algebras. He began with a study of finite division algebras in [**105**], [**115**], [**116**], and [**117**]. In these papers he determined all three and four-dimensional finite (non-associative) division algebras over a field of characteristic not two, a set of algebras of dimension six, and a method for constructing algebras of dimension $mk$ with a subfield of dimension $m$. In [**126**] he related the theory of ternary cubic forms to the theory of three-dimensional division algebras. His last paper on non-associative algebras, [**268**], appeared in 1937 and contained basic results on algebras of degree two.

Reference has already been made to Dickson's first book on linear algebras. In that text he gave a proof of his result that a real Cayley division algebra is actually a division algebra. He presented the Cartan theory of linear associative algebras rather than the Wedderburn theory but stated the results of the latter theory in his closing chapter without proofs. The present value of this book is enhanced by numerous bibliographical references.

Dickson defined cyclic algebras in a Bulletin abstract of vol. 12 (1905–1906). His paper, [**160**], on the subject did not appear until 1912 where he presented a study of algebras of dimension 16.

Dickson's work on the arithmetics of algebras first appeared in [**204**]. His major work on the subject of arithmetics was presented in [**213**] where he also gave an exposition of the Wedderburn theory. See also [**237**] and [**238**].

The text [**231**] is a German version of [**213**]. However, the new version also contains the results on crossed product algebras which had been published in [**223**], and contains many other items of importance.

5. *Theory of numbers.* Dickson always said that mathematics is the queen of the sciences, and that the theory of numbers is the queen of mathematics. He also stated that he had always wished to work in the theory of numbers and that he wrote his monumental three-volume *History of the theory of numbers* so that he could know all of the work which had been done in the subject. His first paper, [**28**], contained a generalization of the elementary Fermat theorem which arose in connection with finite field theory. He was interested in the existence of perfect numbers and wrote [**166**], and [**167**] on the re-

lated topic of abundant numbers. His interest in Fermat's last theorem appears in [**190**], [**136**], [**137**], [**138**], and [**144**]. During 1926–1930 he spent most of his energy on research in the arithmetic theory of quadratic forms, in particular on universal forms.

Dickson's interest in additive number theory began in 1927 with [**229**]. He wrote many papers on the subject during the remainder of his life. The analytic results of Vinogradov gave Dickson the hope of proving the so-called ideal Waring theorem. This he did in a long series of papers. His final result is an almost complete verification of the conjecture made by J. A. Euler in 1772. That conjecture stated that every positive integer is a sum of $J$ $n$th powers where we write $3^n=2^nq+r$, $2^n>r>0$, and $J=2^n+q-2$. Dickson showed that if $n>6$ this value is correct unless $q+r+3>2^n$. It is still not known whether or not this last inequality is possible but if it does occur the number $g(n)$ of such $n$th powers required to represent all integers is $J+f$, or $J+f-1$, according as $fq+f+q=2^n$, or $fq+f+q>2^n$, where $f$ is the greatest integer in $(4/3)^n$.

6. *Miscellaneous.* We close by mentioning Dickson's interest in the theory of matrices which is best illustrated by his text, *Modern algebraic theories.* His geometric work in [**179**], [**181**], [**182**], [**183**], [**184**], [**185**], and [**186**] must also be mentioned, as well as his interesting monograph [**219**] on differential equations from the Lie group standpoint.

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