THE APRIL MEETING IN CHICAGO

The five hundred thirteenth meeting of the American Mathematical Society was held at the University of Chicago, Chicago, Illinois, on Friday and Saturday, April 22–23, 1955. Attendance was approximately 220, including 203 members of the Society.

By invitation of the Committee to Select Hour Speakers for Western Sectional Meetings, Professor A. H. Taub addressed the Society on the topic *Determination of flows behind shock waves* at 2:00 P.M. on Friday, April 22.

There were five sessions for contributed papers, two on Friday morning, two on Friday afternoon, and a final session which included late papers on Saturday morning. Presiding officers for the various sessions were Professors Tibor Radó, M. E. Shanks, Ralph Hull, G. B. Price, and L. M. Graves.

The files of the Mathematical Sciences Employment Register were available to interested parties during the course of the meeting.

The Society was entertained by a tea in the Common Room of Eckhart Hall on Friday afternoon, an occasion at which the ladies of the local Department of Mathematics acted as hostesses.

Abstracts of the papers presented follow. The name of a paper presented by title is followed by "*t*." In the case of joint authorship, the name of the person presenting the paper is followed by (p).

ALGEBRA AND THEORY OF NUMBERS

530. A. A. Albert (p) and Mrs. M. S. Frank: *New classes of simple Lie algebras.*

Let $B_n = F[x_1, \cdots, x_n]$ be the algebra of all polynomials in $x_1, \cdots, x_n$ with coefficients in a field F of characteristic p subject only to the condition that $x_1^p = \cdots = x_n^p = 0$. Then it is shown that the set of all derivations $A = (a_1, \cdots, a_n)$ with divergence equal to the sum of coordinates is a simple Lie algebra of dimension $(n-1)p^n$ over F. The set of all derivations $D(\phi)$ defined by $a_i = \partial\phi/\partial x_{i+m}$, $a_{i+m} = -\partial\phi/\partial x_i$, where $n = 2m$ and the leading coefficient of ϕ is zero, forms a Lie algebra of dimension $p^{2m}-2$. This algebra is simple but has the same dimension as a classical matric algebra. It is proved that it is isomorphic to the classical algebra if and only if $m=1$ and $p=3$. The algebra suggests the definition of a class of Lie algebras defined by the multiplication table $u_\alpha u_\beta = f(\alpha, \beta)u_{\alpha+\beta}$ where the subscripts range over the elements of an elementary p-group which is abelian and of order p^n, and where f is a functional. Using this definition we construct algebras which yield simple Lie algebras of dimensions p^m, p^m-1, and p^m-2 for every integer $m>1$ and every odd prime p. (Received March 3, 1955.)

531. J. R. Büchi (p) and J. B. Wright: *Invariant theory in groups.* Preliminary report.

A number of examples are presented illustrating different types of theories derived from group-theory. The examples suggest a key-concept, which is important in organizing these algebraic theories into a hierarchy, corresponding to Klein's hierarchy of geometries. (Received April 4, 1955.)

532t. A. A. Grau: *Relative irreducibility of polynomials in a skewfield*. Preliminary report.

Let a^α be the image of an element a under an automorphism α of a skewfield S. The set of formal polynomials $f(x) = \sum a_i x^i$ with coefficients a_i in S form a Euclidean ring $S_\alpha[x]$ if $a^\alpha x = xa$. The polynomial $f(x)$ is α-irreducible if it cannot be expressed as a product of nontrivial factors in $S_\alpha[x]$. An α-irreducible polynomial may determine a two-sided ideal $\mathfrak{a}$ in $S_\alpha[x]$, which is maximal as a left ideal, so that $S_\alpha[x]/\mathfrak{a}$ is a skewfield in which $f(x)$ has a solution. The case where such an ideal is not determined remains to be considered. (Received March 8, 1955.)

533. Franklin Haimo: *Some extensions of normal subgroups in a class of semi-direct products.*

Let H be an abelian group, let K be a group, and let G be a semi-direct product of H and K where H is normal in G. If V is a normal, n-nilpotent subgroup of K, conditions are found which allow one to extend V to a normal, n-nilpotent subgroup of G. In particular, if the circle composition (Jacobson, *Lectures in abstract algebra*, I) of the automorphisms of H generated by any two elements of K is the identity, then the members of the ascending central series of the normal subgroup V extend easily to the corresponding members of the ascending central series of the extension of V to a normal subgroup of G. Conditions are found on a single element k of K so that the normal subgroup of K generated by k will have the above extension property. (Received March 9, 1955.)

534. D. R. Hughes: *Planar division neo-rings.*

The axioms of a division ring can be weakened by discarding the demand that addition be associative and commutative, and demanding instead that the addition form a loop and that the resulting system retain the property of coordinatizing a projective plane (see Paige, *Neofields*, Duke Math. J. vol. 16 (1949)). Such a system will be called a planar division neo-ring (PDNR); examples of infinite PDNRs which are not division rings are known. If a PDNR is finite and has power-associative multiplication, then its addition is shown to be commutative and to have the inverse property. The chief result is the following: if p is a prime divisor of the order of a finite PDNR whose multiplication is associative and commutative, then the mapping $x \to x^p$ is an automorphism. Mainly by means of this result, it can be shown that all PDNRs with associative multiplication and order $\leqq 300$ actually have prime-power order. Non-prime-power integers which cannot be rejected by any known means exist however; the smallest known to the author is 2501. (Received March 7, 1955.)

535. Arno Jaeger: *On the replacibility of partial differential equations by ordinary ones in fields of prime characteristic.*

Let F be a separably generated algebraic function field of n independent indeterminates of prime number characteristic $p > 0$ with a separating transcendence basis $(x_1, x_2, \cdots, x_n)$ over its ground field K. Let D_i $(i = 1, 2, \cdots, n)$ be the partial derivations of F over K uniquely defined by $D_i(x_j) = \delta_{ij}$ (Kronecker delta), and let C be the field of all elements of F which are constant under each partial derivation. Then it is shown that there exist derivations d over C, for instance $d = \sum_{i=1}^{n} x_i^{p+1} D_i$, such that $(d, d^p, d^{p^2}, \cdots, d^{p^{n-1}})$ is a basis for the F-module of all derivations of F over C. Hence all partial differential equations in the D_i can be expressed as ordinary differential equations in d. (Received March 4, 1955.)

536*t*. J. A. Kalman: *A common abstraction of Boolean algebras and l-groups*. Preliminary report.

A distributive lattice with an involution $x \to x'$ such that $x \cap x' \leqq y \cup y'$ identically will be called a *normal i-lattice*. *Results*. The operation $x \triangle y = (x \cup y) \cap (x' \cup y')$ is associative in any normal i-lattice. Every normal i-lattice is an i-sublattice (sublattice which always contains x' with x) of a normal i-lattice with a zero (element 0 such that $0' = 0$). A normal i-lattice with a zero 0 may be recovered from its sublattice of "positive" elements if and only if 0 is meet-irreducible. The only subdirectly irreducible normal i-lattices are the chains with one, two, or three elements. Each of the following conditions is necessary and sufficient that a normal i-lattice L be a Boolean algebra: (a) $x \cup x' = y \cup y'$ identically; (b) if $a \triangle x = b \triangle x$ for some x, then $a = b$ (L need not be assumed distributive); (c) $x \cap (y \triangle z) = (x \cap y) \triangle (x \cap z)$ identically. An additive l-group G (e.g. vector lattice) is a normal i-lattice with $x' = -x$: the l-ideals in G are then just the normal subgroups of G which contain with any x also all $x \triangle y$ for y in G. Every normal i-lattice is an i-sublattice of a vector lattice: two proofs are given, both requiring the axiom of choice. (Received March 7, 1955.)

537*t*. J. A. Kalman: *Some results in lattice theory*. Preliminary report.

I. A lattice may be defined as a set closed under associative operations $\cup$ and $\cap$ which satisfy the four absorption laws: $a \cap (a \cup b) = a \cup (b \cap a) = (b \cup a) \cap a = (a \cap b) \cup a = a$. II. $|a \cup c - b \cup c| + |a \cap c - b \cap c| = |a - b|$ is an identity in l-groups. G. Birkhoff (*Lattice theory*, 1st ed., Theorem 7.8) discovered this identity for vector lattices, but (Ann. of Math. vol. 43 (1942) p. 309) he was unable to generalize it to noncommutative l-groups. III. A number of identities for abelian l-groups are proved. Samples: (i) $2(x \triangle y) = |x-y| - |x+y|$ ($\triangle$ as in the previous paper); (ii) $2(x \triangle y)^+ = |x| + |y| - |x+y|$; (iii) $|x \triangle y| = |x| \cap |y|$ (valid in all normal i-lattices: $|x| = x \cup x'$); (iv) $|x+y| + |x-y| = |x+|y|| + |x-|y||$; (v) $||x+y| - |x-y| + 2y| = |x+|y|| - |x-|y|| + 2|y|$. IV. A vector lattice may be defined as a real linear space R together with a mapping $x \to |x|$ of R into R which satisfies (III (iv), III (v), (a) $|kx| = |k||x|$ for all real k, and (b) $|x| = 0$ implies $x = 0$. If the postulate (b) is omitted, $R \cong S \times T$ (direct union) with S a vector lattice and $|t| = 0$ for all t in T. Other definitions of vector lattices in terms of a postulated absolute are given. (Received March 7, 1955.)

538. L. A. Kokoris: *Simple power-associative algebras of degree two*.

A gap in the structure theory of power-associative algebras is closed by proving the following theorem. Let A be a simple commutative power-associative algebra of degree two over a center F of characteristic zero. Then A is a Jordan algebra. (Received January 6, 1955.)

539. P. J. McCarthy: *Sufficient conditions for a genus of indefinite ternary quadratic forms to contain only one class*.

This paper continues the work of A. Meyer (see B. W. Jones, Canadian Journal of Mathematics vol. 4 (1952) pp. 120–128, vol. 5 (1953) pp. 271–272; B. W. Jones and D. B. Marsh, Bull. Amer. Math. Soc. Abstract 61-3-393). Let f be a primitive, indefinite, ternary quadratic form with integral matrix A. Let Ω be the g.c.d. of the two-rowed minor determinants of A, and let the integer Δ be determined by $|A| = \Omega^2\Delta$. Then f is in a genus of one class provided it satisfies one of the following sets of conditions. (1) (Ω, Δ) divides 6, $\Omega \not\equiv 0 \not\equiv \Delta \pmod 4$, $|A| \not\equiv 0 \pmod{81}$, and $c_3(f) = 1$ (where $c_3(f)$ is the Hasse-Pall invariant of f with respect to 3). (2) $\Omega \equiv 8 \pmod{16}$, $\Delta \equiv 2 \pmod 4$, and $(\Omega, \Delta) = 2$. (3) $\Omega \equiv 4 \pmod 8$, $\Delta \equiv 2 \pmod 4$, and $(\Omega, \Delta) = 2$. (4) $\Omega \equiv 8 \pmod{16}$, Δ is odd, $(\Omega, \Delta) = 1$, and the generic character of f with respect to 4 is equal to $(-1)^{(\Delta-1)/2}$. The proofs of these theorems make no use of the theory of automorphs of ternary quadratic forms. (Received March 7, 1955.)

540. R. B. Reisel: *A generalization of the Wedderburn-Malcev Theorem to infinite-dimensional algebras.*

A is an associative algebra over the field F having (Jacobson) radical N, such that $\bigcup_{k=1}^{\infty} N^k = 0$, and such that A/N is locally separable. A is assumed to be complete in the natural topology determined by the powers of N. It is then proved that if A/N has countable dimension over F, there exists a subalgebra S of A such that $A = S \oplus N$ (vector space direct sum). For the uniqueness result, this dimensionality restriction is not needed, but it is assumed that for every positive integer n, the A/N-module N^n/N^{n+1} is complete with respect to a topology in which a fundamental system of neighborhoods of zero is composed of the centralizers of finite-dimensional separable subalgebras of A/N. Then it is shown that if there exist two subalgebras S and S^* of A such that $A = S \oplus N = S^* \oplus N$, there exists $r \in N$, with quasi-inverse r', such that the mapping $x \rightarrow x - xr - r'x + r'xr$ is an isomorphism of S onto S^*. An example is given to show the necessity of the additional hypothesis on the radical in the uniqueness result when A/N is infinite-dimensional. (Received March 4, 1955.)

541t. W. E. Roth: *On the characteristic polynomial of the product of several matrices.*

If A_i, $i = 1, 2, \cdots, r$, are $n \times n$ matrices with elements in the field F, and whose characteristic polynomials are $m_i(x) = m_{i,0} + m_{i,1}x + m_{i,2}x^2 + \cdots + m_{i,r-1}x^{r-1}$, where $m_{i,j}$, $j = 0, 1, \cdots, r-1$, are polynomials in x^r with coefficients in F, and if for every h, $k = 1, 2, \cdots, r$, $A_h - A_k = \alpha_{hk}D$, where the rank of D does not exceed unity and α_{ij} is a constant in F, then the characteristic polynomial of the product, $A_1A_2A_3 \cdots A_r$, is given by $m(x)$, where $m(x^r)$ is the determinant, $|\mu_{i,j}|$, and $\mu_{i,j} = m_{i,j-i}x^{j-i}$ if $j \geqq i$ and $\mu_{i,j} = m_{i,r+j-i}x^{r+j-i}$ if $j < i$. (Received February 28, 1955.)

542. H. A. Simmons: *A class of maximum numbers associated with a symmetric equation in n reciprocals.*

Our first paper on this subject appeared in Trans. Amer. Math. Soc. vol. 34, pp. 876–907. Equation (21) of that article was beyond the scope of the procedure there and in three later papers. However, in the present paper, the terms "Kellogg solution" and "E-solution" are used with the meanings they had in the first paper, and Theorems 2 and 3 on page 887 of that article are extended to cover the case of the equation (21) just mentioned; that is, in previous language, "the remarkable properties" of the Kellogg solution of equation (21) have been established. Our main new developments are use of a special form in which to represent any E-solution of (21), definition of the pseudo-Kellogg solution relative to any given E-solution of (21) or to certain associated derived sets, and use of essentially our previous transformations to carry sets toward or actually into pseudo-Kellogg solutions. (Received March 9, 1955.)

543. Leonard Tornheim: *Inessential discriminant divisors of normal quartic fields.*

Let K be an algebraic number field generated by a root of the irreducible equation $x^4+Ax^2+B=0$ where A and B are rational integers and no square dividing A has its square dividing B. All normal quartic fields are included. Necessary and sufficient conditions on A and B are given for 3 to be an inessential discriminant divisor. Also the highest power of 3 possible is the first. (Received March 9, 1955.)

544. L. M. Weiner: *Note on S-matrices.*

Let $u=(\alpha_1, \alpha_2, \cdots, \alpha_n)$ be an n-dimensional vector over a field F subject to the condition $\sum_{i=1}^{n} \alpha_i \neq 0$, and let $A=(a_{ij})$ be an n by n matrix over F. Then A is called an S-matrix for the vector u if $\sum_{i=1}^{n} \alpha_i a_{ij} = \sum_{i=1}^{n} \alpha_i a_{ji} = S(A)$ for $j=1, 2, \cdots, n$. If one redefines multiplication of matrices such that when $A=(a_{ij})$ and $B=(b_{ij})$, then $C=AB=(c_{ij})$ with $c_{ij}=\sum_{k=1}^{n} \alpha_k a_{ik} b_{kj}$, it is seen that the set of all S-matrices for a fixed vector u forms a subalgebra R of the total matric algebra of degree n for which $S(A+B)=S(A)+S(B)$, $S(\alpha A)=\alpha S(A)$ for α in F, and $S(AB)=S(A)S(B)$. The algebra R is the direct sum of the one-dimensional ideal M consisting of all matrices A for which $a_{ij}=a$, a constant, and the $(n-1)^2$-dimensional ideal N consisting of all matrices A for which $S(A)=0$. If $\alpha_k \neq 0$, then a basis for the ideal N is given by the matrices A_{ij} $(i, j=1, 2, \cdots, k-1, k+1, \cdots, n)$ where A_{ij} is the matrix which has $\alpha_i\alpha_j$ in the kth row and kth column, $-\alpha_k\alpha_i$ in the kth row and jth column, $-\alpha_k\alpha_j$ in the ith row and kth column, and α_k^2 in the ith row and jth column. (Received February 18, 1955.)

545. Oswald Wyler: *On ranks in regular rings.* Preliminary report.

In an earlier paper (Compositio Math. vol. 9 (1951) pp. 193–208), the author has defined a general notion of rank for elements of rings, especially regular rings. In the present paper, an ordering of ranks in a regular ring $\mathfrak{R}$ is studied. If Ra denotes the rank of an element a of $\mathfrak{R}$, and if a and b are in $\mathfrak{R}$, we say that $Ra \leqq Rb$ if there are elements p, q in $\mathfrak{R}$ such that $a=pbq$. Then $Ra \leqq Rb$ if and only if there is c in $\mathfrak{R}$ such that $Rb=Ra+Rc$. This definition leads to a quasi-ordering of ranks in $\mathfrak{R}$. If the lattice of principal left ideals of $\mathfrak{R}$ satisfies weak continuity conditions, the relation $Ra \leqq Rb$ defines a (partial) ordering of ranks in $\mathfrak{R}$. (Received March 9, 1955.)

Analysis

546. W. A. Beck. *Some commutation relations in a Hilbert space.*

Let N, M, and B denote (bounded) operators in a Hilbert space $\mathcal{H}$. Consider the operator $C=BN-MB$. Suppose N is a normal operator. An extension of a method of proof used by Fuglede (*A commutativity theorem for normal operators*, Proc. Nat. Acad. Sci. U.S.A. vol. 36 (1950) pp. 35–40) shows that the commutation relations $CN=NC$ and $MN=NM$ imply that $BN=NB$ if certain restrictions are imposed on the spectra of N and M, namely, (i) $\lambda \in$ ptsp N implies $\lambda \notin$ ptsp M, and (ii) $\lambda \in$ Closure (sp $N-$ptsp N) implies $\lambda \notin$ sp M. (This work was supported in part by the National Science Foundation research grant NSF-G481.) (Received March 10, 1955.)

547. J. S. Frame: *A hypergeometric solution of the equation* $x^n = px - q$.

If $p, q > 0$ and $u = q^{n-1}p^{-n} < (n-1)^{n-1}n^{-n}$, the smaller positive root x of the equation $x^n = px - q$ can be expressed explicitly in terms of the generalized hypergeometric series $f_a = f_a(n, q) = \sum_{k=0}^{\infty} C_k^{nk+a}u^k$ by each of the three formulas $px/q = nf_0/[1+(n-1) \cdot f_0] = f_1/f_0 = q^{-1}\int_0^q f_0 dq$. The proof of this theorem is based on the lemma $\sum C_k^{nk+a} \cdot t^k(1-t)^{nk-k+a} = (1-nt)^{-1}$ for integral n, $a \geqq 0$, which is proved by using a contour integral in the complex plane. For the cubic case $n = 3$, the functions f_0 and f_1 are the hypergeometric functions $F(1/3, 2/3, 1/2, q^2/4)$ and $F(4/3, 2/3, 3/2, q^2/4)$ whose ratio can be expressed as a hypergeometric continued fraction. All three roots of the irreducible cubic are then expressed to at least five significant figure accuracy by using certain simple remainder estimates for the fourth convergents of two hypergeometric continued fractions. (Received March 9, 1955.)

548. Jesus Gil de Lamadrid: *On mappings and their derivatives in locally convex topological vector spaces.*

Let E and F be two such spaces, $\mathcal{G}(E, F)$ the space of all mappings of E into F, let $\mathcal{L}(E, F) \subset \mathcal{G}(E, F)$ consist of all linear continuous mappings, and τ be a topology of $\mathcal{G}(E, F)$. For $f \in \mathcal{G}(E, F)$, $\Delta_t(x_0)f \in \mathcal{G}(E, F)$ is defined as $\Delta_t(x_0)f(h) = (f(x_0+th) - f(x_0))/t$, $t \neq 0$, and $f'(x_0) = \lim_{t\to\infty} \Delta_t(x_0)f \in \mathcal{G}(E, F)$. With Michal [Bull. Amer. Math. Soc. vol. 43 (1937) pp. 394–401], $f'(x_0)$ is called the (τ) derivative of f at x_0. For normed E, and F the field R of reals, f' becomes the gradient of f of E. H. Rothe [Duke Math. J. vol. 15 (1948) pp. 421–431]. It is shown that $f': E \to \mathcal{G}(E, F)$ generalizes the ordinary idea of derivative $(E = R)$ and shares with it many important properties. For instance, f' determines f within a constant. These ideas are applied to generalizing certain results of Rothe [op. cit.], relating the continuity properties of $f: E \to R$ to those of its gradient f'. The main result states that if $f'(E) \subset \mathcal{L}(E, F)$, τ is the bounded-open topology and f' is compact, then f (restricted to bounded sets) is continuous in a topology, in strength (fineness) between the weak and the strong topologies of E—weak and strong in the sense of Dieudonné [Bull. Amer. Math. Soc. vol. 59 (1953) pp. 495–512]. These results extend to mappings $f: K \to F$, for $K \subset E$ and K having some suitable properties. (Received March 9, 1955.)

549*t*. Casper Goffman and G. M. Petersen: *A class of consistent summability methods.*

A set of regular summability methods which are mutually consistent and together sum all bounded real sequences is given. Indeed, the methods in the set may all be taken to be stronger than any given regular method. (Received February 14, 1955.)

550. Casper Goffman and G. M. Petersen: *Submethods of a matrix summability method.*

A submethod of a regular matrix method is one obtained by deleting a set of rows from the matrix. A one-one correspondence is established between the submethods of a method A and the interval (0, 1]. For every bounded sequence not summed by A, the set of submethods which sum it is of the first category but may be of measure 0 or 1. The set of submethods equivalent to A is of the first category, but A has equivalent methods B and C such that the set of submethods of B equivalent to B is of measure

0 and the set of submethods of C equivalent to C is of measure 1. (Received February 14, 1955.)

551. E. L. Griffin, Jr.: *Isomorphism of substantial rings of operators.*

Let M, $\tilde{M}$ be substantial rings of operators on Hilbert spaces H, H of arbitrary dimension, with commutants M', $\tilde{M}'$ respectively (substantial refers to the property of having no type III part). As in a previous paper, *Some contributions to the theory of rings of operators*, Trans. Amer. Math. Soc. vol. 75 (1953) pp. 471–504, one considers essentially bounded coupling operators C, $\tilde{C}$ of the above rings. If ϕ is an isomorphism (not necessarily adjoint preserving) of M onto $\tilde{M}$ carrying C into $\tilde{C}$, then there exists a 'inear mapping W which takes a dense linear subset of H in a one-to-one manner onto a dense linear subset of $\tilde{H}$ and which induces ϕ. This implies that such isomorphisms preserve weak and σ-weak convergence of uniformly bounded directed sets in arbitrary substantial rings. The method of proof is similar to that of the paper quoted above. (Received March 9, 1955.)

552*t*. Peter Henrici: *On generating functions of the Jacobi polynomials.*

Every solution of $(D)\partial^2u/\partial x^2+\partial^2u/\partial y^2+(2\mu+1)x^{-1}\partial u/\partial x+(2\nu+1)y^{-1}\partial u/\partial y=0$ which is regular at (0, 0) and even in x and in y admits a series expansion of the form $\sum a_n\rho^nP_k^{(\nu,\mu)}(\tau)$, where $\rho=x^2+y^2$, $\rho\tau=x^2-y^2$, and may therefore be regarded as a generating function of the Jacobi polynomials. The special solution $u(x, y)$ $=\omega^{-1}(\omega+1+\rho)^{-\mu}(\omega+1-\rho)^{-\nu}F(-\kappa,\kappa+1;\ \mu+1;\ 2^{-1}\omega^{-1}(\omega-1-\rho))F(\kappa,-\kappa+1;\ \nu+1;$ $2^{-1}\omega^{-1}(\omega-1+\rho))$, where $\omega=(1-2\rho\tau+\rho^2)^{1/2}$, contains several of the known generating functions as special cases. Its coefficients a_n can be represented by generalized hypergeometric sums. Letting $\tau=2\xi\nu^{-1}-1$, $\kappa^2=-\nu\eta$, $\nu\rightarrow\infty$ yields the Hardy-Hille series of products of Laguerre polynomials. By specializing τ one obtains identities of Cayley-Orr type (see Bailey, Cambridge tract No. 32, p. 84) between the coefficients of the Taylor expansions of certain products of hypergeometric functions. (Received March 11, 1955.)

553*t*. Peter Henrici: *Addition theorems for general Legendre and Gegenbauer functions.*

The classical addition theorems of Heine, Hobson and R. Lagrange for the Legendre functions with upper index zero are extended to general Legendre functions. A typical result is the following: $\rho_1'^{-\nu}P_\mu^{-\nu}(\rho_1)=2^\nu\Gamma(\nu)\sum_{n=0}^\infty(\nu+n)(\mu+\nu+1)_n(\nu-\mu)_n$ $\cdot P_\mu^{-\nu-n}(\rho)P_\mu^{-\nu-n}(\tau)C_n^\nu(\eta)$, where μ, ν are arbitrary, $\rho_1=\rho\tau-\rho'\tau'\eta$, $\rho'=(\rho^2-1)^{1/2}$, $\tau'=(\tau^2$ $-1)^{1/2}$, $\rho_1'=(\rho_1'^2-1)^{1/2}$, the expansion being valid provided $\Re\rho\geqq0$, $\Re\tau>0$, ρ not real and $\leqq1$, and η such that ρ_1 lies in the interior of the ellipse with foci at $\rho\tau\pm\rho'\tau'$ and passing through -1. Similar expansions are proved for the Legendre functions of the second kind and for the general Gegenbauer function. Numerous known results, among them Gegenbauer's addition theorems for the Bessel functions, are obtainable from these formulae as special or limiting cases. The proofs make use of a one-to-one correspondence, given by $f(w)=(1-w)^{-\mu-\nu-1}u(c(1-w)^{-1},\ -icw(1-w)^{-1})$, between the solutions $u(x, y)$ of (D) (see preceding abstract) regular at $(c, 0)$ $(c\neq0)$ and the analytic functions $f(w)$ regular at 0. (Received March 11, 1955.)

554. R. D. James: *Generalized primitives and n-convex functions.*

A generalized integral of Perron type which, starting with a function $f(x)$, goes directly to a primitive $F(x)$ of order n, was defined in a previous paper [Trans. Amer. Math. Soc. vol. 76 (1954) pp. 149–176]. In the present paper it is shown that $F(x)$ has generalized derivatives $D^{n-2k}F(x)$ for $1 \leqq k \leqq [n/2]$ and that $D^n F(x) = f(x)$ almost everywhere. These generalized derivatives may be regarded as primitives of order $n-2k$. They are not, in general, continuous but they cannot have ordinary discontinuities. The proof requires some differentiability properties of convex functions of order n, that is, functions whose divided differences of order n are all non-negative. (Received March 7, 1955.)

555. J. H. B. Kemperman: *Bounds on polynomials.*

Let $f(w) = \sum_0^n (a_j \cos jw + b_k \sin jw)$, a_j and b_j real, $|f(u)| \leqq 1$ for u real. Then for u, v real: (a) $|f(u+iv)|^2 \leqq \cosh^2 nv - (1-f(u)^2)$. Equality only for $v=0$ or $f(w) = \cos(nw-\alpha)$, α real. Let $P(z) = \sum_0^n a_j z^j$, a_j real, (b) $|P(z)| \leqq 1$ for $-1 \leqq z \leqq +1$; N. G. de Bruijn conjectured that for $k=0, 1, \cdots, n$ and any complex z, (c) $|P^{(k)}(z)| \leqq T_n^{(k)}(z^*)$. Here, $z^* = (|z-1|+|z+1|)/2$, $T_n(z) = \cos(n \text{ arc} \cos z)$. It is shown that (c) holds for $k=0$, $k=1$ and (obviously) $k=n$. Equality for $k=1$ only if $P(z) = \pm T_n(z)$ and $z = \pm z^*$. The case $k=0$ follows from (a) putting $z = \cos w$. A. C. Schaeffer and G. Szegö, Trans. Amer. Math. Soc. (1941), proved a.o. (c) for z real when (b) is replaced by the weaker assumption (d) $|P(\cos m\pi/n)| \leqq 1$ $(m=0, 1, \cdots, n)$. Let β_k be the largest zero of $T_n^{(k-1)}(z)$ $(\beta_1 = \cos \pi/2n$, $\beta_2 = \cos \pi/n$, $0 < \beta_{k+1} < \beta_k)$. It is shown that even when (b) is replaced by (d) one has $|P^{(k)}(z)| \leqq |T_n^{(k)}(z)| \leqq T_n^{(k)}(z^*)$ for $|z| \geqq \beta_k$ $(k=0, 1, \cdots, n; \beta_0 = 1)$. For $k=0$ this was already proved by P. Erdös, Bull. Amer. Math. Soc. (1947). (Received March 9, 1955.)

556t. M. S. Webster: *A recurrence relation for ultraspherical polynomials.*

The ultraspherical polynomials are uniquely determined by the nonlinear recurrence relation given by Thiruvenkatachar and Nanjundiah [Proc. Indian Acad. Sci. Sect. A vol. 33 (1951) pp. 373–384].

557. F. M. Wright: *Some sufficient conditions for a determinate Hamburger moment sequence.*

Let $\{\mu_n\}$ $(n=0, 1, 2, \cdots)$ be a given positive definite Hamburger moment sequence. Using certain relationships between the moments and the coefficients in the J-fraction expansion of the formal power series $\sum_{n=0}^{\infty} \mu_n/z^{n+1}$, together with Carleman's inequality, Schwarz's inequality, and a sufficient condition in terms of the coefficients in the J-fraction expansion of $\sum_{n=0}^{\infty} \mu_n/z^{n+1}$ for $\{\mu_n\}$ to be a determinate Hamburger moment sequence, the author obtains an infinite sequence of sufficient conditions in terms of the moments themselves for $\{\mu_n\}$ to be a determinate Hamburger moment sequence. Another infinite sequence of sufficient conditions, each simpler but weaker than the corresponding condition of the above sequence, is then given. It is shown how the famous Carleman criterion for a determinate Hamburger moment sequence follows from the preceding and hence is weaker than any of the sufficient conditions proved thus far in this paper. Finally, another sufficient condition in terms of the moments themselves for $\{\mu_n\}$ to be a determinate Hamburger moment sequence is given, but the author does not know how this condition compares with those proved earlier in this paper. (Received March 7, 1955.)

558. R. E. Zindler: *The qualitative behavior of integral curves of systems of three differential equations near a singular point.*

Integral curves of differential equations of the form $dx_i/dt = U_i(x_i)$ are studied for $i=1, 2, 3$ and where the U_i are continuous and either all vanish or one or more are infinite at the origin or the limit as $r=(x^2+y^2+z^2)^{1/2}\to 0$ of at least one of them is not independent of the direction of approach. It is assumed that no other singularities exist for $r \leqq r_0$. The concept of an exceptional direction is defined and tests are provided to determine whether a direction is exceptional. The equations are rewritten in spherical coordinates $dr/dt=P_1(r, \phi, \psi)$, $r\ d\phi/dt=P_2(r, \phi, \psi)$, $r \sin \phi\ d\psi/dt =P_3(r, \phi, \psi)$ and it is further assumed that the $\lim s_{r\to 0} P_i\Delta^{-1}$ where $\Delta=(\sum U_i^2)^{1/2}$ are independent of the direction of approach. Numerous theorems are stated in regard to the behavior of curves in the neighborhood of an exceptional direction with various conditions placed on the qualitative behavior functions $K=P_1\Delta^{-1}$ and $K_{ij}=P_iP_j^{-1}$. It is shown that in the neighborhood of each nonexceptional direction there exists a cone about the direction from which no curve can enter the origin. (Received March 8, 1955.)

559*t*. R. E. Zindler: *A note on L. E. Reizin's paper "Behavior near to a singular point of integral curves of a system of three differential equations.*

An examination of L. E. Reizin's paper (Math. Reviews vol. 15, p. 311) showed that in applying the conditions of Theorem 2, Reizin concluded that $m=1$. A study of the conditions of the theorem revealed that m could be any positive integer. The proof of Theorem 2 is made in the same manner as Reizin's when the integral condition on the function $B(r)$ is modified to be $\int_0^{r_0} B(r)r^{-m-1}dr < +\infty$. In a manner suggested by Dr. J. L. Brown, Jr., the existence of a suitable function $B(r)$ is shown if functions f and g are Hölder continuous in the radial variable r. (Received March 8, 1955.)

Applied Mathematics

560. P. C. Hammer (p) and A. W. Wymore: *Numerical evaluation of multiple integrals.* Preliminary report.

There are established formulas for numerical integrations over adequately symmetrical regions in n-space for $n \geqq 2$. For example, one formula, based on fourteen points in the solid sphere, gives exact integrals for the general fifth degree polynomial in three variables. This formula being valid under affine transformations then applies to integrals over arbitrary ellipsoids. Generation of formulas for cartesian product regions such as cylinders, squares, and cubes is shown to be readily done provided formulas for the factor spaces are known. Hitherto the full use of affine transformations and cartesian products seems not to have been made. Tests of formulas at the University of Wisconsin Numerical Analysis Laboratory have indicated that many will be satisfactory for machine calculations and some will be useful for desk calculation. (Received March 7, 1955.)

561. Jacob Korevaar: *Distributions defined by fundamental sequences.* II. *The definite integral.*

Define $f^{(-1)}(t)=\int_0^t f(u)du$, $f^{(-2)}(t)=\int_0^t f^{(-1)}(u)du$, etc. A sequence of functions $f_n(t)$, integrable for $t\geqq 0$, $n=1, 2, \cdots$, is called fundamental when to every $A>0$ there is a positive integer p such that $\{f_n^{(-p)}\}$ converges uniformly on $[0, A]$. A fundamental sequence $\{f_n\}$ defines a distribution ϕ on the half-line $t\geqq 0$. The derivative of a distribution was defined in I (*Distributions and their Laplace transforms* $\cdots$, Bull. Amer. Math. Soc. Abstract 61-3-428). To every distribution ϕ and every $A>0$ there are an integrable function $F(t)$ and an integer $r\geqq 0$ such that $\phi=[F]^{(r)}$ on $[0, A)$; $[F]^{(r)}$ denotes the rth derivative of the distribution $[F]$ associated with the function F (see I). A distribution ϕ is said to be integrable over $[a, b]$ $(a>0)$ when it may be given by a fundamental sequence $\{f_n\}$ such that $\{f_n^{(-1)}\}$ is uniformly convergent in some neighborhood of a and in some neighborhood of b. One defines $\int_a^b\phi(t)dt=\lim_{n\to\infty}\int_a^b f_n(t)dt$. The Dirac distribution δ_τ is given by $f_n(t)=n$ on the interval $(\tau, \tau+1/n)$, $f_n(t)=0$ outside that interval. Hence $\int_a^b\delta_\tau(t)dt=1$ whenever $a<\tau<b$. A distribution ϕ is shown to be integrable over $[a, b]$ if and only if there is a continuous function $G(t)$ such that in some neighborhood of a and in some neighborhood of b one has $\phi=[G]'$; $\int_a^b\phi(t)dt$ is equal to $G(b)-G(a)$. (Received March 10, 1955.)

562*t*. Jacob Korevaar: *Distributions defined by fundamental sequences.* III. *Convergence, multiplication and division.*

A sequence of functions $f_n(t)$ integrable for $t\geqq 0$ is said to be convergent to the distribution ϕ on (a, b) $(a\geqq 0)$ when there are an integer $p\geqq 0$, a function $F(t)$ integrable for $t\geqq 0$, and a sequence of polynomials $P_n(t)$ of degree $\leqq p-1$ such that $f_n^{(-p)}(t)-P_n(t)\to F(t)$ as $n\to\infty$, uniformly or dominatedly on $[a, b]$, while $\phi=[F]^{(p)}$ on (a, b). When the $f_n(t)$ are continuous and possess a piece-wise continuous derivative $f_n'(t)$ on $[a, b]$, then $f_n\to\phi$ on (a, b) implies $f_n'\to\phi'$ on (a, b). It follows that every trigonometric series $a_0/2+\sum_{n=1}^{\infty}(a_n\cos nt+b_n\sin nt)$ with $|a_n|+|b_n|<Mn^k$, $n=1, 2, \cdots$, is convergent on every interval (a, b). One defines the product ϕg of a distribution ϕ given by a convergent sequence $\{f_n\}$ as above and a function $g(t)\in C^k[a, b]$ with $k\geqq p$ by the sequence $\{f_n g\}$ which is convergent on (a, b). When ϕ is given as $[H]^{(r)}$ on (a, b), $H(t)$ integrable for $t\geqq 0$, $r\leqq k$, then an equivalent definition is $\phi g=[Hg]^{(r)}-C_{r,1}[Hg']^{(r-1)}+\cdots+(-1)^r[Hg^{(r)}]$. Example: $\delta_\tau=[U_\tau]'$ where $U_\tau(t)$ is the unit step function, hence $\delta_\tau(t)g(t)=g(\tau)\delta_\tau(t)$ whenever $g\in C^1$ or even $g\in C^0$. Let $\phi g=0$ on (a, b). When $g(t)>0$ on (a, b), then $\phi=0$ on (a, b); when ϕ is not equal to 0 on any subinterval of (a, b), then $g=0$ on (a, b). Division is studied. Example: the equation $\phi(t)\cdot(t-\tau)=\omega(t)$ $(a<\tau<b)$ always has infinitely many solutions $\phi(t)$ on (a, b). Any two solutions differ by $c\delta_\tau(t)$. (Received March 10, 1955.)

Geometry

563*t*. John DeCicco: *Some theorems on the projective geometry of surfaces.*

A set of six functions 1, m, n, λ, μ, ω, of class C^2 in a two-dimensional region represent the Christoffel symbols of the second kind of a surface S if and only if they obey a certain system of five partial differential equations, one of which is of the first order, and the remaining ones are of the second order. Necessary conditions are obtained that a second order differential equation of the cubic type represents that of the ∞^2 geodesics of a surface S. In particular, a new generalization of Beltrami's theorem on geodesic mappings of surfaces is obtained. Thus, in projective maps, we have the two independent projective invariants: $U=-G\partial K/\partial x+F\partial K/\partial y$, $V=F\partial K/\partial x-E\partial K/\partial y$, where E, F, G, are the fundamental magnitudes of first order and K is the Gaussian curvature of a surface S. (Received January 4, 1955.)

Logic and Foundations

564. B. A. Galler: *Cylindric and polyadic algebras.* Preliminary report.

Halmos (Proc. Nat. Acad. Sci. U.S.A. vol. 40 (1954) pp. 296–301) developed the theory of polyadic algebras as an algebraic analogue of the first-order predicate calculus. Tarski (Bull. Amer. Math. Soc. Abstracts 58-1-85, 58-1-86) developed the cylindric algebras for the same purpose. It is shown in this paper that the cylindric algebras are essentially the polyadic algebras which contain a "binary predicate" analogous to the identity predicate of the first-order predicate calculus. (Received March 9, 1955.)

565*t*. P. C. Hammer: *The connectedness concept in general topology.*

A set-valued set function g defined for all subsets of a space M is an *inclusion preserving enlargement* (i.p.e.) *function* if gX contains X and if X contains Y implies gX contains gY universally. Let g_1 and g_2 be two i.p.e. functions. A pair X, Y of proper subsets of M are said to be g_1g_2*-separated* if the intersection of g_1X and g_2Y and the intersection of g_2X and g_1Y are empty. A set X is g_1g_2*-connected* provided no proper dichotomy of the set is g_1g_2-separated. Many of the usual theorems concerning connected sets as well as certain stronger ones are shown to hold for this generalization of connectedness. Let S be a class of pairs (U, V) of sets such that (U, V) in S implies (V, U) in S and the intersection of U and V is empty. A pair X, Y of sets is S*-separated* if there exists (U, V) in S such that U contains X and V contains Y. The connectedness based on this concept is appropriate in many theorems on connected sets. (Received March 7, 1955.)

566*t*. A. R. Schweitzer: *A bond between the foundations of geometry and transformation group theory.* I.

Kinematics is interpreted as a bond connecting the foundations of geometry and Lie's theory of transformation groups. In the foundations of geometry kinematics is represented by metrical properties. The latter have been axiomatically expressed by Hilbert in terms of equality (congruence) of linear segments (*Grundlagen der Geometrie*, 7th ed., Leipzig and Berlin, 1930, p. 11) by F. Schur in terms of motion as a defined concept on the basis of descriptive-projective properties (*Grundlagen der Geometrie*, Leipzig and Berlin, 1909) by Pieri in terms of motion as an undefined concept (Turin, 1899) and by others. (Received March 8, 1955.)

567*t*. A. R. Schweitzer: *A bond between the foundations of geometry and transformation group theory.* II.

In continuation of the preceding paper the author first discusses transition from kinematics as represented by metrical properties in the foundations of geometry to Lie's theory of transformation groups with references to Hilbert (loc. cit. p. 178) F. Schur (loc. cit. pp. iii–iv, 40) H. Weyl, *Mathematische Analyse des Raumproblems* (Berlin, 1923, p. 30), G. Kowalewski, *Einführung in die Theorie der kontinuierlichen Gruppen* (New York, 1950, pp. 20, 21–30, 97, 145). Secondly, the author considers kinematics as encompassed by Lie's theory with reference to S. Lie, *Theorie der Transformationsgruppen*, Leipzig, 1893, Part III. In his treatise Kowalewski finds it convenient to refer to R. Lipschitz's essay, *Untersuchungen über die Summen von*

Quadraten (loc. cit. p. 20) and to the quaternion (loc. cit. pp. 21–30). The author discusses these references in relation to his article in Math. Ann. vol. 69 (1910). (Received March 8, 1955.)

568t. A. R. Schweitzer: *On the foundations of metrical geometry.*

The author constructs a complete set of axioms for the foundations of geometry in terms of two undefined relations between n-simplexes symbolized by $\alpha_1\alpha_2 \cdots \alpha_{n+1}K\beta_1\beta_2 \cdots \beta_{n+1}$, $\alpha_1\alpha_2 \cdots \alpha_{n+1}E\beta_1\beta_2 \cdots \beta_{n+1}$ $(n=1, 2, \cdots)$ and respectively expressing sameness of orientation and equality (congruence). In the present paper metrical geometry in terms of the latter relation is considered. The n-simplex $\alpha_1\alpha_2 \cdots \alpha_{n+1}$ is isosceles relatively to a given linear segment, say $\alpha_1\alpha_2$, if and only if $\alpha_1\alpha_2\alpha_3 \cdots \alpha_{n+1}E\alpha_2\alpha_1\alpha_3 \cdots \alpha_{n+1}$. The linear segment $\alpha_1\alpha_2$ is the "base" of the isosceles n-simplex $\alpha_1\alpha_2 \cdots \alpha_{n+1}$; the equal $(n-1)$-simplexes $\alpha_1\alpha_3\alpha_4 \cdots \alpha_{n+1}$, $\alpha_2\alpha_3\alpha_4 \cdots \alpha_{n+1}$ are "sides" and the $(n-2)$-simplex $\alpha_3\alpha_4 \cdots \alpha_{n+1}$ is the "top." From an isosceles n-simplex a set of "n-simplicial dividers" is obtained by omitting the interior points of the base; the dividers are said to lie on the associated n-simplex and two sets of dividers are equal if and only if the associated isosceles n-simplexes are equal. The author develops foundations of metrical geometry in terms of an undefined relation of equality between two sets of n-simplicial dividers. Two linear segments are equal if and only if there exist equal simplicial dividers which they subtend. (Received March 8, 1955.)

STATISTICS AND PROBABILITY

569t. H. S. Konijn: *On some tests for treatment effects in paired replicates.*

Several methods are considered for testing whether or not a certain treatment causes a shift in the distribution of some measured characteristic. Among the tests the distributions of which are studied in detail, Wilcoxon's is most suitable for detecting small shifts, if few a priori assumptions regarding the distribution of the measurements are justified. When the members of each pair of objects have been drawn from a finite collection of objects, the sign test, even though possessing strong minimax properties, can be exceedingly inefficient in detecting small shifts; in other cases the asymptotic efficiency is more than 1/2. In passing, tests for the two-sample location problem and some of their properties are reviewed. (Received March 9, 1955.)

570. J. F. Nash: *Definability, randomness, and indistinguishable random processes.*

One can define a concept of definability via the class of ordinals for which a Gödel constructible enumeration of the preceding ordinals exists. From this a concept of a random sequence (of 1's and 0's) can be defined. If this class of ordinals is assumed (nonconstructively) enumerable, then random sequences exist (in the idealized sense). Another idea comes from regarding a random process as a machine for generating a random sequence of 1's and 0's. Two such machines are indistinguishable if distinction cannot be made from comparison of output sequences, one from each. Or, formally, if they correspond to two measures on the space of sequences, each continuous with respect to the other. If the machines have the probability of making the nth sequence digit 1 independent of past choices a simple criterion for indistinguishability obtains.

The series formed by the squared difference of the probabilities of 1 at the nth digit for the two processes, divided by the minimum of these probabilities and the two complements, should diverge. From this idea one can obtain results of the large numbers type by comparison of the coin process with an indistinguishable one. (Received March 15, 1955.)

571. J. M. Shapiro: *A condition for existence of moments of infinitely divisible distributions.*

Let $F(x)$ be an infinitely divisible distribution and let $G(u)$ be the bounded nondecreasing function given by the Lévy-Khintchine formula for the representation of the characteristic function of $F(x)$. It is shown that $\int_{-\infty}^{\infty} x^{2k} dF(x) < \infty$ is equivalent to $\int_{-\infty}^{\infty} u^{2k} dG(u) < \infty$. A simple relation is given between the moments of $G(u)$ and those of $F(x)$ in terms of the semi-invariants of $F(x)$. (Received March 3, 1955.)

Topology

572*t*. R. H. Bing: *An upper semicontinuous decomposition of E^3 into points and tame arcs whose decomposition space is not E^3.*

It is found that one of the useful theorems for E^2 does not generalize to E^3. An example is given of an upper semicontinuous decomposition G of E^3 such that each element of G is either a point or a tame arc but the decomposition space is not topologically equivalent to E^3. The elements of G are so tame that no horizontal plane intersects any one of them in two points. In terms of continuous transformations this says that there is a map f of E^3 onto a metric space X topologically different from E^3 such that for each point x of X, $f^{-1}(x)$ is either a point or a tame arc. (Received March 7, 1955.)

573. L. E. Pursell: *Fixed ideals and automorphisms of certain function rings.*

Let $S(X)$ be any ring of functions from a topological space X to a division ring D satisfying the following: (i) $Z(f)$ is closed for $f \in S$, (ii) if $x \notin$ a closed set F, $\exists f \in S \ni Z(f)$ contains a neighborhood of F but not x, (iii) if $f(x) \neq 0$ for all x in a closed set F, $\exists g \in S \ni (fg)(x) = 1$ for $x \in F$, and (iv) for $x \in X$, $\exists f, g \in S \ni x = Z(f) - Z(g)$. Define $A(f) = \{g \in S \mid fg = 0\}$ and $H(f) = \{g \in S \mid g \neq 0$ and $f/A(g)$ is a unit in $S/A(g)\}$. An ideal I in S is fixed if and only if $\exists g, h \in S \ni H(g) \not\supset H(h)$ but $H(gf) \supset H(hf)$ for each f in I. Under an isomorphism i: $S(X) \rightarrow S'(X')$ the image of a (maximal) fixed ideal is a (maximal) fixed ideal. This one-to-one correspondence between maximal fixed ideals determines a one-to-one correspondence from X onto X' which is a homeomorphism $\phi(i)$. If $A(S(X))$ is the automorphism group of $S(X)$ and $H(X)$ is the homeomorphism group of X, then ϕ is a homomorphism of $A(S(X))$ into $H(X)$. (Received March 7, 1955.)

574*t*. L. E. Pursell: *Fixed ideals and automorphisms of certain rings of real-valued continuous functions.*

Properties (i)–(iv) of the preceding abstract are satisfied by the ring $C(X, R)$ of all real-valued continuous functions on a normal space X with points which are G-delta sets, by the ring $C_0(X, R)$ of all real-valued continuous functions on X with compact supports if X is a locally-compact space with points which are G-delta sets, and by the ring $C^r(M)$ of all r-continuously differentiable functions on a differentiable manifold M with a neighborhood-finite coordinate covering. For $C(X, R)$ and $C_0(X, R)$, the homomorphism ϕ is an isomorphism from the automorphism group onto $H(X)$. For any subring S of $C(X, R)$ satisfying (i)–(iv) and (v) (for each $x \in X$, $\{f(x) | f \in S\} = R$) the homomorphism ϕ is an isomorphism into $H(X)$; any automorphism α of S can be extended to $C(X, R)$; and $f(x) = (\alpha f)(\phi(\alpha)x)$. (Received March 7, 1955.)

575. E. H. Rothe: *Mapping degree in Banach spaces and spectral theory.*

An attempt is made to define the mapping degree in Banach spaces directly and not by approximation from finite dimensional spaces as is done in the theory of Leray and Schauder as well as in that of Nagumo. Due to the lack of a definition of an orientation in Banach spaces, already the case of a linear map $f(x)$ requires consideration. For such maps, the definition of the degree is given in terms of that part of the spectrum of $F(x) = x - f(x)$ which is real and >1 (essentially using a theorem by Leray-Schauder as definition), and the homotopy theorem for the degree becomes a theorem in spectral perturbation theory. To be able to make full use of this latter theory, the paper deals with complex Banach spaces, and the results are carried over to real Banach spaces which can be imbedded in the complex spaces considered. Finally, the degree is defined and its homotopy property proved for nonlinear maps of a very smooth type. (Received February 14, 1955.)

576. D. E. Sanderson: *An extension of a deformation theorem of Alexander's.*

J. W. Alexander proved that any homeomorphism of an n-cell onto itself which is the identity on the boundary can be deformed onto the identity by an isotopy which is fixed on the boundary (Proc. Nat. Acad. Sci. U.S.A. vol. 9 (1923) pp. 406–407). An extension of this theorem for $n \leqq 3$ can be given as follows: *Let h be any orientation-preserving homeomorphism of E^3 onto itself which is the identity on a closed subset K of E^3 and maps each component of $E^3 - K$ onto itself. Then h can be deformed onto the identity by an isotopy which is fixed on K.* Using a recent result by M. K. Fort (Proc. Amer. Math. Soc. vol. 5 (1954) pp. 456–459), a theorem announced by the author (Bull. Amer. Math. Soc. Abstract 59-6-567), and Alexander's theorem, an isotopy is obtained which is fixed on K and deforms h onto a homeomorphism f which is piecewise linear on $E^3 - K$. Next, a shellable triangulation (for definitions, see abstract referred to above) of each component of $E^3 - K$ is obtained with the aid of Fort's result and, finally, f is deformed onto the identity on each simplex in the order of shelling. The desired isotopy results. (Received February 25, 1955.)

577. Jerome Spanier: *Generalized almost complex structures.* Preliminary report.

An m-dimensional differentiable manifold M is *almost complex* if the structural group of its tangent bundle is reducible to $U(n)$ ($m = 2n$ or $2n+1$). If $m = 2n+1$, a reduction to $O(2n)$ can always be made. Such an almost complex structure may be regarded as a cross section in a suitable bundle associated to the tangent bundle of M. Examples of odd-dimensional almost complex manifolds are the odd-dimensional spheres and the tangent sphere bundle of any manifold. Two almost complex structures are *equivalent* if the cross sections they determine are homotopic by a vertical homotopy. The following are proved: (1). S^{2n+1} admits infinitely many inequivalent almost complex structures if and only if $2n+1\equiv 3 \pmod 4$. (2). If $p:E\rightarrow B$ is a fiber space with almost complex base B and fiber F admitting an almost complex structure invariant under the structural group G, then E is almost complex. (2) is used to prove: (3). The Stiefel manifold $V_{n,k}=O(n)/O(n-k)$ is almost complex if either n or k is even. (1) depends on the lemma: If $p:E\rightarrow S^n$ is any fiber space admitting a cross section, then two cross sections are homotopic if and only if they are homotopic by a vertical homotopy. (Received March 7, 1955.)

578*t*. A. W. Wymore: *On weak compactness in functional analysis.*

Let $C(S)$ be the set of continuous functions on a compact Hausdorff space S. Three types of compactness (countable, sequential and bi-) for subsets of $C(S)$ in two topologies (point-open and weak) are analyzed. The fundamental equivalences in this context are obtained by intrinsic methods. In particular, the Osgood theorem is studied. (Theorem (Osgood, 1897): Let $\{x_n\}$ be a norm-bounded sequence in $C(S)$ such that $\lim_n x=0$ for every $t\in S$. Then $\lim_n x=0$ weakly in $C(S)$.) This theorem is shown to be equivalent to important assertions in the theory of weak compactness for which it had formerly been used as a tool (*e.g.*, The Osgood Theorem: The point-open closed convex hull of a point-open compact set is point-open compact). Corresponding theorems in Banach spaces and locally convex spaces are obtained as corollaries. (Received March 7, 1955.)

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