THE ANNUAL MEETING IN PITTSBURGH

The sixty-first Annual Meeting of the American Mathematical Society was held at the University of Pittsburgh on Monday through Wednesday, December 27–29, 1954, in conjunction with meetings of the Mathematical Association of America, which extended through Thursday, December 30; of the Association for Symbolic Logic on Wednesday, December 29; and of the Society for Industrial and Applied Mathematics on Tuesday, December 28. The registration was 658 including 546 members of the Society.

The twenty-eighth Josiah Willard Gibbs Lecture, entitled *Asymptotic phenomena in mathematical physics*, was delivered by Professor K. O. Friedrichs of the Institute of Mathematical Sciences, New York University, on Monday evening. Professor G. T. Whyburn, President of the Society, presided.

By invitation of the Committee to Select Hour Speakers for Annual and Summer Meetings, Professor Samuel Eilenberg of Columbia University addressed the Society on *Homological algebra and dimension* on Monday afternoon, and Professor Lipman Bers of the Institute of Mathematical Sciences, New York University, addressed the Society on *Pseudo-analytic functions* on Tuesday afternoon. Professor Saunders MacLane presided at the former address, and Professor T. H. Hildebrandt at the latter.

On Wednesday afternoon, December 29, the Frank Nelson Cole Prize in Algebra was awarded to Professor Harish-Chandra of Columbia University for his papers on representations of semisimple Lie algebras and groups, and particularly for his paper *On some applications of the universal enveloping algebra of a semisimple Lie algebra*, which appeared in volume 70 of the Transactions.

The ladies of the Department of Mathematics of the University of Pittsburgh entertained at tea on Monday afternoon in the Faculty Club of the Cathedral of Learning. A conducted tour of the Mellon Institute of Industrial Research was held on Tuesday afternoon, and visits were made to other points of interest in Pittsburgh.

A banquet was held at the Hotel Webster Hall on Wednesday evening. Professor J. S. Taylor of the University of Pittsburgh was the toastmaster. The speakers were Dean S. C. Crawford, College of Arts and Sciences, University of Pittsburgh, Professor Harold W. Kuhn, President of the Society for Industrial and Applied Mathematics, Professor W. V. Quine, President of the Association for Symbolic Logic, Professor E. J. McShane, President of the Mathematical Association of America, and Professor L. W. Cohen, Associate Secretary of the American Mathematical Society. A resolution of thanks to the University of Pittsburgh was presented by Professor C. B. Allendoerfer and adopted unanimously.

The Berkeley sessions of the Sixty-first Annual Meeting were held at the University of California, Berkeley, California, on Thursday and Friday, December 30 and 31, 1954, in conjunction with the meetings there of the American Association for the Advancement of Science. All sessions of the Society were joint sessions with section A of the A.A.A.S. The registration was 64, including 57 members of the Society.

The Program Committee for Section A of the A.A.A.S. and the Committee to Select Hour Speakers for Far Western Sectional Meetings of the Society invited two hour speakers. Professor Hans Lewy of the University of California, Berkeley, delivered on Thursday morning an address entitled *Reflection laws for linear elliptic equations*, while on Friday morning Professor H. F. Bohnenblust of the California Institute of Technology spoke on *Transition points in differential equations*. Professor Lewy was introduced by Professor J. W. Green and Professor Bohnenblust by Professor R. E. Langer.

On Thursday evening a dinner was arranged at the Shattuck Hotel for the mathematicians attending the meetings of the A.A.A.S., the Society, and the Institute of Mathematical Statistics. The principal speaker at the dinner was Dr. C. B. Tompkins of the Numerical Analysis Research at U.C.L.A. The Department of Mathematics at Berkeley entertained the visitors at tea following the meetings on Friday.

The annual Business Meeting of the Society was held on Wednesday, December 29, 1954. The Secretary reported that at this time the ordinary membership of the Society is now 4693, including 466 nominees of institutional members and 36 life members. The membership of the Society showed a greater increase in 1954 than in the last few preceding years. There are also 132 institutional members. The total attendance at all meetings in 1954 was 2197; the number of papers read was 823; there were 20 hour addresses; 1 Gibbs Lecture and 10 papers at an Applied Mathematics Symposium. The number of members attending at least one meeting was 1515.

At the annual election, in which over one thousand votes were cast, the following officers were elected:

Vice President, Professor Emil Artin.

Treasurer, Dean A. E. Meder, Jr.

Secretary, Professor E. G. Begle.

Associate Secretary, Professor R. D. Schafer.

Member of the Editorial Committee of the Proceedings, Professor Shizuo Kakutani.

Member of the Editorial Committee of the Transactions and Memoirs, Professor Samuel Eilenberg.

Member of the Editorial Committee of the Colloquium Publications, Professor Salomon Bochner.

Member of the Editorial Committee of the Mathematical Reviews, Professor J. L. Doob.

Member of the Editorial Committee of the Mathematical Surveys, Professor R. J. Walker.

Member of the Committee on Printing and Publishing, Professor E. F. Beckenbach.

Members of the Board of Trustees, Professors G. A. Hedlund, Deane Montgomery, and Dean Mina Rees.

Members-at-large of the Council, Professors E. F. Beckenbach, David Blackwell, R. H. Fox, K. O. Friedrichs, and G. de B. Robinson.

The Council met on Tuesday evening, December 28, 1954.

The Secretary announced the election of the following forty-four persons to ordinary membership in the Society:

Mr. Allan Ingvald Benson, Los Alamos Scientific Laboratory;
Professor Edward Charles Blom, State Teachers College, Fredonia, New York;
Professor William Antonio Carito, Boston College;
Professor Kenneth Scott Carman, Kansas Wesleyan University;
Mr. Wilton Roy Cooper, Columbia University;
Lieutenant Richard Edwin Cornish, Fort George G. Meade, Maryland;
Professor Frank Bigley Crippen, Fordham University;
Miss Betty Charles Detwiler, Washington University;
Mr. Lester Eli Dubins, University of Chicago;
Professor James Watson Ellis, Florida State University;
Dr. Marc Francis Fontaine, The Beacon Laboratories of The Texas Company, Beacon, New York;
Mr. Talmage Yates Hicks, International Business Machines Corp., Houston, Texas;
Mr. Lee Harmon Hook, University of Minnesota;
Professor Stephen Kulik, University of South Carolina;
Professor Norman Douglas Lane, McMaster University;
Mr. Melchiore Louis LaSala, Army Chemical Center, Maryland;
Mr. Bernard S. Levine, City College of New York;
Pfc. Richard Lawrence Liboff, Army Chemical Center, Maryland;
Mr. Louis Floyd McAuley, University of Maryland;
Dr. Robert Richard McCready, Cleveland-Hopkins Airport, Cleveland, Ohio;
Dr. Garner McCrossen, National Bureau of Standards, Boulder, Colorado;
Mr. Robert Gibson McDermot, University of Pittsburgh;
Mr. Jack Nicholas Medick, Cornell Aeronautical Laboratories;
Mr. Victor Julius Mizel, Massachusetts Institute of Technology;

Mr. George Aaron Paxson, United States Army;
Mr. Paul Anthony Penzo, University of Pittsburgh;
Professor William Morris Perel, Georgia Institute of Technology;
Dr. William Wesley Peterson, International Business Machines Corp., Poughkeepsie, New York;
Mr. Stanley Preiser, Nuclear Development Associates, White Plains, New York;
Mr. Henry Rainbow, Shell Oil Company, Houston, Texas;
Professor John William Riner, St. Peters College;
Professor Hortense Corbett Rogers, Winthrop College;
Pvt. Eugene Paul Rozycki, United States Army;
Professor Hanno Rund, University of Toronto;
Miss Joan P. Salatino, Marquette University;
Mr. Charles B. Shaw, Jr., Lockheed Aircraft Corp.
Mr. Thomas Harold Starks, Convair, Fort Worth, Texas;
Mr. John Kenneth Sterrett, Eglin Air Force Base, Florida;
Dr. George Herbert Swift, Jr., Duke University;
Mr. Robert Earl Thomas, Iola Senior High and Junior College, Iola, Kansas;
Mr. Robert J. Wernick, Army Chemical Center, Maryland;
Mr. Fred Walter Wolock, Iona College;
Professor Nelson Paul Yeardley, Thiel College.

It was reported that the following one hundred ninety-four persons had been elected to membership on nomination of institutional members as indicated:

University of Alabama: Mr. Theodor D. Sterling.

University of British Columbia: Mr. Eugene Butkov and Mr. Robert Charles Thompson.

Brown University: Mr. Abdulrazak Ali Huswan, Mr. Harry Lighthall, Mr. Donald Gene Malm, Professor Leonard C. Maximon, and Mr. J. Rainer Maria Radok.

California Institute of Technology: Mr. John Williams Lamperti, Mr. Lee Meyers Sonneborn, and Mr. Lloyd Richard Welch.

University of California, Berkeley: Mr. Israel Jacob Abrams, Mr. Satya P. Agarwal, Mr. David Wilson Bressler, Mr. Theodore Thomas Frankel, Mr. Richard Carlton MacCamy, Mr. Marvin Rosenblum, Mr. Warren Bernard Stenberg, Mr. Balkrishna Vasudeo Sukhatme and Mr. Adil Mohamed Yaqub.

University of California, Los Angeles: Mr. Robert Kenneth Froyd and Mr. Kenneth Myron Hoffman.

College of The City of New York: Mr. Irwin Stanley Bernstein and Mr. Ronald Jacobowitz.

Columbia University: Mr. Norman Alling, Mr. Yeaton Hopley Clifton, Mr. George Claggett Francis, Mr. Paul Alexander Gillis, Mr. Jacob Eli Goodman, Mr. Wendell Lyons Jones, and Mr. Laurence Edward Sigler.

Cornell University: Mr. Louis deBranges, III, Mr. Lonnie Cross, Mr. Stanislaw Leja, Mr. Richard E. Schwartz, and Mr. Paul Strauss.

Duke University: Mr. Robert Bruce Jackson, Jr.

Harvard University: Mr. Edward F. Assmus, Mr. Leonard E. Baum, Mr. Ira Ewen, Mr. Robert Gold, Mr. Solomon Wolf Golomb, Mr. Stevens Heckscher, Mr. John Arnold Kalman, Mr. Ralph Mack Krause, Mr. Karl Martin Kronstein, Mr. Henry Siggins Leonard, Jr., Dr. Donald Howard Menzel, Dr. Marvin Lee Minsky, Mr. Thomas Wilson Mullikin, Mr. Mason Miller Phelps, and Miss Lisa A. Steiner.

University of Illinois: Mr. Richard Solomon Ballance, Mr. William Clarence Bennewitz, Mr. Kenneth Allyn Brons, Mr. Yuan Shih Chow, Mr. Aubyn Freed, Mr. John Graham, Mr. Willard Donald James, Mr. Morris Wolfe Katz, Mr. Eugene Edmund Kohlbecker, Mr. Raymond Peter Polivka, Mr. Burnett Henry Sams, and Mr. Norman Emil Sexauer.

Institute for Advanced Study: Dr. Alfred V. Froelicher, Dr. William Martin Huebsch, Dr. Masatake Kuranishi, Mr. Christer Lech, Dr. Paul Malliavin, Dr. Georges H. Reeb, Dr. Peter Roquette, and Dr. Kosaku Yosida.

Iowa State College: Mr. Fowler Redford Yett.

Johns Hopkins University: Mr. William Henry Adams, Jr., Mr. Theodore Albert Bickart, Mr. George Backus Brown, Mr. Charles Vernon Coffman, Mr. Daniel Isaac Fivel, Mr. Gabriel M. Gerstenblith, Mr. Thomas Lewis Gibson, Jr., Mr. John Allan Henneberger, Mr. Donald William Kydon, Mr. Frank S. Levin, Mr. David Bowen McCandlish, Mr. Ganesan Sri Ram, Mr. Ray Ronald Rudolph, Mr. Jose Luis Soto-Alarcón, Mr. William Richard Webster, Mr. George Wend, and Professor Robert W. Zwanzig.

University of Kansas: Mr. George Alfred Ladner, and Mr. Eugene Kay McLachlan.

Kenyon College: Mr. Erwin Herman Knapp.

Lehigh University: Mr. Thomas Francis Green.

University of Maryland: Mr. James Henry Bramble, Mr. Hubert Whitman Lilliefors, and Mr. Charles Leonard Waldman.

Massachusetts Institute of Technology: Mr. Oma Hamara, Mr. Noel Justin Hicks, Mr. Harold Frazyer Mattson, Jr., Mr. Nevin William Savage, Mr. Gustave Solomon, and Mr. Walter E. Weissblum.

Michigan State College: Mr. James William Caltrider, Mr. Yousel Alavi, and Miss Elizabeth Karol Banks.

University of Michigan: Mr. Ervin Roy Deal, Dr. Richard C. W. Kao, Mr. James Edwin Keisler, Dr. Kyung Whan Kwun, Mr. Earl Edwin Lazerson, Mr. Gerald Otis Losey, and Mr. Stephen Smale.

University of Minnesota: Mr. Herbert Fantle, Mr. William Ashton Harris, Jr., Mr. Gerald Arthur Heuer, Mr. Roy Alfred Jorgensen, Jr., Mr. Richard Kent Juberg, Mr. Sherman James O'Neill, Mr. Roger Noel Pederson, and Mr. William Daniel Serbyn.

University of Missouri: Mr. John Homer DeHardt.

Northwestern University: Dr. José María Gonzáles-Fernández, and Dr. Walter Thomas Kyner.

Ohio State University: Mr. Henry Hofman Diehl, Miss Betty Lou Bernhardt, Mr. Carl Christopher Maneri, Mr. Donald Edward Robison, and Mr. Thomas Aloys Willke.

Oklahoma Agricultural and Mechanical College: Mr. John Leroy Folks and Mr. Oliver Paulsanders.

University of Oregon: Mr. Calvin Thomas Long.

University of Pennsylvania: Mr. Samuel Abraham Hoffman and Mr. Wallace Smith Martindale, 3rd.

Princeton University: Mr. Dean Paul Benzecri, Mr. William Browder, Mr. Albert William Currier, Mr. George Howard Dinsmore, Jr., Mr. James Hugo Griesmer, Dr. Kiyoshi Ito, Dr. Ioan Mackenzie James, Mr. Harold Melvin Kaplan, Mr. James A. Lechner, Dr. Kenneth Rogers, Mr. Norman Shapiro, Mr. Richard Gordon Swan, and Mr. David Leon Yarmush.

Purdue University: Mr. William Austin Beck and Mr. Leonard Irvin Holder.

The Rice Institute: Mr. Robert Melvin McLeod and Mr. John Raymond Swaffield.

Rutgers University: Mr. Allen Howard Miller.

College of Saint Thomas: Mr. Hubert Ronald Walczak.

University of Southern California: Mr. Russell Verner Benson, Mr. Albrecht Ferling, Mr. Thomas William Kampe, and Mr. Jack Bard Stutesman.

Stanford University: Mr. James Sterling Ayars, Mr. Gerald Leland Davey, Mr. Adriano Mario Garsia, Mr. John Walker Gray, and Mr. Cyril Joseph Murphy.

Syracuse University: Professor James B. Munz and Mr. Wilbert Neil Prentice.

University of Toronto: Mr. Donald Trevor Bean, Mr. Hermes Andrew Eliopoulos, Mr. William Kahan, Mr. Rudolph Oscar Robinson, and Mr. Ralph Wormleighton.

University of Utah: Mr. Byron Leon McAllister.

University of Virginia: Mr. Walter Francis Davison, Mr. John Jay Greever, III, and Mr. Edward Malcolm Wyatt.

University of Washington: Mr. Robert Trull Ives.

Washington University: Mr. Richard Allen Askey, Mr. Paul Richard Beesack, Dr. Edgar Henry Brown, Jr., and Mr. Ning Sheng Fan.

University of Wisconsin: Mr. Eugene Lang Albright, Mr. John A. Fibiger, Mr. Harry Melvin Friedman, Mr. Thomas I. Gilroy, Mr. Horace Clark Hearne, Jr., Mr. Orville John Marlowe, Mr. Robert Kirkpatrick Meany, and Dr. Roy Clifford Townsend Smith.

Yale University: Mr. Victor Elihu Bach, Mr. Leon Greenberg, Mr. Leo Hellerman, Mr. David Carter McGarvey, Mr. Allen Lewis Morton, Jr., Mr. Gian Carlo Rota, Mr. Dallas Wilton Sasser, and Miss Maria Josefa Wonenburger.

The Secretary announced that the following had been admitted to the Society in accordance with reciprocity agreements with various mathematical organizations: Deutsche Mathematiker Vereinigung: Professor Friedrich Lösch, Technical University of Stuttgart, Stuttgart, Germany, and Professor Oskar Perron, University of Munich, Munich, Germany; Indian Mathematical Society: Mr. Srinivasa Swaminathan, University of Madras; London Mathematical Society: Professor Abraham Robinson, University of Toronto; Société Mathématique de France: Professor Sunouchi Gen-ichirô, Tôhoku University, Sendai, Japan, and Professor Tadao Tannaka, Tôhoku University, Sendai, Japan; Suomen Matemaattinen Yhdistys: Professor Gunnar Johannes af Hällström, Abo Akademi, Abo, Finland, and Mr. Carl Gustaf Wolff, Ekehäs Samlyuum, Ekenäs, Finland.

The following appointments by the President were reported: as a committee to nominate a representative of the Society on the Policy Committee for Mathematics: J. R. Kline, Chairman, Marston Morse, and Antoni Zygmund; as a joint committee of the Society and the Mathematical Association of America on the Employment Register: J. S. Frame, Chairman, H. M. Bacon, T. R. Hollcroft, Morris Ostrofsky, and J. A. Ward; as Chairman of the Committee on Places of Meetings for 1955: E. R. Lorch; as a new member of this Committee: C. B. Morrey (Committee now consists of E. R. Lorch, Chairman,

W. M. Whyburn, and C. B. Morrey); as a representative of the Society on the Advisory Board of the Applied Mechanics Reviews for a term of three years beginning July 1, 1954: Dr. Eleazer Bromberg; as a committee on arrangements for the 1955 Summer Meeting to be held at the University of Michigan: Wilfred Kaplan, Chairman, P. S. Dwyer, H. M. Gehman, P. S. Jones, W. J. LeVeque, R. M. Thrall, R. L. Wilder, and J. W. T. Youngs; as members of Committees to Select Hour Speakers (terms to expire December 31, 1956): Summer and Annual Meetings: D. H. Lehmer (Committee now consists of E. G. Begle, Chairman, Wilfred Kaplan, and D. H. Lehmer); Eastern Sectional Meetings: Lipman Bers (Committee now consists of R. D. Schafer, Chairman, Nelson Dunford, and Lipman Bers); Western Sectional Meetings: P. V. Reichelderfer (Committee now consists of J. W. T. Youngs, Chairman, M. E. Shanks, and P. V. Reichelderfer); Far Western Sectional Meetings: Ivan Niven (Committee now consists of J. W. Green, Chairman, J. L. Kelley, and Ivan Niven); Southeastern Sectional Meetings: G. B. Huff (Committee now consists of J. H. Roberts, Chairman, W. V. Parker, and G. B. Huff); as Chairman of the Committee on Applied Mathematics for 1955: F. J. Murray; and as members for terms of three years beginning January 1, 1955: A. S. Householder and F. J. Murray (Committee now consists of F. J. Murray, Chairman, R. E. Bellman, Garrett Birkhoff, R. S. Burington, A. S. Householder, and Shizuo Kakutani); as Chairman of the Committee on Visiting Lectureships for 1955: Einar Hille; and as a member for a term or three years beginning January 1, 1955: J. M. Thomas (Committee now consists of Einar Hille, Chairman, P. R. Halmos, and J. M. Thomas); as tellers for the 1954 election: F. M. Stewart and John Wermer.

The following appointments to represent the Society were reported: at the inauguration of Herrick Black Young as President of Western College for Women on October 9, 1954: Professor H. S. Pollard; at the inauguration of Carl Cluster Bracy as President of Mount Union College on October 15, 1954: Professor R. P. Gosselin; at the inauguration of Owen Meredith Wilson as President of the University of Oregon on October 19, 1954: Professor W. E. Milne; at the dedication ceremonies of New York University's Institute of Mathematical Sciences on November 29, 1954: Dr. J. H. Curtiss.

The Secretary reported that Professors H. F. Bohnenblust and Hans Lewy had accepted invitations to deliver addresses at the joint meeting of the Society and Section A of the AAAS at Berkeley, California, December 1954; that Professor Harish-Chandra had accepted an invitation to deliver an address at the February 1955 meeting in New York; that Professor I. S. Cohen had accepted an invitation to deliver an address at the April 1955 meeting in Brooklyn, and that Professor Salomon Bochner had accepted an invitation to deliver the Colloquium Lectures in 1956.

The Council voted to elect Professor André Weil as a representative of the Society on the Board of Editors of the American Journal of Mathematics to complete the term of Professor Samuel Eilenberg, who resigned from this position December 31, 1954.

The Council voted to elect Professor Charles Loewner a representative of the Society on the Board of Editors of the Annals of Mathematics for the period 1955–1957.

The Executive Director reported that the Bulletin of Information was being rewritten. He also reported that so far we have had a sixty percent return on the questionnaires sent out for the National Register of Scientific and Technical Personnel. It is hoped that by a follow-up the response will reach at least eighty percent.

The Council voted, in view of the appointment of Professor Paul Erdös as Visiting Lecturer of the Society in 1955–56, to request the President and Secretary to make inquiries in the appropriate offices in Washington as to the visa of Professor Erdös.

The Bulletin Editorial Committee reported that 607 pages had been used in 1954. The Council voted to recommend to the Board of Trustees that the Bulletin be authorized for 1955 to print 690 pages, which would include 65 pages for the 10 volume index.

The Transactions and Memoirs Editorial Committee reported that the interval between receipt of a manuscript and publication is approximately fifteen months. The Council voted to recommend to the Board of Trustees that three volumes of 550 pages each be published in the Transactions during 1955.

The Proceedings Editorial Committee reported that the interval between receipt of a manuscript and publication was approximately eight months. The Council voted to recommend to the Board of Trustees that 1006 pages be authorized for the 1955 Proceedings. Professors Walter Rudin and R. H. Bruck were reported as new Associate Editors for the Proceedings.

The Council voted to recommend to the membership of the Society an amendment to the by-laws which would increase the number of editors of the Proceedings from three to four.

On recommendation of the Colloquium Editorial Committee the Council voted to invite Professor Norman Steenrod as the Colloquium Speaker for 1957.

The Mathematical Reviews Editorial Committee reported that 1012 pages had been published in 1954. The subscription list as of

November 1954 was 2353.

The Committee on the Employment Register reported that over 100 listings appeared in the Employment Register at this meeting, about half of which listed academic positions.

The Council voted to accept an invitation from the University of Washington to hold the 1956 Summer Meeting at Seattle, August 20–25, and to accept an invitation from the University of Rochester to hold the 1956 Annual Meeting at Rochester, December 27–29. The Council voted to request the Presidents of the Society and the Mathematical Association of America to appoint a committee to investigate the possibility of holding the annual meeting at a time other than the week between Christmas and New Years.

The Council approved the following times and places of sectional meetings in 1955: April 22–23, University of Chicago; October 29, University of Maryland; November 18–19, University of Tennessee.

The Council voted to co-sponsor a Midwestern Conference on Solid and Fluid Mechanics to be held at Purdue University, September 8–10, 1955.

The Council voted to recommend to the membership of the Society an amendment to the by-laws which would make the term of office of the Trustees five years instead of the present two, one Trustee being elected each year.

There were sixteen sessions for contributed papers at Pittsburgh, presided over by Professors R. D. Anderson, Reinhold Baer, J. L. Brenner, Bernard Friedman, O. G. Harrold, J. J. L. Hinrichsen, R. E. Johnson, W. T. Martin, D. D. Miller, E. E. Moise, C. G. Mumford, Everett Pitcher, P. C. Rosenbloom, Lowell Schoenfeld, Seymour Sherman, R. M. Thrall. The sessions for contributed papers at Berkeley were presided over by Professors G. C. Evans, D. H. Lehmer, Charles Loewner, and J. L. Kelley.

The abstracts of the papers are appended. Those having the letter "*t*" following the abstract number were read by title. Those having the letter "B" following the abstract number were read at the Berkeley sessions. Where a paper, presented in person, has more than one author, the symbol (p) follows the name of the author who presented it. Dr. Kyner was introduced by Professor S. P. Diliberto, Dr. Stoll by Professor W. H. Gottschalk, Dr. McAuley by Professor Dick Wick Hall, Mr. Arms by Dr. R. L. Jeffery, Dr. Klingenberg by Professor J. W. T. Youngs, Professor Allen by Professor L. W. Johnson, Dr. Saworotnow by Professor C. T. Taam, Professor Kulik by Professor W. L. Williams, Dr. Rogers by Professor Mary P. Dolciani, and Dr. Swift by Professor Edwin Hewitt.

ALGEBRA AND THEORY OF NUMBERS

166. J. E. Adney, Jr.: *On the power of a prime dividing the order of a group of automorphisms.*

Let G be a group of finite order, $A(G)$ its group of automorphisms, and p a prime. I. N. Herstein and the author (Amer. Math. Monthly vol. 59 (1952)) proved if p^2 divides the order of a group G, then at least p divides the order of the group of automorphisms $A(G)$. W. Scott (Proc. Amer. Math. Soc. vol. 5 (1954)) extended this result to the following: If p^3 divides the order of G, then p^2 divides the order of $A(G)$. The main result of this paper is the following theorem. If G has order $p^n m$, $(m, p)=1$, and the Sylow subgroup Sp associated with the prime p is abelian, then at least p^{n-1} divides the order of $A(G)$. The proof requires the use of the transfer of G into Sp. The author knows of no counter-example to the general conjecture: if p^n is the highest power of p that divides the order of G, then at least p^{n-1} divides the order of $A(G)$. (Received November 12, 1954.)

167. Maurice Auslander: *Commutator subgroup of free groups.*

Let F be a non-abelian free group, R a proper normal subgroup of F, and $[R, R]$ the commutator subgroup of R. The following theorems are proved. (1) The center of $F/[R, R]$ is a proper subgroup of $R/[R, R]$. (2) If $G=F/R$ is a finite group such that $G\neq[G, G]$, then the center of $F/[R, R]$ is not $\{1\}$. An immediate consequence of (1) is that if F is a non-abelian free group and R is a normal subgroup such that $[R, R]=[F, F]$, then $F=R$. The proofs of (1) and (2) are entirely cohomological in nature, the principal tool being the cup-product reduction theorem. (Received November 12, 1954.)

168*t*. Maurice Auslander: *On the dimension of modules and algebras. III. Global dimension.*

This paper is concerned with the homological dimension of a ring Λ with unit (see H. Cartan and S. Eilenberg, *Homological algebra*, Princeton University Press, for notations and definitions). First, two results of a general nature are proved. (1) It is shown that l.gl.dim $=$ sup l.dim A, where A ranges over all left Λ-modules generated by a single element. (2) If Λ is both left and right Noetherian, l.gl.dim $\Lambda=$ r.gl.dim Λ. The rest of the paper is concerned with more specialized results. Assume Λ satisfies the descending chain condition for left ideals. (3) If N denotes the radical of Λ, then l.gl.dim $\Lambda=$ l.dim Λ/N. (4) If every simple left Λ-module is isomorphic to a left ideal of Λ, then l.gl.dim $\Lambda=0$, ∞ (completely primary rings and quasi-Frobenius algebras are examples of rings satisfying the hypothesis of (4)). (5) If Λ is commutative, then gl.dim $\Lambda=0$, ∞. Assume Λ is a commutative ring. (6) If every ideal of Λ is generated by a single element, then gl.dim $\Lambda=0$, 1 if and only if Λ has no nilpotent elements; otherwise, gl.dim $\Lambda=\infty$. (7) If Λ is Noetherian, then gl.dim $\Lambda\leqq 1$ if and only if Λ is the direct sum of a finite number of Dedekind rings. (Received November 12, 1954.)

169*t*. H. W. Becker: *Alliston's problem: Hero triangles with* 1, 2 *or* 3 (?) *sides squares.*

These are resp. of type I, II, or III. Corresponding to various general solutions for $H\triangle$, there are various gen. sols. for type I. It is conjectured that: general solution for type II is a chimera; type III is impossible. Any Pyth. tet. $v^2+x^2=u^2+z^2=t^2$,

$v^2-z^2=u^2-x^2=y^2 \to$ a type II $H\triangle u^2$, v^2, $x^2+z^2=$ base; alt. $=2txyz/(x^2+z^2)$. A corollary of the impossibility of type III would be the imp. of Martin vectors (Pyth. tets. with $x^2+z^2=\square$). Working from a table, or the 10 known basic parametric solutions, of $4abcd(a^2-b^2)(c^2-d^2)=\square=(\text{alt.})^2$, Bull. Amer. Math. Soc. vol. 60 (1954) p. 250, or their transforms, one derives all type II of Alliston's first method, Math. Snack Bar (1936) p. 119. Or else, try $(a^2+b^2)^2$, $(c^2+d^2)^2e^2$, $a^4-6a^2b^2+b^4\pm 4cd(c^2-d^2)e^2$; $4ab(a^2-b^2)=(c^4-6c^2d^2+d^4)e^2$, as $a, b, c, d, e=16, 9, 2, 1, 120$. Alliston's next method, top p. 120, generalizes to $(4r^4+s^4)^2$, $(4r^4+s^4)^2+\{2rs(2r^2-s^2)\}^2$, $\{2rs(2r^2+s^2)\}^2$; $4rs(4r^4+s^4)(2r^2-s^2)$. His last method may be formulated $(r^4-2r^2s^2+9s^4)^2$, $(r^4-s^4)\cdot(r^4-81s^4)$, $\{2rs(r^2+3s^2)\}^2$; $8rs(r^2-s^2)(r^2-3s^2)(r^2-9s^2)$. Working from a table or the general solution of $T^2=Y^2+X^2+Z^2$, a type I is $(X^2+Z^2)^2$, $(T^2+Y^2)(X^2-Z^2)$, $2(TX\pm YZ)(YX\mp TZ)$; $(T^2-Y^2)(X^2-Z^2)$. The base $=\square$ for the upper and lower signs resp. if $T, Y, X, Z=3, 2, 2, 1$ and $7, 6, 3, 2$. $(T^2+Y^2)(X^2-Z^2)=\square$ when $T, X, Y, Z=9r^4\pm 8r^2s^2+4s^4$, $4rs(3r^2\pm 2s^2)$, so the $H\triangle$ sides $\boxed{S}$ are $(81r^8-56r^4s^4+16s^8)^2$, $\Pi(9r^4\pm 16r^2s^2+4s^4)^2$, as 41^2, 87^2, 5920 or 9200; 369. (Received November 12, 1954.)

170*t*. H. W. Becker: *3-D Diophantine vectors with elements squares*. Preliminary report.

Solutions of $t^2=x^2+y^2+z^2$ (t, x odd) are type I, II, III or IV according as 1, 2, 3 or 4 of the elements t, x, y, z are $\boxed{S}$. Euler conjectured that no type IV exist; none exist with $t<10^4$, Morgan Ward, Duke Math. J. vol. 15 (1948) p. 827. Parametric solutions of type III. 1, 2, 3 (with respectively $t, x, y \neq \boxed{S}$) were given by Diophantus, Fauquembergue, and Escott, Dickson's *History* II, p. 657–658; but these cannot contribute to a type IV sol. Type II. 1, 2, 3, 4 sols. have resp. t, x; t, y; x, y; y, z both $\boxed{S}$. Para. sols. are resp. $t, x, y, z=\{m^2+n^2\pm(p^2+q^2)\}^2$, $2(mp+nq)(m^2+n^2+p^2+q^2)+2(mq-np)(m^2+n^2-p^2-q^2)$, $2(mq-np)(m^2+n^2+p^2+q^2)-2(mp+nq)(m^2+n^2-p^2-q^2)$; the same, except $x=(m^2+n^2)^2-(p^2+q^2)^2+4(mq-np)(mp+nq)$, $y=\{2(mp+nq)\}^2$; $9r^4+48r^3s+16r^2s^2-128rs^3+264s^4$, $(3r^2+8rs-8s^2)^2$, $\{2s(3r+4s)\}^2$, $8s^2(6r^2+16rs-31s^2)$, adapted from Gerardin, L'Int. des Math. vol. 26 (1919) p. 17; $m^4+n^4\pm 4p^4$, $(2pm)^2$, $(2pn)^2$. The above para. sols. for type II.1, 2 might be general, hence contributory to the settlement of type IV. The above para. sols. for type II. 3, 4 are not general, Ɖ resp. 65, 25, 36, 48 and 69, 67, 16, 4. Sols. $q^2, m^2, n^2, 2p^2 \to q^4$, $m^4+n^4-4p^4$, $(2pm)^2$, $(2pn)^2$, a type III.2→III.3 transform. Thus 9, 1, 4, 8 →81, 47, 16, 64; 49, 9, 36, 32→2401, 353, 2304, 576. In type I.1, 2, 3 sols., t, x, y resp. $=\square$. Special para. sols. are indicated by Bills, Dickson ibid. p. 666, and R. D. Carmichael, *Diophantine analysis* (1915) p. 46, which generalize readily to t, x, or $y=k$th power. (Received November 12, 1954.)

171. S. G. Bourne: *On semirings*. Preliminary report.

A semiring is a system consisting of a set S and two binary operations in S called addition and multiplication such that (a) S together with addition is a semigroup, (m) S together with multiplication is a semigroup, (d) The left and right hand distributive laws $a(b+c)=ab+ac$ and $(b+c)a=ba+ca$ hold. Semirings arise naturally when we take the totality of endomorphisms E of an arbitrary commutative semigroup $\mathfrak{S}$. An additive idempotent of a semiring S is an element a such that $2a=a$ and a multiplicative idempotent is an element m such that $m^2=m$. An additive identity, called zero, is an element 0 such that $0+s=s+0=s$, for all s in S. If S possesses a zero, then $a\cdot 0$ and $0\cdot a$, if defined, are additive idempotents of S, which are not necessarily equal. If m_1 and m_2 are distinct multiplicative idempotents of a semiring

S with zero, such that $m_1m_2+m_2m_1=m_1m_2+m_2=m_2m_1+m_2=0$, then $m_3=m_1+m_2$ is a multiplicative idempotent orthogonal to m_2 and $m_3m_1+m_2=m_1m_3+m_2=m_1$. (Received November 12, 1954.)

172t. Barron Brainerd: *A commutative semigroup theorem.*

Let A be a commutative semigroup containing an identity e. If K is a sub-semigroup of A, then the following condition in A defines a congruence relation ϕ_K: $a\equiv b \bmod \phi_K$ if and only if there exist elements l and k in K such that $al=bk$. ϕ_K is called a reversible congruence relation by some authors. Let $O(\phi_K)$ be the kernel of the congruence relation ϕ_K. If $O(\phi_K)=K$ for all sub-semigroups K of A, then A is a group. This result follows from the fact that the set $A'=\{a \mid \text{for all } x, ax\neq e\}$ is a sub-semigroup of A. (Received October 22, 1954.)

173t. Leonard Carlitz: *Some class number formulas for quadratic forms over GF [q, x].*

Byers (Duke Math. J. vol. 21 (1954) pp. 445–461) has proved several class number formulas by setting up a correspondence between classes of definite quadratic forms and of classes of bilinear forms. In the present paper these as well as more general results are proved by using a "singular series" similar to that for sums of squares in $GF[q, x]$ (Duke Math. J. vol. 14 (1947) pp. 1105–1120). If $h(\Delta)$ represents the number of classes of discriminant Δ, then for example the sum $\sum h(\Delta-\alpha_1U_1^2-\cdots-\alpha_sU_s^2)$ is evaluated; here $\alpha_1, \cdots, \alpha_s\in GF(q)$ and the summation is over all $U_i\in GF[q, x]$ of degree $<m$ and deg $\Delta=2m+1$. (Received November 8, 1954.)

174t. Leonard Carlitz: *Some class number relations.*

Using some well-known theta formulas due to Kronecker and Hurwitz, a number of class number relations are obtained that seem to be new. Typical formulas are $4\sum_{i+j=n}F(4i+1)F(4j+1)=(-1)^nE_2(2n+1)$ and $\sum_{i+j=n}F(8i+2)F(8j-2)=2E_2^1(n)$; here $E_2(n)$ denotes the sum of the squares of the divisors of n which are of the form $4k+1$ diminished by the sum of the squares of the divisors of n which are of the form $4k+3$; $E_2^1(n)$ denotes the sum of the squares of the divisors of n whose conjugates are of the form $4k+1$ diminished by the sum of the squares of the divisors of n whose conjugates are of the form $4k+3$. (Received November 8, 1954.)

175t. Leonard Carlitz and J. H. Hodges: *Distribution of bordered symmetric, skew and hermitian matrices in a finite field.*

Let $q=p^f$, $p>2$, and let A, M, U denote matrices with elements in $GF(q)$. If A is symmetric and non-singular of order m and U is $m\times t$ put $M=\left[\begin{smallmatrix}A & U\\ U' & O\end{smallmatrix}\right]$. Then the number of matrices M of rank $m+r$ and given invariant is determined. Similar results are obtained in the skew and hermitian cases. (Received November 8, 1954.)

176t. Eckford Cohen: *The finite Goldbach problem in algebraic number fields.* II.

Let F be a finite extension of degree n of the rational field and let A be a proper ideal of F. This paper establishes the necessary and sufficient conditions for the representability of all elements of the residue class ring $R(A)$ as sums of at most s primes in $R(A)$. In conjunction with the theorem contained in Bull. Amer. Math. Soc. Abstract 59-3-271, this theorem completes the generalization of results proved previously in the rational case (Proc. Amer. Math. Soc. vol. 5, pp. 478–483). The extended results show the rational field to be quite special for this particular problem. A term referred to as the *finite Goldbach property* is introduced to describe fields whose residue class rings possess a property analogous to the statement of the Goldbach conjecture. This property is shown to hold if and only if 2 is the nth power of a prime ideal of F, and in particular if F is the rational field. (Received November 8, 1954.)

177. W. E. Deskins: *On finite Abelian groups having isomorphic group algebras.*

Perlis and Walker, in their paper, *Abelian group algebras of finite order*, Trans. Amer. Math. Soc. vol. 68 (1950) pp. 420–426, gave conditions that two Abelian groups have isomorphic group algebras over fields having characteristic relatively prime with the order of the groups. Here we consider the problem where the field characteristic divides the order of the groups. Let G and H be abelian groups of order gp^a, $(g, p)=1$. Then $G=G_0\times G_1$, $H=H_0\times H_1$, where G_0 and H_0 are of order g and G_1 and H_1 are of order p^a. Let F be a field of characteristic p. *Theorem. The group algebras of G and H over F are isomorphic if and only if both of the following conditions are satisfied*: (a) *G_0 and H_0 have isomorphic group algebras over F.* (b) *G_1 and H_1 are isomorphic groups.* This reduces the problem to the case solved in the Perlis-Walker paper. (Received November 5, 1954.)

178. Mary P. Dolciani: *An analogue of the Hardy-Littlewood method.*

Let $K[t]$ be the domain of all polynomials in an indeterminate t with coefficients in a fixed field K of p elements, p odd. p is assumed to be a prime, for a more general assumption leads only to a parallel discussion. Let $S(X)=A_1X_1^2+\cdots+A_mX_m^2$ be a quadratic form in 5 or more variables with each A_i in $K[t]$. For any fixed element C of $K[t]$, $I(2n)$ is the number of solutions of $S(X)=C$ in polynomials X_i, such that for each i the degree in t of $A_iX_i^2$ is less than the even integer $2n$. $S(X)$ is said to represent C congruentially relative to an irreducible polynomial P of $K[t]$, if the congruence $S(X)\equiv C \pmod{P^j}$ is solvable for each positive integer j, where, if $C=0$, at least one X_i of the solution must be prime to P. The following result is proved: $I(2n)=p^{n(m-2)}G+O(p^{nm/2})$ where $G=G(A_1, \cdots, A_m, C)>0$, provided $m\geqq 5$ and S represents C congruentially relative to all irreducible polynomials P. Following the basic idea of the Hardy-Littlewood method, $I(2n)$ is expressed as a multiple sum which is divided into two parts, one of which leads to the main term involving a singular series and the other to an error term. The proof requires the consideration of certain exponential sums and the singular series. (Received November 12, 1954.)

179t. L. K. Durst: *On some numbers of Pólya.*

For $s=3$ and any integer $n>10g_2 3$, the numbers $N_n^{(s)}$ (cf. Pólya, J. Symbolic Logic vol. 5 (1940) pp. 98–103) have been evaluated. The result is: $N_n^{(3)}=\sum_k \{[k/3]+\sum_r[(k-3r)/2]\}$, $2\leqq k\leqq n$, $0\leqq r\leqq [k/3]$. (Received October 25, 1954.)

180. M. P. Epstein: *On Liouvillian extensions.* Preliminary report.

Let F be an ordinary differential field of characteristic 0 with field of constants C. Kolchin considered a Liouvillian extension G of F using the assumption that C is algebraically closed. He obtained the equivalence of 10 conditions on the group of automorphisms of G over F with 10 conditions on the generators of G over F. Here the situation is considered after removal of the assumption that C is algebraically closed. It is seen that Kolchin's equivalences fail (in general) and that one set of conditions on the group is necessary while a different (but similar) set is sufficient. These serve to characterize G "up to" a possible algebraic extension. (Received December 8, 1954.)

181. Trevor Evans: *The isomorphism problem for multiplicative systems.*

A general method is exhibited for determining whether or not two finitely generated and related multiplicative systems are isomorphic. Here, by multiplicative system, is meant loop, quasigroup, groupoid, inverse property loop, etc. This decision procedure is a consequence of the following theorem. Two finitely generated and related multiplicative systems are isomorphic if, and only if, the minimal closed sets of generators and relations obtained from the sets of generators and relations defining the two multiplicative systems are isomorphic. The concept of a minimal closed set of generators and relations obtained from a finite generator and relation presentation of a multiplicative system is a simple extension of the concept of a closed set of generators and relations introduced in *Multiplicative systems*, I, Proc. Cambridge Philos. Soc. vol. 47 (1951) pp. 637–649. (Received November 12, 1954.)

182. Leonard Gillman and Melvin Henriksen (p): *Concerning adequate rings and elementary divisor rings.* II.

A *commutative* ring S with unit is called an *Hermite* ring if for every matrix M over S there exists a nonsingular matrix K such that KM is a triangular matrix, an *elementary divisor* ring if there exist nonsingular K, L such that KML is diagonal. S is *adequate* if all finitely generated ideals are principal, and if for every $a, b \in S$, with $a \neq 0$, there exist $r, s \in S$ such that $a = rs$, $(b, r) = S$, and, for every non-unit divisor t of s, $(b, t) \neq S$. (See Bull. Amer. Math. Soc. Abstract 60-4-463, and Kaplansky, Trans. Amer. Math. Soc. vol. 66 (1949) pp. 464–491.) *Theorem* 1. S is Hermite *iff* for every $a, b \in S$, there exist a_1, b_1, d such that $a = a_1 d$, $b = b_1 d$, and $(a_1, b_1) = S$. *Theorem* 2. An Hermite ring S is an elementary divisor ring *iff* for all $a, b, c \in S$, with $(a, b, c) = S$, there exist $p, q \in S$ such that $(pa, pb+qc) = S$. *Theorem* 3. Every adequate Hermite ring is an elementary divisor ring. Examples are given of rings S of continuous functions in which (i) S is not adequate, but is an elementary divisor ring, (ii) every finitely generated ideal of S is principal, but S is not Hermite, (iii) S is Hermite, but not an elementary divisor ring. This work was supported in part by the National Science Foundation. (Received November 12, 1954.)

183. Leonard Gillman (p) and Melvin Henriksen: *On rings of continuous functions in which every finitely generated ideal is principal.* II.

Let X be a completely regular space; βX = Stone-Čech compactification of X; C = ring of all continuous real functions on X; C^* = subring of bounded functions; $Z(f)$ = set of zeros of $f \in C$; $\overline{P}(f)$ = closure (in X) of $\{x \in X : f(x) > 0\}$; $Z(f)^\beta$ = closure of $Z(f)$ in βX; for $p \in \beta X$, $M^p = \{f \in C : p \in Z(f)^\beta\}$; $N^p = \{f \in C$: there exists a neighborhood Σ of p such that $f(\Sigma \cap X) = 0\}$. It is known that $M \subset C$ is a maximal ideal *iff* there is a (unique) $p \in \beta X$ such that $M = M^p$ (see Proc. Amer. Math. Soc. vol. 60 (1954) p. 448, Theorem 1). The following improve Bull. Amer. Math. Soc. Abstract 60-4-462. *Theorem* 1. For every maximal ideal M^p of C, the intersection of all the prime ideals contained in M^p is N^p. *Theorem* 2. The following are equivalent. (a) Every finitely generated ideal of C is principal. (b) For all $f \in C$, $(f, |f|)$ is principal. (c) For all

$f, G \in C$, $(f, g) = (|f| + |g|)$. (d) For all $f \in C$, the sets $\overline{P}(f)$, $\overline{P}(-f)$ are completely separated. (e) For all $f \in C$, $f = k|f|$ for some $k \in C$. (f) For every $p \in \beta X$, the ideal N^p is prime (see Theorem 1). (g) Every finitely generated ideal of C^* is principal. (h) For every $f \in C$, every bounded non-negative continuous function defined on $X - Z(f)$ has a continuous extension over X. This work was sponsored in part by the National Science Foundation. (Received November 12, 1954.)

184. Emil Grosswald: *On a theorem of Erdös and Szekeres.*

For any integer $h > 0$, denote by M_h the set of all integers n such that, if $p|n$, then also $p^h|n$; let $M_h(x)$ stand for the number of integers $n \leqq x$ in M_h. Erdös and Szekeres (Acta Univ. Szeged. vol. 7 (1934) pp. 95–102) proved that $M_h(x) = A_h x^{1/h} + O(x^{1/(h+1)})$, where A_h depends on h alone. This result can be improved to $M_h(x) = A_h x^{1/h} + B_h x^{1/(h+1)} + O(x^{1/(h+2)})$, where $A_h = \prod_p \{1 + \psi_1(p)\}$, $B_h = 6\pi^{-2}\zeta(h/(h+1)) \cdot \prod_p \{1 + \psi_2(p)\}$. Here $\zeta(s)$ stands for the Riemann zeta-function, $\psi_1(p) = p^{-(h+1)/h} \cdot (1 - p^{-(h-1)/h})(1 - p^{-1/h})^{-1}$, and $\psi_2(p)$ has a similar expression. Formally, this result follows immediately, applying Cauchy's theorem on residues to the function $f_h(s)x^s/s$, where $f_h(s) = \prod_p (\sum_{\nu=0}^{\infty} p^{-\nu h s})(1 + \sum_{\mu=1}^{h-1} p^{-(h+\mu)s}) = \sum_{n=1}^{\infty} a_n n^{-s}$ is the generating function for the integers in M_h. It remains to justify the procedure, by showing that, for $c = 1/(h+2)$, $\int_{c-iT}^{c+iT} f_h(s) x^{it} s^{-1} ds$ stays bounded when $T \to \infty$. This is achieved by considering the function $F_h(s) = \prod_p \{(\sum_{\nu=0}^{\infty} p^{-\nu s}) \prod_{\mu=1}^{h-1} (1 + p^{-(h+\mu)s})\} = \sum_{n=1}^{\infty} b_n n^{-s}$. The sum $N_h(x) = \sum_{n \leqq x} b_n = A_h' x^{1/h} + B_h' x^{1/(h+1)} + C_h' x^{1/(h+2)} + o(x^{1/(h+2)})$ can be obtained by elementary methods; a comparison with the results of contour integration applied to $F_h(s)x^s/s$ then shows that $\int_{c-iT}^{c+iT} F_h(s) x^{it} s^{-1} ds$ stays bounded for $c \geqq 1/(h+2)$. The conclusion now follows, by proving that $f_h(s) = \alpha_h F_h(s) + \beta_h F_{h+1}(s) + \gamma_h F_{h+2}(s) + G_h(s)$, with α_h, β_h, γ_h constants and $G_h(s)$ regular for $s > 1/(h+3)$. (Received October 15, 1954.)

185. Franklin Haimo: *Some identities in semi-direct products.*

Certain identities related to normal nilpotent subgroups of semi-direct products with abelian normal factor are constructed. Work of W. H. Mills is extended. (Received November 15, 1954.)

186t. Isidor Heller: *A characterization of determinants.*

If V is a finite-nonzero-dimensional vectorspace over a field L and W is the set of linear transformations on V, the determinant $\Delta(A)$ associates an element of L to each A of W. The present note formulates a characterization of Δ in terms of linear transformations, avoiding the customary detour over matrices or inner product. Notation. I and 0 denote the identity and the null transformation respectively. E denotes an idempotent element of W which is not 0 and which is not the sum of two idempotent elements distinct from E and 0 (in other words, E is a projection of rank 1). E and $I - E = F$ are called an elementary pair. For the function Δ from W to L the following relations are postulated to hold for any A, B and an elementary pair E, F of W, and for any λ of L: I. $\Delta(AB) = \Delta(A) \cdot \Delta(B)$, II. $\Delta(A+B) = \Delta(FA+B) + \Delta(A+FB)$, III. $\Delta(\lambda A) = \lambda\Delta[(\lambda F + E)A]$, IV. $\Delta(A) \not\equiv 0$. It is then shown that the above relations uniquely determine Δ. (Received October 25, 1954.)

187B. Edwin Hewitt (p) and H. S. Zuckerman: *Semicharacters of semigroups.*

Let G be a semigroup (i.e., a set with an associative binary operation). A semicharacter of G is a bounded complex function χ on G such that χ is not identically 0 and $\chi(x)\chi(y)=\chi(xy)$ for all $x,y\in G$. Let G be finite and commutative. Let $G'=\{x\mid x\in G,\ x^m=x$ for some pos. int. $m\}$. Let H be the set of all idempotents in G, and, for $a\in H$, let $T_a=\{x\mid x^s=a$ for some pos. int. $s\}$. Then $T_a\cap G'$ is a group. A function χ on G is a semicharacter if and only if there exists an $a_0\in H$ and a character ψ of $T_{a_0}\cap G'$ such that $\chi(x)=0$ for $x\in T_a$ and $a_0a\neq a_0$ and $\chi(x)=\psi(a_0x)$ for $x\in T_a$ and $a_0a=a_0$. Let $G^\frown$ denote the set of all semicharacters of G with ordinary multiplication as a binary operation (not necessarily defined for all pairs). $G^\frown$ is a semigroup if and only if H has a unit. A finite commutative semigroup Q is isomorphic to some $G^\frown$ if and only if $Q'=Q$ and Q has a unit. G is isomorphic to $G^{\frown\frown}$ if and only if $G'=G$ and G has a unit. The effects on $G^\frown$ of adding a unit and zero to G are determined, as well as the effect of admitting 0 as a semicharacter. Thus, let $G^-=G^\frown\cup\{0\}$. Then $G^{--}\cong(G_e')_z$, where the subscripts e and z denote the addition of a unit and a zero, respectively. This work is connected with results obtained independently by Š. Schwarz (Proc. Int. Cong. Math. 1954). (Received October 20, 1954.)

188. J. H. Hodges: *Representations by bilinear forms in a finite field.*

Let $GF(q)$, where $q=p^f$ and p is a rational prime, denote the Galois field of order p^f. This paper is concerned with the problem of determining the number $N_t^s(A,B)$ of pairs of matrices U, V, such that $UAV=B$, where U is $s\times m$, V is $n\times t$, A is $m\times n$ of rank r, B is $s\times t$ of rank ρ, and all matrices have elements in $GF(q)$. A reduction formula is first obtained, giving $N_t^s(A,B)$ in terms of $N_{t-\rho}^{s-\rho}(A_\rho,0)$, where A_ρ is $(m-\rho)\times(n-\rho)$ of rank $r-\rho$. $N_{t-\rho}^{s-\rho}(A,0)$ is then computed directly, giving $N_t^s(A,B)$ explicitly. A connection is indicated between $N_t^t(A,B)$, with A square and nonsingular, and bilinear forms in $GF[q,x]$. Finally, a certain exponential sum $H(B,z)$, which for $B=0$ reduces to the number of $s\times t$ matrices of rank z, is evaluated and applied to the solution of several matric equations. (Received October 28, 1954.)

189. Arno Jaeger: *On representations of differential polynomials with change of field characteristic.*

Let I be the ring of integers, R the field of rationals, and J be an integral domain over I, integrally closed in its quotient field Q and satisfying $J\cap R=I$. Assume that Q has a finite degree of transcendency over a subfield C, let Δ be a nontrivial derivation in Q over C satisfying $\Delta J\subseteq J$, and denote by $\Omega(\Delta, J)$ the ring of differential polynomials in Δ over J. If P is the prime ideal of J generated by the rational prime number p the operator Δ induces a derivation δ in J/P as well as a two-sided ideal X of relations in the ring $\Omega(\delta, J/P)$ of differential operators in δ over J/P. $\Omega(\delta, J/P)/\mathrm{X}$ is isomorphic with a ring M_p of matrices over the subring $J^p(C\cap J)/P$ of J/P having prime number characteristic, and consequently the canonical mappings $\Omega(\Delta, J)\to\Omega(\delta, J/P)/\mathrm{X}$ lead to representations of $\Omega(\Delta, J)$ by matrices. Now local investigations in the rings M_p are sufficient to determine many properties of the elements of $\Omega(\Delta, Q)$, such as irreducibility and solvability. (Received November 15, 1954.)

190. R. E. Johnson: *Structure theory of faithful rings.* I. *Closure operations on lattices.*

A faithful ring is a ring that has no nonzero right or left annihilator. This paper gives the lattice-theoretic background of subsequent papers on the structure theory of faithful rings. It has to do mainly with properties of the lattice $C(\mathcal{L})$ of all closure operations on a complete lattice $\mathcal{L}$ satisfying the chain condition $(\bigcup_i A_i)\cap A$

$=\bigcup_i(A_i\cap A)$ for every $A\in\mathcal{L}$ and every chain $\{A_i\}\subseteq\mathcal{L}$. If $I(\mathcal{L})$ is the lattice of all subsets of $\mathcal{L}$ that contain I and are $\cap$-closed, and if, for $a\in C(\mathcal{L})$, $\mathcal{L}^a$ is the set of all $A\in\mathcal{L}$ such that $A^a=A$, then $C(\mathcal{L})$ and $I(\mathcal{L})$ are dual isomorphic under the correspondence $a\rightarrow\mathcal{L}^a$. The set $C_m(\mathcal{L})$ of all elements of $C(\mathcal{L})$ that are also $\cap$-endomorphisms of $\mathcal{L}$ is a complete sublattice of $C(\mathcal{L})$. If each $A\in\mathcal{L}^a$ contains an atom of $\mathcal{L}^a$, a is called homogeneous. The homogeneous elements of $C_m(\mathcal{L})$ form a dual ideal of $C_m(\mathcal{L})$. If $\mathcal{L}$ and $\mathcal{M}$ are two lattices and x is a $\cap$-homomorphism of $\mathcal{M}$ into $\mathcal{L}$, necessary and sufficient conditions are given for $ax=xb$, where $a\in C(\mathcal{L})$ and $b\in C(\mathcal{M})$. If for each $A\in\mathcal{L}^c$, $A\neq I$, there exists a nonzero $B\in\mathcal{L}$ such that $A\cap B=0$, then c is called reducible. If $\mathcal{L}$ is modular, and if $\mathcal{L}$ has a reducible $c\in C_m(\mathcal{L})$, then c is unique. Furthermore, if c is homogeneous, $\mathcal{L}^c$ is an atomic lattice. (Received November 10, 1954.)

191. Irving Kaplansky: *Any orthocomplemented complete modular lattice is a continuous geometry.*

The paper is devoted to proving the theorem stated in the title. The method is first to introduce coordinates from a regular ring, with the usual exception of things like non-Desarguian projective planes. The ring will possess an involution that makes it resemble closely a ring of operators. The arguments then needed are slight modifications of known ones—except for the proof of "finiteness." To prove finiteness it is argued that otherwise there is an infinite set of matrix units giving rise to infinite matrices. It turns out that the rows and columns can be arbitrarily prescribed. This bizarre state of affairs leads ultimately to a contradiction. (Received November 4, 1954.)

192. Erwin Kleinfeld: *Primitive alternative rings.*

An alternative ring R is defined to be primitive in case it contains a regular maximal right ideal which contains no proper two-sided ideal of R. It is then shown that a primitive alternative ring is either associative, in which case the well known Jacobson theory applies (Amer. J. Math. vol. 67 (1945) pp. 300–320), or else a Cayley-Dickson algebra of dimension 8 over its center. The proof depends on the author's previous result (Ann. of Math. vol. 58 (1953) pp. 544–547) that a simple alternative ring S is either associative or a Cayley-Dickson algebra, S being defined as simple provided it has no proper two-sided ideal and is not nil. Incidental to the proof is the following easily established result: A non-associative, alternative ring which has no proper two-sided ideal also has no proper one-sided ideal. This verifies the known fact that a Cayley-Dickson algebra has no proper one-sided ideal. It also adds weight to the conjecture that there are no alternative, non-associative nil rings without proper ideals. (Received August 30, 1954.)

193*t*. Erwin Kleinfeld: *Semi-simple alternative rings.*

The radical N of an alternative ring R is defined to be the intersection of the regular maximal right ideals of R. Then using the above classification of primitive, alternative rings (Bull. Amer. Math. Soc. Abstract 61-2-192), as well as Jacobson's density theorem it is easily established that N is also the intersection of the right-primitive ideals of R and hence an ideal. A ring with zero radical is called semi-simple. The quotient ring R/N is then semi-simple and consequently a subdirect sum of Cayley-Dickson algebras and associative, primitive rings. Another characterization of N, in terms of quasi-regular elements, has been given by Bailey Brown (Proc.

Amer. Math. Soc. vol. 2 (1951) pp. 114–117). Irving Kaplansky obtained the above structure theory under the additional assumption of π-regularity and conjectured the present result (Portugaliae Math. vol. 10 (1951) pp. 37–50). An immediate application shows that squares of associators and commutators of a semi-simple alternative ring are in its nucleus. For arbitrary alternative rings one can only assert that fourth powers of commutators are in the nucleus at the present time (Ann. of Math. vol. 58 (1953) pp. 544–547). (Received August 30, 1954.)

194*t*. L. A. Kokoris: *On u-stable commutative power-associative algebras.*

A stable simple commutative power-associative algebra of degree two over a center whose characteristic is prime to 30 is known to be a Jordan algebra (cf. A. A. Albert, *On commutative power-associative algebras of degree two,* Trans. Amer. Math. Soc. vol. 74 (1953) pp. 323–343). It is also known (cf. the above reference and *Power-associative commutative algebras of degree two,* Proc. Nat. Acad. Sci. U. S. A. vol. 38 (1952) pp. 534–537) that simple u-stable, that is, stable with respect to the idempotent u, algebras of degree two and characteristic $p>5$ need not be Jordan algebras. This paper extends these results by showing that a u-stable central simple commutative power-associative algebra of degree two and characteristic zero is a Jordan algebra. (Received November 5, 1954.)

195. D. J. Lewis: *Cubic congruences.*

Let $\mathfrak{O}$ be the ring of integers of an algebraic number field, and $\mathfrak{m}$ any ideal in $\mathfrak{O}$. Let $F(X)=\sum_{i=1}^{n} a_i x_i^3$, where the a_i are in $\mathfrak{O}$. It is shown that the congruence $F(X)\equiv 0 \pmod{\mathfrak{m}}$ has a solution in $\mathfrak{O}$ which is nontrivial modulo every prime factor of $\mathfrak{m}$, provided $n\geqq 7$. Furthermore 7 is the smallest such integer. The result is obtained first for $\mathfrak{m}=\mathfrak{p}^s$, where $\mathfrak{p}$ is a prime ideal. Here, except for when $\mathfrak{p}$ divides 3, the usual methods apply. The final conclusion is then obtained by means of the approximation theorem. This result is comparable with that of the author's in *Cubic homogeneous polynomials over $\mathfrak{p}$-adic fields,* Ann. of Math. vol. 56 (1952) pp. 473–478. (Received November 12, 1954.)

196*t*. B. N. Moyls and M. D. Marcus: *Field convexity of a square matrix.*

Let C be a complex n-square matrix. Set $f(z)=(Cz, z)$ for $\|z\|^2=\sum_{i=1}^{n} z_i\bar{z}_i=1$; $F(C)=$range of f; $P(C)=$smallest convex polygon in the plane containing all the eigenvalues of C; $N^n=$set of all complex n-square normal matrices; $Q^n=$set of all complex n-square matrices with $P(C)=F(C)$. It is known that $F(C)$ is convex and $P(C)\subset F(C)$. Results: (i) $n\leqq 4$ implies $Q^n=N^n$ and for $n>4$, $Q^n\neq N^n$; (ii) If at most one eigenvalue lies in the interior of $P(C)$ then $C\in Q^n$ implies $C\in N^n$. By Schur's Lemma we need only consider triangular matrices. For such we give nasc. that $C\in Q^n$ for arbitrary n in terms of a quadratic form associated with C. (Received November 12, 1954.)

197. E. T. Parker: *A bound on transitivity of groups.*

A theorem of Miller (Bull. Amer. Math. Soc. vol. 22, pp. 68–71) is generalized: If $n=kp+r$, p a prime, k and r integers, $k<p^2$, $k<r(r-2)/2$, $r\geqq 13$, then each $(r+1)$-fold transitive permutation group of degree n contains the alternating group of degree n. The argument is begun like Miller's. Lemmas proved are: (1) No transitive

group of degree p or p^2 (p a prime), with a normal p-subgroup transitive on all the letters, has a composition-factor isomorphic with any alternating group of degree >5. (2) For $r \geqq 13$, the alternating group of degree r has no subgroup of index strictly between r and $r(r-1)/2$. It is shown that the hypothesis implies existence of an element of the group fixing enough letters to satisfy the hypothesis of a theorem of Bochert (Math. Ann. vol. 40, pp. 176–193). Then it may be shown, for example, that any group of degree $n > 6592$ is less than $16\pi^{1/3}/5$ times transitive or contains the alternating group. (Received November 12, 1954.)

198. G. W. Patterson: *A uniqueness theorem concerning generalized notation for integers.*

Consider an infinite sequence of nonzero integers $\{b_n\}$, $n \geqq 1$, and another infinite sequence of sets of integers $\{R_n\}$, $n \geqq 0$. For each b_i, the corresponding set R_{i-1} is a complete set of residue class representatives in the ring $I/(b_i)$, where I is the ring of integers. By iterating a simple generalization of the division algorithm, any integer, a, can be expressed $a = q_{n+1} \prod_{i=1}^{n+1} b_i + \sum_{i=1}^{n} r_i \prod_{k=1}^{i} b_k$, with $r_i \in R_i$ for $0 \leqq i \leqq n$. The infinite sequence $\{r_n\}$ obtained in this way is called the expansion of a. It may happen that for some n, q_{n+1} vanishes. In such a case we say that a has a finite or terminating expansion (with respect to $\{b_n; R_n\}$). The present theorem concerns the uniqueness of such an expansion: $\sum_{i=0}^{n} r_i \prod_{k=1}^{i} b_k = \sum_{i=0}^{n} r_i' \prod_{k=1}^{i} b_k$ & $\bigwedge_{i=0}^{n-1} (r_i, r_i' \in R_i)$: $\rightarrow \bigwedge_{i=1}^{n} (r_i = r_i')$, for all $n \geqq 0$, where $\bigwedge_{i=0}^{n}$ signifies the conjunction of the statements of the form indicated. Verbally, two terminating expansions are identical if their corresponding coefficients are selected from the same complete sets of residue classes. The proof is by induction on n and utilizes the fact that $r_i, r_i' \in R_i \rightarrow (r_i = r_i' \pmod{b_i}$ $\rightarrow r_i = r_i')$. If $b_i = b$ and $R_i = (k \mid 0 \leqq k \leqq b_i)$, then we obtain the uniqueness theorem for ordinary positional notation; decimal for $b = 0$, binary for $b = 2$. If $b_i = 5$ for i even and $b_i = 2$ for i odd, with R_i as before, the theorem applies to biquinary notation. If $b_i = b$ (odd) but $R_i = (k \mid -b+1 \leqq 2k \leqq b-1)$, then the theorem applies to symmetric notation. (Received November 12, 1954.)

199*t*. Rimhak Ree: *A class of simple Lie algebras of characteristic p.* Preliminary report.

Let $\mathfrak{A}$ be an associative, commutative algebra over a field Φ of characteristic p, and let $D_1, D_2, \cdots, D_m$ be derivations on $\mathfrak{A}$ such that $D_iD_j = D_jD_i$. Denote by $\mathfrak{L} = \mathfrak{L}(\mathfrak{A}; D_1, \cdots, D_m)$ the subalgebra of the derivation algebra of $\mathfrak{A}$ consisting of all derivations of the form $f_1D_1 + \cdots + f_mD_m$, where $f_i \in \mathfrak{A}$. This class of Lie algebras $\mathfrak{L}$ contains the generalization of the Witt-Zassenhaus-Jacobson algebras mentioned by Kaplansky [Bull. Amer. Math. Soc. vol. 60 (1954) pp. 470–471]. If $\mathfrak{A}$ is a field of finite degree over Φ and if $\sum f_iD_i = 0$ implies $f_i = 0$, $i = 1, 2, \cdots, m$, then $\mathfrak{L}$ is shown to be simple. If Φ is not algebraically closed, this result gives a class of new simple Lie algebras. Other simple Lie algebras of the form $\mathfrak{L}(\mathfrak{A}, D_1, \cdots, D_m)$ are being investigated. (Received November 15, 1954.)

200. Irving Reiner: *Automorphisms of the symplectic modular group.*

The symplectic modular group Γ_{2n} consists of all integral $2n \times 2n$ matrices M satisfying $MFM' = F$, where F is the matrix of the bilinear form $\sum_{i=1}^{n} (x_iy_{n+i} - x_{n+i}y_i)$. It is proved that every automorphism of Γ_{2n} is given by $X \rightarrow f(X) \cdot AXA^{-1}$, where A is an integral matrix satisfying $AFA' = \pm F$, and f is a character of Γ_{2n}, that is, a homomorphism into $\{\pm 1\}$. In the course of the proof, canonical forms are obtained for involutions under similarity transformations in Γ_{2n}, and certain types of involutions are characterized by inner properties. (Received November 1, 1954.)

201t. Irving Reiner: *Characters of the symplectic modular group.*

The symplectic modular group Γ_{2n} consists of all integral $2n\times 2n$ matrices M satisfying $MFM'=F$, where F is the matrix of the bilinear form $\sum_{i=1}^{n}(x_iy_{n+i}-x_{n+i}y_i)$. It is shown that Γ_{2n} has no nontrivial characters (defined to be homomorphisms into $\{\pm 1\}$) for $n>2$, and that Γ_2 and Γ_4 each have exactly one nontrivial character. The proof depends on a new identity connecting generators of Γ_{2n}, and on some known results on symplectic groups over $GF(p)$. (Received November 1, 1954.)

202. C. E. Rickart and P. C. Curtis, Jr. (p): *A characterization of AW* algebras.*

In an unpublished paper Loomis has considered B^* algebras which satisfy the following axiom: if $\{x_i\}$ is a uniformly bounded family of elements of the algebra A such that $x_ix_j^*=x_i^*x_j=0$ for all i, j, $i\neq j$, then there exists an element x in A with the following property: if y is an element of A and $yx_i=0$ for all $i\neq j$, then $yx=yx_j$ and if $x_iy=0$ for all $i\neq j$ then $xy=x_jy$. Loomis has established that B^* algebras satisfying this axiom are a subclass of AW^* algebras as defined by Kaplansky (Ann. of Math. vol. 53). In this note it is shown that the Loomis axiom is satisfied in every AW^* algebra. (Received November 12, 1954.)

203. R. F. Rinehart: *The derivative of a matric function.*

The only definition of the derivative of a function of a square matrix which has appeared in the literature is: $f^I(A)=\lim_{h\to 0}\{[f(A+hE)-f(A)]/h\}$ where h is a scalar and E is the identity matrix. This definition restricts severely the manner in which the "incremental matrix" shall approach zero. The following less restrictive definition is proposed. Let $f(z)$ be a scalar function of a complex variable and let $f(A)$ be defined. Let $H\neq 0$ be a matrix commutative with A. If $f(A+H)$ is defined for all H whose elements are sufficiently small in absolute value, if for arbitrary such H, $f(A+H)-f(A)$ can be expressed as $H\cdot M$, and if $\lim_{H\to 0} M$ exists independently of the manner in which $H\to 0$ subject to the above conditions, then this limit is defined to be the derivative, $f^I(A)$. The following theorem is proved for $f^I(A)$ so defined. If $f(z)$ is a function of a complex variable and $f(A)$ is defined, then a necessary and sufficient condition that $f^I(A)$ exist is that $f(z)$ be analytic at the characteristic roots of A. Further, $f^I(A)=g(A)$, where $g(z)=f'(z)$. (Received November 8, 1954.)

204. G. de B. Robinson (p) and R. M. Thrall: *The content of a Young diagram.*

The purpose of this paper is to throw light on the *graph* $G[\lambda]$ of a Young diagram $[\lambda]$, discussed elsewhere, and the significance of Frobenius' notation for a partition in this connection. Consider a doubly infinite matrix $G=(g_{ij})$ where $g_{ij}=j-i$, and imagine a given $[\lambda]$ superimposed upon G so that the (i,j) node of $[\lambda]$ covers g_{ij}. The *content* of $[\lambda]$ corresponds to the set of elements of G covered by $[\lambda]$. The authors obtain the necessary and sufficient condition that a given content should be *admissible*, i.e. should correspond to a diagram $[\lambda]$, and show how to construct $[\lambda]$ when its content is given. Replacing the g_{ij} by their non-negative residues modulo q, they obtain a criterion that $[\lambda]$ should be a q-core in terms of Frobenius' notation, the criterion having already been given in Young's case. If one represents the horizontal and vertical translation of $[\lambda]$ over G by two operators T, S, one may adapt the familiar partition generating function to yield the content of $[\lambda]$ for all λ. (Received November 8, 1954.)

205*t*. Kenneth Rogers: *Indefinite binary Hermitian forms.*

For an indefinite binary Hermitian form $f(x, y)$ of determinant $-d\neq 0$ the inhomogeneous minimum $M(f)$ with respect to $k(i)$ is defined, and it is shown that $M(f)<2d^{1/2}/5$ except for f equivalent to a multiple of $e^{i\pi/4}\bar{x}y+e^{-i\pi/4}x\bar{y}$, $x\bar{x}, -3y\bar{y}$, $x\bar{x}-y\bar{y}$ or $x\bar{y}+\bar{x}y$, $x\bar{x}-21y\bar{y}$, $|x+((1+i)/2)y|^2-3y\bar{y}/2$ or $x\bar{x}-6y\bar{y}$: for these $M(f) =(d/2)^{1/2}$, $(d/3)^{1/2}$, $d^{1/2}/2$, $2(d/21)^{1/2}$, $(d/6)^{1/2}$. If $|f|$ takes arbitrarily small nonzero values for integers x, y of $k(i)$, then $M(f)=0$. The method of proof is first to show that $M(f)<2d^{1/2}/5$ if Gaussian integers x, y exist with $0<|f(x, y)|<d^{1/2}/6$. Null and nonnull forms are then treated, the latter by the detailed knowledge of the possible homogeneous minima of $f(x, y)$ obtained by Professor Oppenheim and the author. Except for a small range this method leads to a finite set of forms whose $M(f)$ is found or estimated. Standard reduction transformations are used, as well as an analogue of Cassels' method for obtaining the Markoff chain. (Received November 9, 1954.)

206. Alex Rosenberg: *The Cartan-Brauer-Hua theorem for matrix and local matrix rings.*

Let $R=A_n$ be an $n\times n$ matrix ring over a ring with unit A, and let Z be the center of R. Let S be a subring of R setwise invariant under all inner automorphisms of R and not in Z. If $n\geqq 3$, $S\supset\{Ie_{ij},\ i\neq j;\ I(e_{ii}-e_{jj});\ [A, I]e_{ii}\}$ for some nonzero (two-sided) ideal I of A. In particular if A is simple this means $S=R$. If the only nilpotent ideals of A are in the center this implies that either $S\supset K_n$ for some nonzero ideal K of A or $S\supset$ all matrices with trace 0 and entries from an ideal of square zero in the center of A. If $n=2$ and $2a\neq 0$ for each a in A and if A is generated by its quasi-regular elements or commutative the same results hold. If now R is locally matrix of degree $\geqq 3$ in the sense that every finite set of elements is contained in a subring which is a matrix ring of degree $\geqq 3$ over a simple ring with unit, then the only subrings setwise invariant under all quasi-inner automorphisms are subrings of the center and R. These results generalize those of Hattori (Jap. J. Math. vol. 21 (1952) pp. 120–128) and Kasch (Arch. Math. vol. 4 (1953) pp. 182–190). (Received October 20, 1954.)

207. Alex Rosenberg and Daniel Zelinsky (p): *Cohomology of infinite algebras.*

Let A be an algebra over a field F and, for any two-sided A-module N, let $H^n(A, N)$ denote the n-dimensional cohomology group of A with coefficients in N as defined by Hochschild (Ann. of Math. vol. 46 (1945) pp. 58–67). Then it is shown that if $H^1(A, N)=0$ for each N, A is finite-dimensional over F. Combined with results of Hochschild (loc. cit.) this shows that $H^1(A, N)=0$ for every N if and only if A is a finite-dimensional separable algebra. Now let A be a field with nonzero transcendence degree over F. Then $H^2(A, N)=0$ for each N if and only if A is a separable finite extension of a rational function field in one variable over F. (Received November 8, 1954.)

208*t*. Abraham Seidenberg: *On separating transcendency bases for differential fields.*

Let F be an arbitrary ordinary differential field of characteristic p, $p\neq 0$, and let $F\langle u_1, \cdots, u_n\rangle$ be a differential extension field of the field F. It is proved that if $F\langle u_1, \cdots, u_n\rangle/F$ is separable, then any transcendency basis of $F\langle u_1, \cdots, u_n\rangle/F$ is also a separating transcendency basis. (Received November 10, 1954.)

209*t*. Seymour Sherman: *Doubly stochastic matrices and complex vector spaces.*

A doubly stochastic (d.s.) matrix is a matrix P such that $P_{ij} \geqq 0$, $\sum_i P_{ij} = \sum_j P_{ij} = 1$ for all i and j. A. Horn has proved (Amer. J. Math. vol. 76 (1954) pp. 620–630): Theorem 1. *If $y = Px$, where x, y are complex n-vectors, and P is a d.s. matrix, and $c_1, c_2, \cdots, c_n$ are any complex numbers, then $\sum_{i=1}^n c_i y_i$ lies in the convex hull of all the points $\sum_{i=1}^n c_i x_{\alpha i}$, $\alpha \in R^n$, where R^n is the set of all the permutations of $(1, \cdots, n)$,* and conjectured the truth of: Theorem 2. *If x, y are complex n-vectors and $c_1, c_2, \cdots, c_n$ are any complex numbers imply that $\sum_{i=1}^n c_i y_i$ lies in the convex hull of the vectors $\sum_{i=1}^n c_i x_{\alpha i}$, $\alpha \in R^n$, then $y = Px$ where P is a d.s. matrix.* This note establishes the validity of Theorem 2. (Received November 12, 1954.)

210. E. C. Smith, Jr.: *A distributivity condition for Boolean algebras.*

Let n be an infinite cardinal number and ν the least ordinal number such that $\bar{\nu} = n$. Let B be a Boolean algebra. B has the property (P_n) if, for every set $\{a_{\xi,\eta} \mid \xi, \eta < \nu\} \subset B$ such that all the $\sum_{\eta<\nu} a_{\xi,\eta}$ and $\prod_{\xi<\nu} \sum_{\eta<\nu} a_{\xi,\eta}$ exist and the latter is >0, there is a function f on $\{\xi \mid \xi < \nu\}$ to $\{\eta \mid \eta < \nu\}$ such that $\prod_{\xi<\kappa} a_{\xi f(\xi)} = 0$ is false for every $\kappa < \nu$. B is an n-quotient algebra if, for some n-additive ideal I in an n-additive field of sets F, B is isomorphic to F/I. Results: I. If B has the property (P_n) and the power of every set of pairwise disjoint elements of B is $<n$, then B is atomistic. II. If B has the property (P_n) and every set of at most n elements of B has a sum in B, then B is an n-quotient algebra. (This generalizes a theorem of Loomis, Bull. Amer. Math. Soc. vol. 53 (1947) pp. 757–760.) III. If $n^m = n$ whenever $0 < m < n$ and B is an n-quotient algebra, then B has the property (P_n). IV. If n is regular, there exists a complete Boolean algebra with the property (P_n) which is not n-distributive. (Received November 1, 1954.)

211. R. R. Stoll: *A rank function for certain semigroups.* Preliminary report.

Let S be a semigroup with the following properties: (i) S contains a zero and a unit element. (ii) Each element of S is regular, i.e., for each element $a \in S$ there exists an x such that $axa = a$. (iii) For each idempotent $e \in S$ there exists an element e' orthogonal to e and such that $xe' = 0$ if and only if $xe = x$ and $e'x = x$ if and only if $ex = 0$. Then R, the set of principal right ideals of S, when partially ordered by inclusion, is a complemented modular lattice. If R has finite length, then the rank of a principal right ideal and consequently of an element in S is defined. (Received November 15, 1954.)

212. E. G. Straus and J. D. Swift (p): *The representation of integers by certain rational forms.*

The results of a previous paper (Arch. Math. vol. 5 (1954) pp. 12–18) are extended to the consideration of representation of integers by quotients $N(x)/D(x)$ where $x = \{x_1, x_2, \cdots, x_n\}$, N and D are at most quadratic in any x_i, and the total degree of D is at least that of N. The solutions of $z = N(x)/D(x)$ are reduced to two classes.

The regular solutions satisfy simpler Diophantine equations obtained by consideration of the critical cone, $D_0(x)=0$, where $D_0(x)$ consists of the terms of highest total degree in $D(x)$. The remaining solutions are called exceptional and under certain divisibility criteria can be shown to be finite in number. These criteria provide that to any lattice point $\{x_1, \cdots, x_n, z\}$ of $z=N/D$ there exists a conjugate lattice point differing only in that one or more of the x_i are replaced by conjugate values obtained by considering the equation $zD-N=0$ as a quadratic in x_i. Where the divisibility criteria are not met, it is frequently possible to show the existence of an infinite number of exceptional solutions. Diophantine equations which can be reduced by subsitution to the form considered may frequently be handled more easily than by other methods. (Received November 4, 1954.)

213. Leonard Tornheim: *The minimum of a product of four homogeneous linear forms.*

Let $L_i(u)=a_{i1}u_1+\cdots+a_{in}u_n$ $(i=1, \cdots, n)$ be n real forms of determinant $\Delta=|\det(a_{ij})|$. Let Δ_n be the greatest lower bound of the Δ for which (A) $|L_1\cdots L_n|<1$ has only the trivial solution $u=0$. Korkine and Zolotareff [Math. Ann. vol. 6 (1873) p. 581] proved that $\Delta_2=5^{1/2}$ and Davenport [Proc. London Math. Soc. (2) vol. 44 (1938) p. 414] that $\Delta_3=7$. The conjecture of Hofreiter [Monatshefte für Mathematik vol. 49 (1941) p. 295] that $\Delta_4=(725)^{1/2}$, the least discriminant of totally real quartic fields, is proved. Furthermore all sets of forms having this property (A) and with $\Delta=(725)^{1/2}$ are equivalent. Also the minimum is isolated. The proof is geometric. All admissible lattices of determinant <26.93 of the region $|x_1x_2x_3x_4|\leqq 1$ and containing (1, 1, 1, 1) are considered. A great multiplicity of cases occur and are eliminated, except for one, by various conditions on the generators. Part of this work was accomplished under a grant from the National Science Foundation. (Received November 12, 1954.)

214. A. W. Tucker: *A skew-symmetric matrix theorem.*

Construct in Euclidean vector n-space: (a) the convex hull of the columns of a real n-rowed matrix A, (b) the convex cone spanned by the columns of a real n-rowed matrix B, and (c) the closed convex set of all vectors $X+Y$, x in a, Y in b. *Either* (1) the set c contains the null vector, i.e. $AP+BQ=0$ for some parameter vectors $P\geqq 0$ and $Q\geqq 0$, the component-sum of P being one, *or* (2) the set c contains a shortest vector ($\neq 0$). Such a vector, transposed into a row U, determines a half-space $UX\geqq 0$ which contains the hull a in its interior and the cone b in its closure, i.e. $UA>0$ and $UB\geqq 0$. This is a geometric form of an "alternative" or "transposition" theorem, related to theorems of Farkas, Stiemke, Motzkin, von Neumann and Morgenstern. Now let $A=S+I$ and $B=(S, I)$, where S is skew-symmetric and I the identity, both of order n. Alternative (1) proves impossible, so (2) must hold. Hence the theorem: A system of homogeneous linear inequalities $US\geqq 0$ with skew-symmetric matrix S has a solution $U(\neq 0)$ such that $U\geqq 0$ and $US+U>0$. Applied to a symmetric zero-sum two-person game in normalized form (to which any normalized zero-sum two-person game can be reduced), this existence theorem yields a form of the von Neumann minimax theorem sharpened to include the theorem of Bohnenblust, Karlin and Shapley on "essential" or "active" pure strategies. Also, through Dantzig's program-game equivalence, it implies the basic theorems of linear programming, including the detailed duality between "tight" and "slack" constraints. (Received November 12, 1954.)

215. George Whaples: *Axiomatic characterization of the classical local fields.*

The fields k which are complete under a discrete rank 1 valuation with a Galois field as residue class field can't be characterized by the fact that local class field theory holds over them, since this theory holds over a much wider set of fields. But they can be characterized by the axioms of local class field theory and the further axioms: (a) The residue class field is of characteristic p; (b) The norm residue symbol maps the group of units of k into a compact subgroup of the Galois group of the maximal Abelian extension of k. (Received November 15, 1954.)

ANALYSIS

216. M. I. Aissen: *Difference equations with periodic coefficients.*

Some standard results concerning solutions of linear homogeneous difference equations with constant coefficients are generalized to analogous results concerning solutions of linear homogeneous difference equations with periodic coefficients. The methods used are also analogous to the methods used for the classical "constant coefficients" case. Some applications are then made to display connections between continued fraction and other approximations to certain quadratic irrationals. (Received November 12, 1954.)

217. R. J. Arms: *Convex bodies, tangential points.*

The author considers a real linear Hausdorff space E satisfying: (i) every point in E is the intersection of a denumerable set of open sets, (ii) addition is continuous with respect to both factors simultaneously, (iii) scalar multiplication is continuous with respect to each factor separately. *Theorem* 1. *Let E be separable. Let K be a convex subset of E having inner points. Let B denote its boundary and T its set of tangential points (i.e. those boundary points through which there is not more than one support to K). Then, relative to B, T is an F_σ set, $B-T$ is a G_δ set, $B-T$ is a set of the first category.* Let E be a normed space. E is said to be *uniformly smooth* if, given $\epsilon>0$, there exists a $\delta=\delta(\epsilon)$ such that x_0, $x\in E$, $|x_0|=1$, $|x|<\delta$ implies $|x_0+x|+|x_0-x|\leqq 2+\epsilon|x|$. It is easily verified that $L^{(p)}$ $(1<p<\infty)$ spaces are uniformly smooth. *Theorem* 2. *If E is uniformly smooth, $B-T$ is a set of the first category relative to B.* Theorem 2 does not hold for all normed spaces. Mazur (see Studia Math. vol. 4, pp. 70–84) states Theorem 1 for normed separable spaces. (Received July 29, 1954.)

218. Nachman Aronszajn: *Computation of eigenvalues in ordinary differential problems.*

The basic principle underlying the application of the generalized Rayleigh-Ritz and Weinstein approximation methods is the following fact: If L is a completely continuous self-adjoint operator in a Hilbert space and if $\mathcal{L}\subset\mathcal{L}'$ are two closed subspaces with dim $(\mathcal{L}'\ominus\mathcal{L})=n<\infty$, and if the eigenvalue problem in the subspace $\mathcal{L}$ (R.R. method) or $\mathcal{L}'$ (W. method) has been solved, then a determinant $D_n(\lambda)$ can be written whose elements are explicitly given as sums of simple fractions in λ and whose poles and zeros determine the eigenvalues in the other subspace. The above statement can be extended to the case when it is only supposed that dim $[(\mathcal{L}\oplus\mathcal{L}')\ominus\mathcal{L}']=m$ and dim $[(\mathcal{L}\oplus\mathcal{L}')\ominus\mathcal{L}]=n$, m, n finite: if the problem has been solved in $\mathcal{L}$, a determinant of order $n+m$ can be written explicitly which gives the solution in $\mathcal{L}'$. Applying this to ordinary differential equations the general result is obtained that if the solution of the eigenvalue problem is known for some boundary conditions, then it can be solved explicitly with any other boundary conditions. This fact is of interest even for equations with constant coefficients for which the eigenvalue problem with periodic boundary conditions is explicitly solvable (by simple exponentials). The classical method in this case for equations of higher order leads to inextricable problems of elimination. (Received November 12, 1954.)

219*t*. W. G. Bade: *On functions of spectral measures.*

A Boolean algebra (B.A.) of projections $\mathfrak{B}$ in a real or complex Banach space is complete if $\mathfrak{B}$ is complete as an abstract B.A. and for each subset $\{E_\alpha\} \subseteq \mathfrak{B}$, $(\bigvee E_\alpha)\mathfrak{X} = \overline{sp}\ \{E_\alpha \mathfrak{X}\}$. We denote by $\mathfrak{A}(\mathfrak{B})$ and $\mathfrak{W}(\mathfrak{B})$ the uniformly and weakly closed algebras generated by $\mathfrak{B}$ respectively. If $\mathfrak{B}$ is σ-complete, then $\mathfrak{W}(\mathfrak{B}) = \mathfrak{A}(\overline{\mathfrak{B}}^s)$, where $\overline{\mathfrak{B}}^s$, the strong closure of $\mathfrak{B}$, is complete. One says $\mathfrak{B}$ has simple spectrum if for some $x_0 \in \mathfrak{X}$, $\mathfrak{X} = \overline{sp}\ \{Ex_0 | E \in \mathfrak{B}\}$. If $\mathfrak{B}$ is complete and has simple spectrum, then $\mathfrak{A}(\mathfrak{B})$ equals the commutant of $\mathfrak{B}$. If $\mathfrak{X}$ is a complex Banach space and A is a scalar type spectral operator of class $\mathfrak{X}^*$ (cf. N. Dunford, *Spectral operators*, Pacific J. Math. vol. 4, no. 3) then each operator in the weakly closed algebra generated by A is again of this type. The proofs use the lemma: If $\mathfrak{B}$ is σ-complete and $x \in \mathfrak{X}$, there exists an $x^* \in \mathfrak{X}^*$ such that $x^*Ex \geqq 0$, $E \in \mathfrak{B}$, and if $x^*Ex = 0$ then $Ex = 0$. Certain of these results extend theorems proved by a different method for reflexive spaces in the Pacific J. Math. vol. 4, no. 3. (Received November 15, 1954.)

220. A. V. Balakrishnan: *An operational calculus for closed operators.*

E. Hille has described (E. Hille, *Functional analysis and semi-groups*, American Math. Soc. Colloquium Publications, vol. 31, p. 303) an operational calculus in which the operand space is a B-space, each operator A is the infinitesimal generator of a one-parameter semigroup of linear bounded operators, and the functions $\phi(\cdot)$ for which the operator extension $\phi(A)$ is to be defined are Laplace-Stieltjes transforms. The present paper extends the calculus to the case where the functions $\phi(A)$ are closed operators, the numerical valued functions $\phi(\cdot)$ being now ratios of Laplace transforms. The extension preserves many of the properties of the previous calculus. (Received November 17, 1954.)

221. J. H. Barrett (p) and F. E. Lilley: *Second order matrix differential equations with symmetric coefficients.*

Let $P(x)$ and $Q(x)$ be symmetric square matrices of continuous functions on $x \geqq a$ and $P(x)$ be positive definite. Consider the second-order matrix differential equation (I) $(PY')' + QY = 0$. A theory of this equation is developed based on the matrix "Wronskian" of two solutions U and V: (II) $\mathrm{Wr}[U, V] = U * PV' - U * 'PV$, where the $(*)$ denotes the transpose of the matrix. A study is made of the asymptotic behavior of (matrix) solutions of (I) including oscillation of solutions. A (matrix) solution of (I) is said to be oscillatory for large x provided that its determinant, $|Y|$, is oscillatory in the usual sense for scalar functions. Theorem: Let $P = (p_{ij})$, $Q = (q_{ij})$, $P^{-1} = (\bar{p}_{ij})$ and $\overline{P}_k$ be the matrix P^{-1} with the diagonal element $\bar{p}_{kk}$ replaced by zero. If for some integer k the scalar equation (III) $(y'/\bar{p}_{kk})' + q_{kk}y = 0$ is oscillatory for large x and $\overline{P}_k$ is positive semi definite for $x \geqq a$, then (I) is oscillatory for large x. (Received November 10, 1954.)

222. R. G. Bartle: *A general bilinear vector integral.*

Let X, Y, and Z be Banach spaces and let there be a continuous bilinear function on $X \times Y$ to Z. Let S be an abstract set and $\mathfrak{S}$ a field of subsets of S. This paper develops a Lebesgue-type theory of integration of (unbounded) functions $f: S \to X$ with respect to a (finitely) additive set function $\mu: \mathfrak{S} \to Y$. This integral has many of the familiar properties. In particular, it is seen that the Vitali and the bounded convergence theorems remain valid if almost everywhere convergence is replaced by convergence in measure. The countably additive case is also studied. The approach is essentially that of F. Riesz [Acta Math. vol. 42 (1920)]. Special cases of this integral reduce to well-known integrals, and a discussion of the relation is made. (Received December 21, 1954.)

223t. R. G. Bartle: *Newton's method in Banach spaces.*

Let f be a map between Banach spaces $\mathfrak{X}$ and $\mathfrak{Y}$. In order to find a solution of $f(x)=0$, one may employ the iterations $x_{n+1}=x_n-[f'(x_n)]^{-1}\cdot f(x_n)$ or $x_{n+1}=x_n-[f'(x_0)]^{-1}\cdot f(x_n)$, $n=1, 2, \cdots$, termed Newton's method, or Newton's modified method. It is shown that if f is in class C' in the sense of T. H. Hildebrandt and L. M. Graves [Trans. Amer. Math. Soc. vol. 29 (1927)] and if $|f(x_0)|$ is sufficiently small, then either of the above methods converge to a locally unique solution. This extends results of L. V. Kantorovič and I. P. Mysovskih [Trudy Mat. Inst. Steklov vol. 28 (1949)] and M. L. Stein [Proc. Amer. Math. Soc. vol. 3 (1952)], although these authors establish faster convergence for the ordinary method. These results appear to be new even in the case of the real numbers. (Received November 12, 1954.)

224. F. E. Browder: *Asymptotic formulae for elliptic eigenvalue problems.*

In extension of the author's previous results, the eigenvalue problem considered is $Ku=\lambda Bu$ with K and B elliptic operators of orders $2m$ and $2r$ respectively $(m>r)$ on the C^m bounded precompact domain D of Euclidean n-space under general boundary conditions, B having no null solutions. If K and B are self-adjoint with self-adjoint boundary conditions, the asymptotic formulae are established for the eigenvalues λ_i, the asymptotic average of the square of the eigenfunction as well as of their various derivatives. All results are expressed in terms of n, m, r, and the characteristic forms $a(x, \xi)$, $b(x, \xi)$ of K and B. Extensions are given to manifolds and to strongly elliptic systems. (Received November 12, 1954.)

225. A. H. Brown: *Absolute equivalence of projections.*

Call two projections E and F on complex Hilbert space *absolutely equivalent* if they are equivalent (in the sense of Murray-von Neumann, *On rings of operators*, Ann. of Math. vol. 37 (1936) pp. 116–229) with respect to every ring containing them. A criterion for absolute equivalence is set forth along with the appropriate generalization of a formula for the implementing partial isometry previously established by sz. Nagy in the case $\|E-F\|<1$. (Received November 15, 1954.)

226t. A. H. Brown: *On an equivalence relation for operators.*

It is shown that rank, co-rank and the (Hermitian) operator A^*A form a complete set of invariants for an operator A under transformations UAV where U and V are unitary, thus generalizing the familiar theorem for finite matrices. The proof is entirely elementary, depending on nothing more profound than polar resolutions. This permits the construction of a straightforward geometric proof that A^*A is a complete set of unitary invariants for an operator A with $A^2=0$ and without normal kernel, a result previously obtained by the author (*The unitary equivalence of binormal operators*, A. Brown, Amer. J. Math. vol. 76 (1954) pp. 414–434) by much more involved methods. (Received November 15, 1954.)

227B. F. H. Brownell: *Extended asymptotic eigenvalue distributions for domains in n-space.*

Let D be a bounded, open, connected set with boundary B in R_n, n dimensional euclidean space for integer $n \geqq 2$. Let λ_j, $0<\lambda_j \leqq \lambda_{j+1}$, be the eigenvalues of the problem $-\nabla^2 u=\lambda u$ on D, $u=0$ on B. It is assumed that B has a finite number of components, that each of these is sufficiently smooth to possess at every point a tangent hyperplane and a unit normal vector, and that this unit normal satisfies a Hölder condition with respect to distance in B. Then, by using the heat equation Green function for D, one can show that $N(y)=\sum_{\lambda_j \leqq y} 1$ has $\int_0^\infty e^{-yt}dN(y)=\sum_{j=1}^\infty e^{-\lambda_j t}=\mu_n(D)/(2\pi^{1/2})^n t^{n/2} -\sigma_n(B)/4(2\pi^{1/2})^{n-1}t^{(n-1)/2}+O(1/t^{(n-1-\eta)/2})$ over $t>0$, with $0<\eta\leqq 1$. From this it is concluded that $N(y)=\mu_n(D)/(2\pi^{1/2})^n\Gamma(n/2+1)y^{n/2}-(\sigma_n(B)/4(2\pi^{1/2})^{n-1}\Gamma((n-1)/2 +1))y^{(n-1)/2}+\tilde{O}(y^{(n-1-\eta)/2} \ln y)$, where this $\tilde{O}$ estimate on the remainder has the sense that a certain Gaussian average, which drops out oscillating parts, has the indicated order. For D an n dimensional parallelopiped, whose boundary of course would not satisfy our smoothness conditions because of the corners, this formula for $N(y)$, with $\tilde{O}$ replaced by O, is a trivial consequence of the standard result for a corresponding lattice point problem of number theory. (Received November 12, 1954.)

228t. F. H. Brownell: *Extended asymptotic eigenvalue distributions for domains in 3-space.*

Let D be a bounded, open, connected set with boundary B in R_3, three-dimensional euclidean space; let λ_j, $0<\lambda_j\leqq\lambda_{j+1}$, be the eigenvalues of the problem $-\nabla^2 u=\lambda u$ on D, $u=0$ on B. Assume that $z\in B$ is given locally with one coordinate of z a function of the other two coordinates which has continuous second partials, and for which these second partials satisfy a uniform Hölder condition of order η, $0<\eta\leqq 1$. Under these conditions one has over $\omega\geqq 1$ the asymptotic result $\sum_{j=1}^\infty 1/\lambda_j(\lambda_j+\omega^2) =(\mu_3(D)/4\pi)(1/\omega)-(\sigma_3(B)/16\pi)$ $(\ln (\omega^2)/\omega^2)+(C/\omega^2)-(1/24\pi)\int_B(q(z)d\sigma_3(z))(1/\omega^3) +O(1/\omega^{3+\eta})$ where μ_3 is 3-dimensional Lebesgue measure, σ_3 is 2-dimensional area measure, and $q(z)$ is twice the mean curvature of B at z (i.e. the trace of the curvature matrix defined as the derivative of the outward unit normal). From this asymptotic behavior follows $N(\lambda)=\sum_{\lambda_j\leqq\lambda} 1=(\mu_3(D)/6\pi^2)\lambda^{3/2}-(\sigma_3(B)/16\pi)\lambda+((1/12\pi^2) \cdot\int_B q(z)d\sigma_3(z))\lambda^{1/2}+\tilde{O}(\lambda^{(1-\eta)/2}\ln\lambda)$. This $\tilde{O}$ estimate on the remainder $R(\lambda)$ has the sense that a certain Gaussian average of $R(\lambda)$, which drops out oscillating parts, is of the indicated order. Stronger smoothness conditions on B yield additional terms in these asymptotic results. (Received October 25, 1954.)

229t. R. C. Buck: *Positive sequences and tauberian conditions.*

A sequence $\{S_n\}$ is said to be of bounded increase if there are numbers M and $\lambda>1$ such that $0\leqq S_k\leqq MS_n$ for all k and n with $n\leqq k\leqq\lambda n$. Theorem: if $(C, 1) \lim S_n=0$ and $\{S_n\}$ is of bounded increase, then $\lim S_n=0$. In this tauberian type theorem, the limit 0 cannot be replaced with any other positive value. Corollary: if $\sum a_n$ converges and $\{a_n\}$ is of bounded increase, then $\lim na_n=0$. (Received November 15, 1954.)

230B. Thelma M. Chaney: *On the existence of invariant means and measures on certain semigroups.*

Let G be a semigroup. A nonvoid set $S\subset G$ is a right ideal if S is closed under right multiplication by elements of G. For a finite semigroup the following are proved: a necessary and sufficient condition for the existence of a right invariant mean on G is the existence of a right ideal in G which is a group; a necessary and sufficient condition for the existence of right invariant measure on G is the existence of a right ideal $S\subset G$ which is a group such that $S'\cap G$ is also a right ideal; structure theorems for all invariant means and measures on G; the existence of more than one right invariant mean on G implies the nonexistence of a left invariant mean; a necessary and sufficient condition for the existence of both a right and left measure on G is that G be a group; if the complex linear space of all linear functionals on G is a semi-simple algebra, then there exists a unique invariant mean on G. By an application of the Markoff-Kakutani fixed-point theorem the existence of an invariant mean on any topological, Abelian semigroup is shown. (Received November 15, 1954.)

231*t*. Harvey Cohn: *Perturbation modular functions.* Preliminary report.

Starting with a fundamental domain in the upper half plane touching the real axis only at cusp-points, one constructs a circle of radius ϵ tangent to the real axis at one cusp point, and with it the images under the corresponding group. The perturbation formed by mapping the remaining z-plane onto itself with fixed point and direction is of the form $\epsilon^2P(z)$ where $P(Sz)(cz+d)^2-P(z)$ is a quadratic in z, depending on $S=(az+b)/(cz+d)$ (where $ad-bc=1$, and S belongs to the group in question). Conversely, under rather broad assumptions, any $P(z)$ satisfying the above transformation is attainable by such a perturbation, and furthermore such a (cusp) perturbation is the most general perturbation producing a $P(z)$ regular in the upper half plane. The decisive formalism is the fact that $P'''(z)[J'(z)]^{-2}$ is a modular invariant and that the fixed point of the perturbation mapping affects $P(z)$ by an added quadratic polynomial. In unpublished work, Schiffer has shown how the theta functions (with poles) can be produced by (interior) variations of the type used in M. Schiffer and D. C. Spencer, *Functionals of finite Riemann surfaces*, Princeton University Press, 1954. (Received November 12, 1954.)

232. V. F. Cowling: *On the summability of Dirichlet series.*

In this paper a study of the application of S_α methods of summabilty introduced by W. Meyer-König (Math. Zeit. vol. 52 (1949) pp. 252–304) to Dirichlet series is considered. It is shown, for example, that if for $0\leqq\alpha\leqq(3-5^{1/2})/2$, $\sum_1^\infty a_nn^{-s}$ is absolutely summable T_α (see the reference above) and S_α at $s=s_0$ then $\sum_1^\infty a_nn^{-s}$ is absolutely summable S_α, $0\leqq\alpha\leqq(3-5^{1/2})/2$, for Re $(s)>$Re (s_0). The application of S_α to other classes of series is also studied. (Received November 15, 1954.)

233. R. B. Deal (p) and E. W. Titt: *The relationship between Green's ideas and Hadamard's finite part as applied to the normal hyperbolic equation with an odd number of independent variables.*

Recently one of the authors presented an alternative method of evaluating Hadamard's finite part in which integration by parts replaced the Taylor expansion. In considering the extension of these ideas to multiple integrals the multiplicity of possible directions of integration presents a problem of uniqueness. This paper presents a method of integrating the normal hyperbolic equation without a formal discipline in finite parts. The source of the idea lies in a closer imitation of the geometry of Green's method in which the ϵ-sphere is an ϵ-hyperboloid and the null sphere is the characteristic cone. Integration by parts is used to extract infinite line integrals from surface integrals over both the ϵ-hyperboloid and the data bearing surface. In fact, these line integrals cancel before passing to the limit and the result is a perfect analogue of the corresponding Green relation. The use of integration by parts to extract the line integrals leads one to the type of integration formula found in McCully and Titt (Journal of Rational Mechanics and Analysis vol. 2 (1953) pp. 443–484). (Received November 15, 1954.)

234. R. B. Deal and E. W. Titt (p): *An application of finite parts to a study of the boundary values for an elliptic equation.*

Our problem is to compute the order of the infinite discontinuity in the boundary values of the normal derivative u_N at a finite jump discontinuity in the boundary values of the solution u. For Laplace's equation over a circular region an explicit formula for the boundary values of u_N in terms of tangential derivatives of u is given in the lecture notes of one of the authors. The formula for the case of discontinuous u is obtained as a limit of the corresponding formula for continuous u since in a direct attack divergent integrals would have entered the work due to the order of the infinite discontinuity in u_N. In the theses of Faircloth and Eason it is found that for an arbitrarily shaped boundary the explicit formula becomes an integral equation. In this paper these formulas are derived by the use of a limiting region rather than limiting boundary values u. The method of the preceding paper by the same authors is used to extract the infinities and the balancing of these infinities in the resulting limiting relation leads to a relationship between a known infinite discontinuity and the unknown infinite discontinuity in u_N. (Received November 15, 1954.)

235. W. F. Donoghue: *Continuous function spaces isometric to Hilbert space.*

A well-known theorem of Banach provides that the space $C(0, 1)$ of continuous functions on the unit interval with norm $\|x\| = \sup_t |x(t)|$ has a subspace isometric to any pre-assigned separable Banach space, in particular therefore to separable Hilbert space. It is shown that if such a subspace is isometric to a Hilbert space of dimension $N>2$, then any $k<N$ linearly independent elements $x_i(t)$, $i=1, \cdots, k$, define a curve in R_k which covers a sphere. No such result holds for $N \leqq 2$. For infinite-dimensional spaces it is shown that the mapping of the unit interval into Hilbert space determined by the reproducing kernel covers a sphere in Hilbert space. (Received November 12, 1954.)

236. R. J. Duffin and D. H. Shaffer (p): *Asymptotic expansion of double Fourier transforms.*

The problem considered is the asymptotic expansion at infinity of the two-dimensional Fourier transform of a function which has a singularity at the origin of the form $x^m y^n (x^2+y^2)^k$ where m and n are positive integers and k is an arbitrary real constant. The addition of a function having all partial derivatives everywhere does not affect the formulae obtained. If $2k+m+n \leqq -2$, convergence of the integral is assured by employing a suitable generalized Fourier transform. In the theory of discrete harmonic functions and discrete analytic functions the fundamental solutions may be defined as two-dimensional Fourier series coefficients of functions with a singular point. As an application, the asymptotic expansions of these fundamental solutions are obtained. (Received November 15, 1954.)

237. Albert Edrei: *On a conjecture of Pólya.*

Let $f(z)$ be a real entire function of finite order ρ (>1) and mean type. The author shows that if $\lim\sup x^{-\rho}\log|f(\pm x)|<0$ as $x\to+\infty$, then every point of the real axis is a point of accumulation of zeros of the successive derivatives of $f(z)$. This result follows from a recent theorem of the author [Bull. Amer. Math. Soc. Abstract 60-2-221]; it throws some light on a conjecture of Pólya [Bull. Amer. Math. Soc. vol. 49, p. 183]. (Received November 9, 1954.)

238t. Ky Fan and I. L. Glicksberg: *Fully convex normed linear spaces.*

A normed linear space X is fully convex (*weakly fully convex*) if every sequence $\{x_n\}$ of elements in X satisfying (a) $\lim \|x_n\|=1$, (b) $\|(x_n+x_m)/2\|\geqq 1$ for $n, m=1, 2, 3, \cdots$ is a Cauchy sequence (converges weakly to some element). Every uniformly convex normed linear space is fully convex. There exist fully convex Banach spaces which are not uniformly convex in any topologically equivalent norm. Every separable reflexive Banach space has a topologically equivalent norm which makes the space weakly fully convex. A normed linear space X is reflexive if and only if every sequence $\{x_n\}$ in X satisfying (a), (b) has a subsequence which converges weakly to some element. G. Birkhoff's mean ergodic theorem for uniformly convex Banach spaces [Duke Math. J. vol. 5 (1939) pp. 19–20] has the following generalization: Let $\{x_n\}$ be a sequence of elements in a fully convex (weakly fully convex) Banach space. If $\|(1/n)\sum_{i=1}^{n} x_{i+j}+(1/m)\sum_{i=1}^{m} x_{i+j}\|\leqq\|(1/n)\sum_{i=1}^{n} x_i+(1/m)\sum_{i=1}^{m} x_i\|$ holds for every three integers, n, m, j, then the sequence $\{(1/n)\sum_{i=1}^{n} x_i\}$ converges (converges weakly) to some element. All above results can be extended if, for any integer $k\geqq 2$, we replace (b) by (b′) $\|(1/k)\sum_{i=1}^{k} x_{n_i}\|\geqq 1$ for $n_1\leqq n_2\leqq\cdots\leqq n_k$. (Received November 10, 1954.)

239. Ky Fan, Olga Taussky, and John Todd (p): *Discrete analogs of inequalities of Wirtinger.*

This paper contains discrete analogs of several known integral inequalities involving functions and their derivatives. All results are concerned with $m(\geqq 3)$ vectors $x_1, x_2, \cdots, x_m$ in a unitary space U^n of arbitrary dimension n, and in each case the best possible constant (which depends on m) is determined. In the three typical results stated below, it is assumed that $\sum_{i=1}^{m} x_i=0$. (i) If $x_{m+1}=x_1$, then $\sum_{i=1}^{m}\|x_i-x_{i+1}\|^2\geqq 4\sin^2(\pi/m)\sum_{i=1}^{m}\|x_i\|^2$. The equality sign holds if and only if there exist two vectors y, z (not necessarily linearly independent) in U^n such that $x_i=(\cos(2i\pi/m))\cdot y+(\sin(2i\pi/m))\cdot z$ for $1\leqq i\leqq m$. (ii) If $x_0=x_1$, $x_{m+1}=x_m$, then $\sum_{i=0}^{m-1}\|x_i-2x_{i+1}+x_{i+2}\|^2\geqq 16\sin^4(\pi/2m)\sum_{i=1}^{m}\|x_i\|^2$. The equality sign holds if and only if there exists a vector $y\in U^n$ such that $x_i=(\cos((2i-1)\pi/2m))\cdot y$ for $1\leqq i\leqq m$. (iii) If $x_{m+1}=x_1$, then $\operatorname{Max}_{1\leqq i\leqq m}\|x_i-x_{i+1}\|\geqq a(m)\cdot\operatorname{Max}_{1\leqq i\leqq m}\|x_i\|$, where the best possible constant $a(m)=4/m$ for even m and $a(m)=4m/(m^2-1)$ for odd m. Result (i) is the discrete analog of Wirtinger's inequality (cf. G. H. Hardy, J. E. Littlewood, G. Pólya, *Inequalities*, Cambridge University Press, 1934, p. 185). Result (iii) is the discrete analog of an inequality of D. G. Northcott [J. London Math. Soc. vol. 14 (1939) pp. 198–202]. (Received November 3, 1954.)

240. M. K. Fort, Jr.: *Research problem number* 22.

Richard Bellman (Bull. Amer. Math. Soc. vol. 60, p. 501) has proposed the following research problem: Solve the functional equation $f(x) = \text{Max}\ (g(x)+f(ax), h(x)+f(bx))$, given that $0<a, b<1$; $h(x), g(x)>0$; $h(0)=g(0)=0$; $h'(x), g'(x)>0$; $h''(x), g''(x)>0$. It is shown that the equation has a unique solution f with the following properties: $f(x)$ is defined for all $x\geqq 0$; f is continuous; $f(0)=0$. If $g'(0)/(1-a) >h'(0)/(1-b)$, there exists $\epsilon>0$ such that $f(x)=g(x)+g(ax)+g(a^2x)+\cdots$ for $0\leqq x\leqq \epsilon$. This condition together with the given equation then determines $f(x)$ for all $x\geqq\epsilon$. If $g'(0)/(1-a)<h'(0)/(1-b)$, a similar solution is obtained. If $g'(0)/(1-a) =h'(0)/(1-b)$, then it can be shown that $f_\lambda(x)\to f(x)$ as $\lambda\to 1$, where f_λ is the solution corresponding to $g, \lambda h, a, b$. (Received November 8, 1954.)

241. Bernard Friedman and Joel Franklin: *A convergent asymptotic representation for integrals.*

A new method of obtaining a series asymptotic in p for integrals of the form $\int_0^\infty e^{-px}x^{c-1}f(x)dx$ is proposed. It is shown that if $f(x)=\int_0^\infty e^{-xt}d\phi(t)$ for Re $x>0$, where $\phi(t)$ is a complex-valued function which is of bounded variation in each finite interval $0\leqq t\leqq T$ and which satisfies the inequality $|\Psi(t)|\leqq M$ for $t\geqq 0$, then the asymptotic series converges to the value of the integral if $p>0$ and Re $c>0$. Numerical calculation seems to show that the first term of the series gives a close approximation to the value of the integral for a wide range of values of p. (Received November 23, 1954.)

242. T. M. Gallie, Jr.: *Essential sets of singularities of Dirichlet series.*

It is known that $z=1$ is an isolated singular point of a uniform function $f(z)$ if and only if there exists an entire function $F(z)$ of minimal exponential type such that $f(z)-\sum_{n=0}^\infty F(n)z^n$ is regular at $z=1$. In the light of this fact Mandelbrojt has suggested the definition: a set E is called an essential set of singularities of a uniform function $f(s)$ with respect to a sequence $\{\Lambda_n\}$ if there exists an $F(z)$ as above such that: (a) $\sum_{n=1}^\infty F(\Lambda_n)e^{-\Lambda_n s}$ is uniform and its set of singular points is E; (b) $f(s) -\sum_{n=1}^\infty F(\Lambda_n)e^{-\Lambda_n s}$, called a regular part of $f(s)$, is regular on E. Let $f(s)$ and $g(s)$ be Dirichlet series whose exponents are complementary with respect to a sequence $\{\Lambda_n\}$. It is proved that if $f(s)$ has an essential set of singularities with respect to $\{\Lambda_n\}$ and if a regular part of $f(s)$ together with $g(s)$ satisfy the hypotheses of Agmon's composition theorem (Journal d'Analyse Mathématique vol. 1 (1951) pp. 232–243), then $g(s)$ can have no accessible singularities (see Pólya, Göttingen Nach., 1927, pp. 187–195) in the composition domain of Agmon's theorem. This result contains results of Mandelbrojt and Pólya (C. R. Acad. Sci. Paris vol. 184 (1927) pp. 502–504). (Received November 12, 1954.)

243. R. K. Getoor: *On semi-groups of unbounded normal operators.*

In some recent investigations of the author semi-groups of unbounded normal operators in Hilbert space arose in a natural fashion. Such semi-groups do not seem to have been considered in the literature although semi-groups of unbounded self-adjoint operators have been considered by A. Devinatz, *A note on semi-groups of unbounded self-adjoint operators*, Proc. Amer. Math. Soc. vol. 5 (1954). The following theorem is proved. Let $N(t)$, $t>0$, be a semi-group of normal operators on a Hilbert space H satisfying: (1) if $D(t)$ is the domain of $N(t)$ then $D=\bigcap_{t>0}D(t)$ is dense in H; (2) if $R(t)$ is the range of $N(t)$ then $R(t)\subset D$ for all $t>0$; (3) if $x, y\in D$ then $(N(t)x, y)$ is continuous at $t>0$ and moreover $(N(t)x, y)\to(x, y)$ as $t\to 0$; under these conditions there exists a complex spectral resolution $K(\lambda)$, $\lambda=\lambda_1+i\lambda_2$, such that if $\Lambda\subset\{\Re(\lambda)<0\}$ then $K(\Lambda)=0$ and $N(t)=\int_{\Re(\lambda)\geq 0}\lambda_1^t \exp(i\lambda_2 t)dK(\lambda)$. (Received November 12, 1954.)

244. A. W. Goodman: *Functions typically-real and meromorphic in the unit circle.*

Let TM denote the class of functions which are typically-real and meromorphic in the unit circle, with $f(z)=z+\sum_{n=2}^{\infty} b_n z^n$ in a neighborhood of the origin. The condition $\Im f(z)\Im(z)>0$ for $|z|<1$ and z not real implies that all the poles are simple and lie on the real axis. By quite elementary means it is possible to find the precise domain in variability of $f(z)$ for each fixed z and $f(z)\in TM$. Further for this class of functions sharp bounds for $|b_n|$ in terms of the distance of the closest pole to the origin are found. (Received November 12, 1954.)

245. R. P. Gosselin: *On the convergence of subsequences of Fourier series in an L^p class.*

The following notation will be used: $s_{m_\nu}(x)$ for the m_νth partial sum of the Fourier series of a function $f(x)$; $\{n_k\}$ for a lacunary sequence of integers, i.e. $n_{k+1}/n_k\geq\lambda>1$; and $[y]$ for the greatest integer $\leq y$. A sequence $\{m_\nu\}$ is said to have upper density 1 if lim sup $(\sigma(n)/n)=1$ where $\sigma(n)$ is the number of terms of the sequence $\leq n$. The following theorems are proved: (i) If $f(x)$ belongs to L^2, with $\{n_k\}$ as above, then there exists a sequence $\{m_\nu\}$ of positive integers containing $[n_{k+1}-n_k/\log n_{k+1}]$ consecutive terms in each interval (n_k, n_{k+1}) such that $s_{m_\nu}(x)$ converges to $f(x)$ almost everywhere. (ii) If $f(x)$ belongs to L^2, then there is a sequence $\{m_\nu\}$ of upper density 1 such that $s_{m_\nu}(x)$ converges to $f(x)$ almost everywhere. There is also a theorem analogous to (i) for functions of class L^p, $1<p<2$. The main tools of proofs are two theorems of Kolmogoroff on the convergence of Fourier series (cf. A. Zygmund, *Trigonometrical series*, Warsaw, 1935, pp. 251–254). (Received November 12, 1954.)

246B. L. W. Green: *Remark on ergodic transformations.*

Let Ω be a compact metric space with a measure μ defined on the Borel field $\mathcal{E}$. Assume that $\{\phi_t\}$ is an ergodic one-parameter group of measure-preserving homeomorphisms of Ω onto itself. It is proved that the physical problem of finding the time average of a given L_1-phase function $f(\omega)$ by discrete observations is "well-set" in the following sense: $(1/n)\sum_{k=0}^{n-1} f(\phi_{t_k}\omega)$ converges to the same constant for "almost all" choices of the sequence $t_0<t_1<\cdots\to\infty$, provided the interval between observations is uniformly bounded. Convergence is a simple consequence of the Random Ergodic Theorem; proof of the constancy of the limit is a little more involved. (Received December 13, 1954.)

247. R. T. Herbst: *The equivalence of linear and nonlinear differential equations.*

The nonlinear equation considered is (*) $y''-(\ln w(x))'y'+q(x)Z(y)=A(y)y'^2+w(x)^2C(y)$, where Z, A, C are given functions satisfying $Z\neq 0$, $Z'-AZ=1$, $ZC'+(3-AZ)C=0$. This paper shows that under suitable continuity conditions (*) can be solved by putting $y=F(u, v)$, where u, v are independent solutions with Wronskian

w of the linear equation $Y'' - (\ln w)'Y' + qY = 0$, and where F satisfies a passive system of total differential equations. Moreover (*) is the only equation of the second order which can be solved by an F which is not homogenous of degree 0. This answers in part the question raised by J. M. Thomas (Proc. Amer. Math. Soc. vol. 3 (1952) pp. 899–903) regarding the determination of equations equivalent to a linear differential equation in the above sense. (Received November 12, 1954.)

248. I. I. Hirschman, Jr.: *Weak quadratic norms.*

Define $N_a[f] = [\int_{-\pi}^{\pi} |f(\theta)|^2 |\theta|^{2a} d\theta]^{1/2}$ $(-1/2 < a < 1/2)$, $L_b[f] = [\int_{-\pi}^{\pi} |f(\theta)|^2 \cdot (\log (|\theta|/2\pi))^b d\theta]^{1/2}$ $(-\infty < b < \infty)$. Let $f(\theta) \sim \sum_1^\infty a_n e^{in\theta}$ (the restriction to Fourier series of power series type being made only for convenience). The author has previously shown that if T is a multiplier transformation, $Tf(\theta) \sim \sum_1^\infty t_n a_n e^{in\theta}$ and if $|t_n| \leqq A$ $(n=1, 2, \cdots)$, $\sum_{2^{n-1}}^{2^n-1} |t_{k+1} - t_k| \leqq A$ $(n=1, 2, \cdots)$, then $N_a[Tf] \leqq AC_1(a) \cdot N_a[f]$ where $C_1(a)$ depends only upon a $(-1/2 < a < 1/2)$. It is shown here that if $|t_n| \leqq A$ $(n=1, 2, \cdots)$, $\sum_{(n-1)^2}^{n^2-1} |t_{a+1} - t_a| \leqq A$ $(n=1, 2, \cdots)$, then $L_b[Tf] \leqq AC_2(b) \cdot L_b[f]$ where $C_2(b)$ depends only upon b $(-\infty < b < \infty)$. (Received November 15, 1954.)

249B. J. C. Holladay: *On convergence to eigenvectors for iterates of homogeneous transformations.*

A theorem due to O. Perron, although usually attributed to Frobenius, states that given a matrix whose elements are positive, there exists a unique positive vector v_0 such that if v is a positive vector, then v_0 is the limit as $j \to \infty$ of $M^j(v)/\|M^j(v)\|$. The main purpose of this paper is to generalize from linearity to homogeneity. Also, positiveness of the transformation is generalized to positiveness of some iterate of the non-negative transformation. For homogeneity of degree α where $0 < \alpha \leqq 1$, one obtains uniform convergence to a unique vector while for homogeneity of degree greater than one, the convergence need not be unique, nor uniform, nor need it even exist. Some observations are made concerning continuity of the dependence of the eigenvector on the transformation. Also, an observation is made concerning positiveness of iterates of non-negative matrices. (Received November 15, 1954.)

250. Meyer Jerison: *Martingale convergence theorems on rings of functions.* Preliminary report.

Let $\{X_n\}$ be an increasing sequence of orthonormal sets of bounded real functions on a finite measure space, such that $X = \bigcup X_n$ is complete in L^2. For $f \in L^1$, define f_n as the function (if it exists) satisfying $\int xf = \int xf_n$ if $x \in X_n$ and $\int xf_n = 0$ if $x \in X - X_n$. Theorem. If the set of linear combinations of functions in X_n is closed under pointwise multiplication (almost everywhere) for each n, then for every $f \in L^1$, f_n exists, is in L^1, and $f_n \to f$ almost everywhere as well as in the L^1 norm. The theorem is proved by combining a measure-theoretic analogue of the Stone-Weierstrass theorem [Segal, Amer. J. Math. vol. 76 (1954) p. 725] with a martingale convergence theorem [Doob, *Stochastic processes*, New York, 1953, p. 331]. (Received November 15, 1954.)

251. M. L. Juncosa and D. M. Young (p): *On the accuracy of finite difference methods for solving the diffusion equation.*

Solutions of the implicit six-point difference equation analogue (J. Crank and P. Nicolson, Proc. Cambridge Philos. Soc. (1947)) of the diffusion equation $u_t = u_{xx}$ with boundary conditions $u(0+, t) = u(1-, t) = 0$, $t \geqq 0$, and $u(x, 0+) = f(x)$, $0 < x < 1$, are shown to converge to the exact solution of the differential equation in R_0: $0 \leqq x \leqq 1$, $t \geqq t_0 > 0$, as $\Delta x = M^{-1} \to 0$ provided $f(x)$ is piecewise continuous and $\Delta t < M^{-1} \cdot (t_0/3 \log M)^{1/2}$. This requirement on Δt allows the use of many fewer time intervals than the usual condition, $\Delta t = O(M^{-2})$. Previous results of the authors (Bull. Amer. Math. Soc. Abstract 60-2-240) on the order of convergence in R_0 are shown to hold under the same conditions on $f(x)$ and Δt. Moreover, if $\Delta t = O(M^{-2})$, if $f(x)$ has a bounded kth derivative ($k = 2$, 3) except at a finite number of values of x, and if $f^{(k-1)}(x)$ is piecewise continuous, then as $M \to \infty$ the error vanishes like $O(M^{-(k-1)})$ uniformly in $0 \leqq x \leqq 1$, $0 \leqq t$. This is also true for the four-point forward difference equation provided $\Delta t M^2 = r$, where r is a constant and $r \leqq 1/2$. For the latter results the methods are similar to those used by Wasow (Journal of Research (1952)) and by Walsh and Young (Journal of Research (1953)). (Received November 15, 1954.)

252. W. B. Jurkat: *An absolute Fatou-Riesz theorem for Laplace integrals.*

In collaboration with A. Peyerimhoff a theorem on the absolute convergence of power series in points of regularity was proved, which corresponds to the well-known theorem of Fatou for ordinary convergence (Archiv d. Math. vol. 4 (1953)). Continuing the collaboration the following theorem for Laplace integrals is now proved, which corresponds to M. Riesz's extension of Fatou's theorem mentioned above: If $b(t)$ is integrable on every bounded interval, $\int_T^\infty |db(t)| < \infty$ for some $T \geqq 0$, and $f(s) = \int_0^\infty e^{-st} b(t) dt$ regular for $s = 0$, then $\int_0^\infty |b(t)| dt < \infty$ holds. There are certain natural generalizations and also applications to Dirichlet series. (Received November 9, 1954.)

253*t*. R. V. Kadison: *Multiplicity theory for operator algebras.*

If $\mathfrak{A}$ is a C^*-algebra and ϕ is a $*$-representation of $\mathfrak{A}$ as an algebra of operators on a Hilbert space, H, the author associates with ϕ a "multiplicity function," f_ϕ, which assigns to 0 a class of ideals of Borel sets in the pure state space, X, of $\mathfrak{A}$, and to each positive real number or infinite cardinal, a, not exceeding the dimension of H, an ideal, N_a, of Borel sets in X (the N_a form a descending chain of ideals). Two $*$-representations ϕ_1, ϕ_2 of $\mathfrak{A}$ are unitarily equivalent (i.e., there exists a unitary transformation, U, of H_1 onto H_2 such that $U\phi_1(A)U^{-1} = \phi_2(A)$ for each A in $\mathfrak{A}$) if and only if their associated multiplicity functions, f_{ϕ_1}, f_{ϕ_2}, are identical. (Received November 12, 1954.)

254B. Tosio Kato: *Notes on perturbation theory.*

In perturbation theory of linear operators there occurs the problem of constructing a regular transformation W of a Banach space which maps the range $\mathfrak{M}_0$ of a projection P_0 onto the range $\mathfrak{M}_1$ of a projection P_1 in a one-to-one fashion, where e.g. $\|P_1 - P_0\| < 1$ is assumed. Such a transformation was first given by Sz.-Nagy in the form $W = P_1[I + P_0(P_1 - P_0)P_0]^{-1/2}P_0 + (I - P_1)[I + (I - P_0)(P_0 - P_1)(I - P_0)]^{-1/2} \cdot (I - P_0)$. In the present note a simpler and symmetric form will be given to this W: $W = [P_1P_0 + (I - P_1)(I - P_0)][I - (P_1 - P_0)^2]^{-1/2}$. Also a different transformation W_1 was given by the author for the case where P_0 and P_1 are connected by a continuously (in the norm) changing projection P_t, $0 \leqq t \leqq 1$, as the solution of the differential equation $W'_t = (P'_tP_t - P_tP'_t)W_t$. In the present note this method is applied to the case where only P_0 and P_1 are given ($\|P_1 - P_0\| < 1$) by first constructing a continuous path P_t and then solving the ensuing differential equation. It is interesting that the result W_1 is again identical with W stated above. (Received November 15, 1954.)

255. N. D. Kazarinoff: *Asymptotic solution with respect to a parameter of a differential equation having an irregular singular point.*

The asymptotic solutions for $|\lambda|$ large of the differential equation (*): $(z-z_0)^\nu \cdot d^2w/dz^2+\lambda(z-z_0)^{\nu/2}P_1(z, \lambda)dw/dz+\lambda^2P_2(z, \lambda)w=0$ with ν real and greater than 2 are considered, the complex variable z being confined to a bounded neighborhood of the irregular singular point z_0. An analogous problem for a class of nth order differential equations containing (*) for integral $\nu \geqq 6$ has been discussed by C. C. Hurd [Tohoku Math. J. vol. 44 (1938) pp. 243–274], but it appears that a more direct investigation of solutions of (*) is possible. This is accomplished by adapting to this situation the classical theory of the asymptotic solution of (*) in the neighborhood of an ordinary point. A simple algorithm is given for the determination of formal solutions of (*) in the form of infinite series in descending powers of λ. Finite segments of these series are shown to approximate solutions of (*). The solutions are found to be subject to the Stokes' phenomenon. (Received November 12, 1954.)

256t. Jacob Korevaar: *Sequences of polynomials whose zeros lie in a given set.*

R denotes a closed set in the z-plane. Any polynomial whose zeros belong to R is called an R-polynomial. The author called a set R regular (Duke Math. J. vol. 18, p. 573) if every entire function is the limit of a sequence of R-polynomials which converges uniformly in every bounded domain. Problem 1. Given a bounded domain D. How should R be in order that every $f(z)$ analytic in D, with its zeros in R, is the limit of a sequence of R-polynomials which converges uniformly in every compact subset of D? Two classes of sets R are known or easily shown to be admissible. A. The sets R which contain a rectifiable Jordan curve going around D (see G. R. MacLane, Duke Math. J. vol. 16, p. 461). B. The regular sets. By solving a second problem the author shows that these are essentially the only possibilities. Problem 2 (posed by Ganelius, Ark. Mat. vol. 3, p. 1). For which sets R does the uniform convergence of a sequence of R-polynomials in $|z|<1$ always imply its uniform convergence in every bounded domain? Answer given in this paper: for all non-regular sets R whose complement is connected and contains part of $|z|<1$. (Received Novembr 15, 1954.)

257. Bertram Kostant: *An extension of the Gårding space.* Preliminary report.

Let G be a connected Lie group, $\mathfrak{g}$ its Lie algebra, and $x\rightarrow U_x$ a strongly continuous representation on a Banach space V. For any $X\in\mathfrak{g}$ let $x(t)=\exp tX$ for all real t, $D_X=[v\in V|\lim_{t\to 0}(U_{x(-t)}-1)/tv=u$ exists] and let $\pi(X)$ be the operator defined on D_X by $\pi(X)v=u$. For a Radon measure μ (resp. a measurable function $f(x)$) for which $\int\|U_x\|d\mu(x)<\infty$ $(\int|f(x)|\|U_x\|dx<\infty)$ define $L_\mu=\int U_x d\mu(x)$ $(L_f=\int f(x)U_x dx)$. Finally let $C_k^0(G)$ be the set of functions of class C_k on G having compact support. Gårding [Proc. Nat. Acad. Sci. U.S.A. vol. 33 (1947) p. 331] has shown that for any $X\in\mathfrak{g}$, if $f\in C_k^0(G)$ $(k\geqq 1)$ then L_f: $V\rightarrow D_X$ and $\pi(X)L_f=L_{Xf}$ where $Xf(x)=\lim [f(x(t)x)-f(x)]/t$ as $t\rightarrow 0$. In the case where V is a Hilbert space and the U_x are unitary the author shows that this holds for any $f\in L_1(G)$ such that $f(x(t)x)$ is locally absolutely continuous in t for each x and $Xf\in L_1(G)$. More generally if μ and ν are bounded Radon measures and $[T_{x(t)}-1]/t\mu\rightarrow\nu$ weakly with respect to $C_\infty^0(G)$ [weakly in the space of distributions on G], then $L\mu$: $V \rightarrow D_X$ and $\pi(X)L\mu = L\nu$ where $T_{x(t)}\mu$ is left translations of μ by $x(t)$. (Received November 15, 1954.)

258. Stephen Kulik: *Application of a new method for the solution of equations.*

A root z_0 of an equation $f(z) = 0$ is expressed as a limit of a function $F_n(z)$ with $n \rightarrow \infty$ and z any number which is closer to z_0 than to any other root of the equation. By this method the largest and smallest roots of an algebraic equation with all real roots can be calculated with stating the lower and upper limits of the approximations; in case of a cubic equation the solutions are expressed in terms of its coefficients. (Received November 10, 1954.)

259. W. T. Kyner, Jr.: *An existence proof for periodic k-surfaces.*

In 1952, S. P. Diliberto proposed a theory of periodic k-surfaces in order to obtain qualitative information about the behavior of solutions of perturbed systems of nonlinear ordinary differential equations. This paper contains the proof of a general existence theorem for these periodic k-surfaces (in the so-called nondegenerate case). G. Hufford, in 1953, proved an existence theorem for periodic 2-surfaces. The first results for periodic k-surfaces, for k any integer, were obtained by M. D. Marcus. Both Hufford and Marcus defined their transformations in terms of differential equations. The above results have been generalized. The problem is first considered as a fixed point theorem in a function space (under a class of transformations that includes those used previously). The elements of the function space can be interpreted as hypersurfaces in some Euclidean space. The hypothesis is used that a certain linear transformation, defined as a translation operator minus a matrix operator, has an inverse. This hypothesis is weaker than that used by either Hufford or Marcus. The proof offered is a generalization of the method used by Hufford. (Received November 15, 1954.)

260B. R. E. Langer: *The asymptotic forms of the solutions of a class of ordinary differential equations of the third order about a multiple turning point.*

The paper gives a derivation of the asymptotic forms, for $|\lambda|$ large, of the solutions of differential equations $L(u) + \lambda^2[b_1(x)u' + b_0(x)u] = 0$, with $L(u) \equiv u''' + a_2(x)u'' + a_1(x)u' + a_0(x)u$, in regions of the complex x in which $b_1(x)$ has a simple zero and is otherwise different from zero. It shows that by transformations of the variables such an equation can be normalized to the extent that its solutions are asymptotically expressible in terms of those of the "base" equation $v''' + \lambda^2 xv' + 3\lambda^2\mu v = 0$, in which μ is a constant. The asymptotic solutions of this base equation are derived for all values of μ by use of the theory of residues. (Received November 10, 1954.)

261. Benjamin Lepson: *On the inner and outer logarithmic capacities of plane sets.*

The identity of the inner and outer Newtonian capacities for analytic sets in Euclidean space of three dimensions has been shown recently by Choquet [C. R. Acad. Sci. Paris vol. 234 (1952) pp. 498 and 784]. One of his principal tools is the property of "strong convexity" for the Newtonian capacity of compact sets, i.e., the inequality (1) $C(A \cup B) + C(A \cap B) \leqq C(A) + C(B)$. In order to obtain, by Choquet's method, a similar relation between the inner and outer logarithmic capacities of plane sets, it seems necessary to verify (1) in some form for plane compact sets. It is known that (1), and in fact even the less restrictive condition of subadditivity, fails to hold when $C(A)$ is taken to be Fekete's transfinite diameter (which is equivalent to logarithmic capacity in the sense of Frostman). However, when the capacity is taken in the sense of De La Vallée Poussin or Carleson (i.e., the reciprocal of Robin's constant), and when both sets are assumed to lie in a fixed circle of unit diameter, the condition of subadditivity does hold [see Nevanlinna, *Eindeutige analytische Funktionen*, p. 120 or Fekete, *On transfinite radius*, Proceedings of the International Congress of Mathematicians 1950, pp. 380–381]. It therefore seems reasonable to attempt to verify (1) under the same assumptions. However, this cannot be done, as is shown by the following counterexample: A and B are semicircles each of unit diameter, $A\cup B$ is a circle of unit diameter, and $A\cap B$ is a line of unit length. The numerical values, which are taken from the recent monograph by Pólya and Szegö, are: $C(A) = C(B) = 1.04737$, $C(A\cup B) = 1.44270$, and $C(A\cap B) = .72135$. (Received November 12, 1954.)

262. Lee Lorch (p) and Peter Szego: *A singular integral whose kernel involves a Bessel function.*

Let $t^\lambda f(t)$ be Lebesgue integrable over $(0, A)$, $A > 1$ and constant, for some constant λ; let $f(1-)$ exist and suppose that $f(t)$ is of bounded variation over $[1, A]$. Then, for $\nu \to \infty$, (i) $\lim \nu \int_0^A f(t) J_\nu(\nu t)dt = f(1-)/3 + 2f(1+)/3$, and (ii) $\lim \nu \int_0^{1+p(\nu)} f(t) \cdot J_\nu(\nu t)dt = [f(1-)/3 + f(1+)/3] \int_0^\sigma [J_{1/3}(t) + J_{-1/3}(t)]dt$, where $\sigma = \lim \nu[2p(\nu)]^{3/2}/3$, provided this last limit exists and $p(\nu) \geqq 0$ for all large ν. As usual, $J_\nu(t)$ is the Bessel function of order ν. Continuity alone of $f(t)$ at $t = 1$ does not suffice for the validity of (i), since the Lebesgue constants for this transform are unbounded, being in fact exactly of order $\nu^{1/2}$. [Thus, the kernel $\nu J_\nu(\nu t)$ behaves, so to speak, like the Fejér kernel in Fourier series when $t < 1$ and like the Dirichlet kernel when $t > 1$. This peculiarity is a consequence of the change in behavior of $J_\nu(\nu t)$ at $t = 1$ from nonoscillatory to oscillatory.] Using (ii), the limit, as $\nu \to \infty$, of $\nu^\lambda \int_0^{j_{\nu 1}} t^{-\lambda} J_\nu(t)dt$ is found for fixed $\lambda \geqq -1/2$; the value of the limit is independent of λ. For certain ranges of fixed λ, ν it is also shown that (iii) $\gamma(\lambda, \nu) < \int_0^{j_{\nu 1}} t^{-\lambda} J_\nu(t)dt < \gamma(\lambda, \nu) + A_\lambda \nu^{-\lambda}$, where A_λ $(\geqq .27)$ depends only on λ and $\gamma(\lambda, \nu) = 2^{-\lambda}\Gamma[(\nu+1-\lambda)/2]/\Gamma[(\nu+1+\lambda)/2]$. (Received November 12, 1954.)

263. G. G. Lorentz: *Properties of Banach lattices.*

Let X be a Banach lattice of measurable functions on a measure space E, for simplicity with separable measure and with $\mu E < +\infty$. Let (a) $f \in X$, $0 \leqq |g| \leqq f$ imply $g \in X$, and (b) $0 \leqq f_n \uparrow f$, $\|f_n\| \leqq M$ imply $f \in X$. Then X is separable if and only if $\|f\chi_e\| \to 0$ for $f \in X$, $me \to 0$. For instance, an Orlicz space L_Φ is separable if and only if $\Phi(2u) \leqq C\Phi(u)$. The author discusses the question under what conditions a Banach lattice of measurable functions X is a dual $X(C)$, i.e. when there is a representation $\|f\| = \sup \int_E |f(x)| C(x)d\mu(x)$, $c \in C$. The necessary and sufficient conditions are (a) and (b′) $\|f\| \leqq \lim\inf \|f_n\|$ if $0 \leqq f_n \uparrow f$. Further, the duals of the spaces $X(C_1C_2)$ are described, where C_1C_2 consists of all products c_1c_2, $c_1 \in C_1$, $c_2 \in C_2$. Essential use is made of the minimax theorem of H. Kneser-Ky Fan. (Received November 15, 1954.)

264. John McCarthy: *Stability of invariant manifolds.*

Let T: $(W, V) \to (W, V)$ be a differentiable homeomorphism where V is a compact differentiable submanifold of the manifold W. V is called an invariant manifold of

T and is said to be stable if for any differentiable family T_λ: $W \to W$ such that $T_0 = T$ there is a continuous family $V_\lambda(|\lambda| < c)$ of submanifolds such that T_λ: $V_\lambda \to V_\lambda$ and $V_0 = V$. Let $\alpha = T|V$. A Riemannian metric is put on W and a 1-1 correspondence between the space of neighboring manifolds of V and the space N of normal vector fields to V is established. T defines a linear map of N into itself which in a pair (β, γ) of coordinate systems covering a point v and its image $\alpha(v)$ is described by a matrix $A_{\beta\gamma}(v)$. It is shown that V is stable if $\|A_{\beta\gamma}(v)\| < 1$ and $\|\alpha'(v)^{-1}A_{\beta\gamma}(v)\| < 1$ for all β, γ, v. (The norms are in the sense of Banach and $\alpha'(v)$ is the Jacobian matrix of $\alpha = T|V$.) This theorem is applied to give necessary and sufficient conditions for stability in some special cases. (Received November 12, 1954.)

265. G. R. MacLane: *A convergence theorem and Riemann surfaces.*

Let G be a Riemann surface over the w-sphere such that (1) there exists a set Γ, of arcs on G, which splits G into two simply-connected components, G' and G'', (2) there exists a 1-1 inversely conformal map of G onto G which carries w into $\bar{w}$ and leaves Γ pointwise invariant, (3) all points of G over ∞ and all algebraic branch points of G lie on Γ, and (4) all singularities of G lie over $\mathfrak{J}w = 0$. Then G is the image of $D = Z - E$ by $w = f(z)$ where Z is the z-sphere, E is a closed proper subset of the real axis, $f(z)$ is meromorphic in D, $f'(z)$ has only real zeros and poles, and any arc boundary value of $f(z)$ is real. Also, $f(z)$ is the limit of a naturally defined sequence of rational functions $R_n(z)$, where $R_n'(z)$ has only real zeros and poles. $R_n(z)$ may be chosen so that D is the maximal domain of convergence. Some results are obtained concerning the possible singularities of $f(z)$. Despite the restrictions on the surface G, the class of functions $f(z)$ is remarkably wide, including, for example, a simple transformation of the modular function. (Received November 12, 1954.)

266t. J. S. MacNerney: *Weakly reproducing kernels.*

Let S be a linear orthogonal complete space, with inner product function Q, and let E be a set. Suppose S_1 is a LOC space of functions from E to S, with inner product function Q_1, and suppose that for each t in E and each x in S the linear function L_{tx}, where $L_{tx}(f) = Q(f(t), x)$, from S_1 to the numbers is continuous. *Theorem* A: For each t in E the linear function L_t, where $L_t(f) = f(t)$, from S_1 to S is continuous. *Theorem* B: There is a function K from $E \times E$ such that (i) for s and t in E, $K(s, t)$ is a continuous linear transformation from S to S with $K(s, t)^* = K(t, s)$ and $K(t, t) \geqq 0$, and (ii) for t in E and x in S the function k, where $k(s) = K(s, t)x$ for s in E, belongs to S_1 and $Q_1(f, k) = Q(f(t), x)$ for all f in S_1. *Theorem* C: For each t in E and each x in S, $|L_{tx}|^2 = Q(K(t, t)x, x)$ and $|L_t|^2 = |K(t, t)|$. (Received November 15, 1954.)

267. Arne Magnus: *On polynomial solutions of a differential equation.*

Let $u = u(x, y)$ and $v = v(x, y)$ denote two polynomials of degrees $m \geqq 2$ and $n \geqq 2$, respectively. If the Jacobian $u_x v_y - u_y v_x = k =$ constant and m and n are relatively prime, then $k = 0$ and there exists a polnomial h of first degree in x and y such that u and v are polymonials in h. The proof employs a formula developed in a previous paper (Proc. Amer. Math. Soc. vol. 5 (1954) p. 258) expressing u in terms of v and a recurrence relation between generalized Legendre polynomials. (Received November 15, 1954.)

268t. J. L. Massera: *Contributions to stability theory.* I.

Lyapunov proved that if A is a real matrix whose characteristic roots have negative real parts and if $U(x)$ is a positive definite algebraic form of degree m in the components of the vector x, there is one and only one negative definite form $V(x)$ of degree m such that $(\partial V/\partial x)\cdot Ax=U$ (where $\partial V/\partial x$ represents the gradient and the dot, scalar product); if at least one characteristic root has a positive real part there is a form V assuming positive values and a number $a>0$ such that $(\partial V/\partial x)\cdot Ax=aV+U$; as a consequence, Lyapunov's theorems on asymptotic stability and on instability admit reciprocals for the class of linear autonomous systems. The following generalization is proved: if the real parts of the characteristic roots are $\leqq 0$ and the elementary divisors of those roots having vanishing real parts are linear, then a quadratic positive definite form $V(x)$ exists such that $(\partial V/\partial x)\cdot Ax$ is negative semi-definite; the reciprocal of Lyapunov's theorem on (simple) stability for linear autonomous systems follows. (Received November 18, 1954.)

269*t*. J. L. Massera: *Contributions to stability theory.* II.

Četaev [Doklady Akad. Nauk SSSR. vol. 1 (1934) pp. 529–531 and Učenye Zap. Kazanskogo Gos. Univ. vol. 98 (1938) pp. 43–58] proved certain instability criteria which are generalized as follows. Let $\dot{x}=f(x, t)$, where x, f are n-vectors and $\dot{x}=dx/dt$, be a system of differential equations defined in the set $S_n(a)\times J$, where $S_n(a)$ is the solid sphere $\|x\|\leqq a$ and J the interval $0\leqq t<+\infty$; f is assumed to be sufficiently regular and $f(0, t)=0$. Let $G\subset S_n(a)\times J$ be an open set and $V(x, t)$ a function defined in the closure of G, bounded and positive in G, $V(0, t)=0$, and such that, in G, $dV/dt=(\partial V/\partial t)+(\partial V/\partial x)\cdot f(x, t)\geqq\beta(\alpha)\cdot\gamma(t)$ whenever $V(x, t)\geqq\alpha>0$; $\beta(\alpha)$ is any positive continuous nondecreasing function and $\gamma(t)>0$, $\int^{+\infty}\gamma(t)dt=+\infty$. Then the solution $x=0$ is unstable if one of the following complementary assumptions is satisfied: (a) $V(x, t)=0$ for any boundary point of G such that $\|x\|<a$ and G has at least one boundary point $(0, T)$, $T>0$; (b) G is bounded by smooth hypersurfaces and at any boundary point (x, t), $0<\|x\|<a$, the vector f points towards the interior of G; moreover, G has a boundary point $(0, T)$, $T>0$; (c) G is bounded by smooth hypersurfaces and at any boundary point (x, t), $0<\|x\|<a$, the vector f points outwardly; moreover, a sequence $t_n\to+\infty$ and a sequence $G_n\subset G$ exist such that $(x, t)\in G_n$ implies $t\geqq t_n$, and the sets $G_n\cup\{(0, t): t\geqq 0\}\cup\{(x, t): \|x\|=a, t\geqq 0\}$ are connected. Theorems (a) and (b) are true in infinite-dimensional spaces. (Received November 18, 1954.)

270*t*. J. L. Massera: *Contributions to stability theory.* III.

The solution $x=0$ of the system $\dot{x}=f(x, t)$ is said to be *uniformly stable* if given $\epsilon>0$ there exists a $\delta(\epsilon)>0$ (independent of $t_0\geqq 0$) such that $\|x_0\|<\delta$ implies $\|F(t, x_0, t_0)\|<\epsilon$ for $t\geqq t_0$, where $x=F(t, x_0, t_0)$ is the solution through (x_0, t_0); it is *uniform-asymptotically stable* if, in addition, there exist a $\delta_0>0$ (independent of ϵ and t_0) and a $T(\epsilon)>0$ (independent of t_0) such that $\|F(t, x_0, t_0)\|<\epsilon$ for any $\|x_0\|<\delta_0$ and $t\geqq t_0+T(\epsilon)$. The following are generalizations of previous results of Malkin [Prikl. Mat. Meh. vol. 18 (1954) pp. 129–138], Barbašin [Mat. Sbornik vol. 29 (1951) pp. 233–280] and the author [Ann. of Math. vol. 50 (1949) pp. 705–721]. 1. If the system is autonomous or periodic, stability (asymptotic stability) implies uniform stability (uniform-asymptotic stability). 2. If f is bounded and of class C^r, $r\geqq 1$, and if $x=0$ is uniform-asymptotically stable, a positive definite function $V(x, t)$ of class C^r exists, having bounded partials, such that dV/dt (see II) is negative definite; if f is periodic in t (independent of t), V may be chosen periodic in t (independent of t). The proof of Theorem 2 is based on a generalization of a lemma proved by the author (loc. cit.). (Received November 18, 1954.)

271*t*. J. L. Massera: *Contributions to stability theory*. IV.

The solution $x=0$ is said to be *asymptotically stable in the large* [Barbašin-Krasovskiĭ, Doklady Akad. Nauk SSSR. vol. 86 (1952) pp. 453–456] if it is stable and furthermore every solution tends to zero as $t\to+\infty$. The following are generalizations of results of Barbašin-Krasovskiĭ. 1. If a positive definite function $V(x, t)$ exists having an infinitely small upper bound (i.e. given $\epsilon>0$ there is a $\delta>0$ such that $V(x, t)<\epsilon$ for $\|x\|<\delta$, $t\geqq 0$) and which is infinitely large (i.e. given $M>0$ there is an $N>0$ such that $V(x, t)\geqq M$ when $\|x\|\geqq N$) and such that dV/dt (see II) is negative definite, then $x=0$ is asymptotically stable in the large. 2. If the system is autonomous or periodic, $f\in C^r$, $r\geqq 1$, $x=0$ is asymptotically stable in the large and if every solution exists in the past, then a positive definite function $V(x, t)$ exists, $V\in C^r$, the partials of V being bounded in any cylinder $\|x\|\leqq a$ (whence V admits an infinitely small upper bound), V infinitely large, such that dV/dt is negative definite (V is independent of or periodic in t, respectively). (Received November 18, 1954.)

272*t*. J. L. Massera: *Contributions to stability theory*. V.

The following theorems precise previous results on stability in the first approximation due to Malkin [Doklady Akad. Nauk SSSR. vol. 76 (1951) pp. 783–784] and Lyapunov [see Četaev, Prikl. Mat. Meh. vol. 12 (1948) pp. 639–642]. 1. Let $f(x)$ be a homogeneous vector function of degree m; if the solution $x=0$ of the system $\dot{x}=f(x)$ is asymptotically stable, a number $M>0$ exists such that for any function g satisfying $\|g(x, t)\|\leqq M\cdot\|x\|^m$, the solution $x=0$ of the system $\dot{x}=f(x)+g(x, t)$ is uniform-asymptotically stable (cf. III). 2. Suppose that the order numbers $\chi_1, \cdots, \chi_n$ (negatives of the "characteristic numbers" of Lyapunov) of the linear system $\dot{x}=A(t)\cdot x$ (A: bounded matrix) satisfy the condition $\chi_i<-\sigma/(m-1)$, $m>1$, where $\sigma=\sum\chi_i-\liminf_{T\to+\infty}(1/T)\cdot\int_{t_0}^{T}\operatorname{tr}A(\tau)d\tau\geqq 0$ is the degree of irregularity of the linear system; then for any g such that $\|g(x, t)\|\leqq\|x\|^m$ (uniformly in t) the solution $x=0$ of the system $\dot{x}=A(t)\cdot x+g(x, t)$ is asymptotically stable. (Received November 18, 1954.)

273*t*. J. L. Massera: *Contributions to stability theory*. VI.

The following theorem includes results by Dini-Hukuwara [Hukuwara, Journal Fac. Sci. Hokkaido Univ. (1) vol. 2 (1934) pp. 13–88] and Caligo [Atti II Congresso Unione Mat. Italiana, Bologna, 1940, pp. 177–185]. If a fundamental matrix solution $R(t)$ of the linear system $\dot{x}=A(t)\cdot x$ satisfies $\|R(t)R^{-1}(t_0)\|\leqq N\cdot e^{-\nu(t-t_0)}$, $\nu>0$, $t\geqq t_0$, and if $\int^{+\infty}[\|B(t)\|-(\nu/N)]dt<+\infty$, the system $\dot{x}=[A(t)+B(t)]\cdot x$ is stable; if the integral above diverges to $-\infty$, the system is asymptotically stable. The condition on R has been previously considered by Malkin, Persidskiĭ and Kreĭn. (Received November 18, 1954.)

274*t*. J. L. Massera: *Contributions to stability theory*. VII.

The solution $x=0$ of $\dot{x}=f(x, t)$ is said to be *totally stable* ("stable under constantly acting perturbations" of the Soviet mathematicians) if given $\epsilon>0$ there is a $\delta>0$ such that $\|G(t, x_0, t_0)\|<\epsilon$ for $t\geqq t_0$, $\|x_0\|<\delta$, where $x=G$ is the general solution of the system $\dot{x}=f(x, t)+g(x, t)$, $\|g(x, t)\|<\delta$. The following are partial reciprocals of a theorem due to Goršin and Malkin [Goršin, Izvestiya Akad. Nauk Kazahskoĭ SSR, Ser. Mat. Meh. vol. 2 (1948) pp. 46–73; Malkin, Prikl. Mat. Meh. vol. 18 (1954) pp. 129–138]. 1. If the solution $x=0$ of the linear system $\dot{x}=A(t)\cdot x$ is totally stable, it is asymptotically stable; 2. If the solution $x=0$ of the autonomous or periodic system $\dot{x}=f(x, t)$ is totally stable, it is asymptotically stable. (Received November 18, 1954.)

275. W. A. Michael: *Singular integral equations with normal kernels.*

Let $K(x, y)$ be a complex-valued kernel, measurable in the square $0 \leqq x, y \leqq 1$, which is *singular* in the sense of Carleman, i.e., $|K(x, y)|^2$ is integrable in each variable separately but not necessarily in (x, y). If $K(x, y)$ is the mean-square limit in the separate variables x and y of a sequence $\{K_n(x, y)\}$ of Fredholm kernels which are *normal*, i.e., $\int_0^1 K_n(x, s)\overline{K_n(y, s)}ds = \int_0^1 \overline{K_n(s, x)}K_n(s, y)ds$ for almost all (x, y), and if the $K_n(x, y)$ satisfy a certain continuity condition, then $K(x, y)$ has spectral representations $K(x, y) \sim \int \lambda dE(x, y; B_\lambda)$ of Carleman type which lead to formulas for solutions of the integral equation $u(x) - \mu\int_0^1 K(x, s)u(s)ds = f(x)$. The spectral function $E(x, y; B)$, defined for the Borel sets B of the complex plane, is obtained by means of a compactness argument inspired by a theorem of Frostman to the effect that every uniformly bounded sequence of measures (on a suitable space) has a subsequence which converges (in a certain sense) to a measure. (Received November 15, 1954.)

276*t*. E. P. Miles, Jr. and Ernest Williams: *General solutions for homogeneous second order partial differential equations in k variables.*

Consider the equation E: $\sum_{i,j=1}^k a_{ij}D_{x_i,x_j}U=0$ with real coefficients having matrix $\|a_{ij}\|$ of rank k. By a suitably chosen nonsingular linear transformation T on the set $\{D_{xi}\}$, E becomes E^*: $\sum_{i=1}^k b_i D_{x_i^*,x_j^*}U=0$ with all b_i either plus or minus 1. The author's basic sets of polynomial solutions for the k variable Laplace or wave equation are readily generalized to provide solutions for any sign distributions in E^* of the form $U^* = \sum_{m=0}^\infty \sum_{i=1}^{C_{m+k-1,k-1}-C_{m+k-3,k-1}} MW_i^m(x_1^*, \cdots x_k^*, E^*)$. Using the relation $x_1^* = \sum_{j=1}^k c_{ij}x_j$, the inverse of the nonsingular linear transformation induced on the x_i's by T, U^* is converted to the desired solution U. (Received November 15, 1954.)

277B. G. M. Muller: *On certain infinite integrals involving Bessel functions.*

By the use of a well-known lemma from the theory of hypergeometric functions, it is possible to express various classes of such integrals in terms of elementary, or at least tabulated, functions. As an illustration, it is shown that the integral $\int_0^\infty K_p(t)I_q(tz)t^r dt$ can be expressed in terms of the complete elliptic integrals of the first and the second kind of parameter z, provided the three numbers q, $(q+r\pm p)/2$ are positive integers or zero. A particular case of this result is used in the evaluation of a certain improper integral. (Received November 8, 1954.)

278. O. G. Owens: *A wave-equation with an integral-condition.*

Let Ω denote the 2-dimensional cartesian plane with coordinates $(x_1, x_2) \equiv (r \cos \theta, r \sin \theta)$; let $F(\theta)$ denote any prescribed function of class C^6 and with period 2π; finally, l will denote an arbitrarily chosen, but hereafter fixed, constant with $0<l<3/2$. Then, there exists a unique $u(x_1, x_2)$ on Ω, which thereon is a solution of the 2-dimensional wave-equation, $u_{x_1x_1}+u_{x_2x_2}+u=0$, and is such that $\int_0^\infty u(r, \theta)r^{l-1}dr = F(\theta)$, the convergence of the integral being uniform in θ. (Received November 12, 1954.)

279*t*. Pasquale Porcelli: *A Stieltjes integral representation for bounded analytic functions.*

The following theorem is proved: If $f(z)$ is analytic and bounded for $|z| < 1$, then there exists a unique continuous function γ (possibly complex valued) of bounded variation on $[0, 1]$ such that $f(z) = [(1+z)/(1-z)]\int_0^1(1/(1+(1-z)^{-2}4zu))d\gamma(u)$, for each $|z| < 1$. The proof involves results of H. S. Wall. (Received November 12, 1954.)

280. R. A. Raimi: *Equicontinuity of linear transformations.*

Let (E, E') and (F, F') be linear systems (Mackey, Trans. Amer. Math. Soc. vol. 57 (1945) pp. 155–207), and let $L(E, F)$ be the space of weakly continuous linear transformations on E to F. For various topologies on E and F, consistent with the linear systems, criteria are given for equicontinuity of a subset $U \subset L(E, F)$, using the Arens-Bourbaki scheme of defining topologies by means of dual sets (e.g. Duke Math. J. vol. 14 (1947) pp. 787–794). If E and F are given weak topologies, U is equicontinuous if and only if $U'y'$ is bounded and finite-dimensional for each $y' \in F'$, where U' is the set of operators adjoint to the elements of U. When E and F are given their k-topologies (Arens, op. cit.), equicontinuity of U may be related to compactness of U' in a suitable topology for $L(F', E')$. The method also produces the theorem: All the topologies of E consistent with the linear system (E, E') may be distinguished by the specification of the equicontinuous subsets of $L(E, E)$. If for two topologies on E, the space of continuous linear transformations is the same, then the stronger will admit the larger equicontinuous sets. (Received November 10, 1954.)

281. L. V. Robinson: *Lie infinitesimal transformation groups of one parameter and differential transforms.*

From differential transforms of the type $e^{t[f(x,y)D_x+g(x,y)D_y]}$, $D_x \equiv \partial/\partial x$, $D_y \equiv \partial/\partial y$, the Lie infinitesimal transformation groups of parameter t are shown to have their origins. The natures and limitations of these are pointed out then. (Received November 15, 1954.)

282*t*. W. G. Rosen: *On invariant means over compact semigroups.*

A *mean* on a topological semigroup Σ is a positive element of norm one in $C(\Sigma)^*$, where $C(\Sigma)$ is the space of real-valued bounded continuous functions on Σ. Let Σ be a compact semigroup. 1. The right and left regular representations of Σ over $C(\Sigma)$ are w^*-continuous. 2. There is a right invariant mean if and only if Σ contains a unique minimal left ideal. 3. There is a two-sided invariant mean if and only if the kernel of Σ (see Numakura, Math. J. of Okayama Univ. vol. 1 (1952) pp. 99–108) is a group. If such a mean exists, it is unique. 4. If Σ has a right invariant mean, then the kernel of Σ is a direct product as regards its topology, its algebraic structure, and its measure. 5. If a right invariant mean on Σ is unique, then it is two-sided invariant. (Received October 21, 1954.)

283*t*. W. G. Rosen: *On invariant means over topological semigroups.* Preliminary report.

A *mean* on a topological semigroup Σ is a positive element of norm one in $C(\Sigma)^*$, where $C(\Sigma)$ is the space of real-valued bounded continuous functions on Σ. Σ is called *weakly continuously representable* (abbreviated WCR) if its right and left regular representations over $C(\Sigma)$ are weakly continuous. Discrete and compact semigroups are WCR. A theorem by Day (Trans. Amer. Math. Soc. vol. 69 (1950) pp. 276–291) is extended from discrete semigroups to WCR semigroups. If Σ is a WCR semigroup, then there is an invariant mean if and only if every bounded w^*-continuous representation of Σ is ergodic, and if and only if the regular representations of Σ over $C(\Sigma)$ are ergodic. Also, if Σ is a WCR semigroup with an invariant mean in $C(\Sigma)^*$, then all invariant means assume the same value at an element $x\in C(\Sigma)$ if and only if x lies in the ergodic subspace of the regular representations. (Received October 21, 1954.)

284*t*. Walter Rudin: *On a problem of Bloch and Nevanlinna.*

If f is bounded and analytic in the open unit disc U, is the derivative f' of bounded characteristic in U? This is the problem referred to in the title. A counter-example (a Blaschke product) has been found by Forstman. It is now shown that the answer is negative even if f is the sum of a power series which converges absolutely on the boundary of U. More precisely, a function $f(z)=\sum c_k z^{n_k}$ is constructed such that $\sum |c_k|<\infty$, $n_k/n_{k-1}\to\infty$, and $f'(re^{i\theta})\to\infty$ as $r\to 1$, for almost all θ. (Received November 8, 1954.)

285. Walter Rudin: *Square roots of absolutely convergent Fourier series.*

Let A be the class of functions defined on the unit circumference C whose Fourier series converge absolutely. A well-known theorem of Wiener and Lévy implies that if $f\in A$ and $f(x)>0$ for all $x\in C$, then $f^{1/2}\in A$. It is shown, by complex variable methods, that there exists a function $f\in A$ such that $f(x)>0$ on C except at one point x_0, and such that $f^{1/2}\notin A$. This leads to the following conjecture: If G is an infinite locally compact abelian group with dual group G^*, there exists a non-negative function f on G which is the Fourier transform of a function $f^*\in L^1(G^*)$, although $f^{1/2}$ is not such a transform. Apart from the case $G=C$, the conjecture is proved if G is the real line, and if G is discrete. (Received November 8, 1954.)

286. P. P. Saworotnow: *On the generalization of the notion of H^*-algebra.* Preliminary report.

An H^*-algebra has a property that the orthogonal complement of every ideal is an ideal of the same kind. The author considers the converse problem. It is proved that if a Banach algebra $\mathfrak{a}$ is a semi-simple dual ring, is a Hilbert space, and has the property that the orthogonal complement of every right ideal is a right ideal, then $\mathfrak{a}$ is a right H^*-algebra sa well as a left H^*-algebra (though the involutions $x\to x^r$ such that $(yx, z)=(y, zx^r)$ and $x\to x^l$ such that $(xy, z)=(y, x^l z)$ may not coincide). The author also shows that every simple algebra of the type described above is isomorphic to the algebra of all linear transformations (a_{ij}) from $L^2(J)$ into $L^2(I)$ of the Hilbert-Schmidt type (where $I=(i)$, $J=(j)$) with the multiplication $(ab)(k, l)=\sum_{j,i} a_{kj}\lambda_{ji}b_{il}$ where (λ_{ji}) is a nondecreasing linear operator from $L^2(I)$ onto $L^2(J)$ which has an inverse transformation. Both results are obtained by proving the structure (Wedderburn) theorems. (Received November 10, 1954.)

287*t*. H. M. Schaerf: *Metric transitivity and existence of means.* Preliminary report.

Let m be a σ-finite content on a σ-algebra S of subsets of a set X. Let H be a group of mappings of X into X which preserve both measurability and null sets of m. A mapping $\bar{h}$ of X into X is termed *H-pieced* if there is both a partition of X into a sequence $\{X_i\}\subset S$ and a sequence $\{h_i\}\subset H$ such that the sequence $\{h_i X_i\}$ is disjoint and $\bar{h}x=h_i x$ for $x\in X_i$ $(i=1, 2, \cdots)$. A number M is called a *uniform* (H, m)*-mean* of a function f defined on X, if, for every number $\epsilon>0$, there is a null set N_ϵ of m and a finite sequence of H-pieced mappings $\bar{h}_1, \cdots, \bar{h}_n$ such that $|M-(1/n)\sum_{i=1}^n f(h_i x)|<\epsilon$ for $x \notin N_\epsilon$. *Theorem*: In order that every bounded measurable function on X have a uniform (H, m)-mean it is necessary and sufficient that H be metrically transitive for m. If there is a finite measure invariant under H and weaker than m, then each uniform mean is unique. (Received November 26, 1954.)

288t. H. M. Schaerf: *Metric transitivity in product spaces*. I. Preliminary report.

Generalizing the usual terminology, a binary relation R on a σ-algebra S of subsets of a set X will be called *metrically transitive* for a measure m on S if every set $A \in S$ which m-almost contains every R-image of any of its subsets is either a null set or the complement of a null set of m. Let S be the cartesian product of a family of σ-algebras S_i of subsets of sets X_i. Then every measure m_i on S_i of the form $m_i(E)=m(E\times F)$ with $E \in S_i$, $F \in \mathrm{P}_{j\in I-(i)}S_j$, will be called a *projection of m on S_i*. Let R_i be a binary relation on S_i which preserves finite partitions. Write $(\mathrm{P}_{i\in I}A_i)R(\mathrm{P}_{i\in I}B_i)$ if there is $j\in I$ with $A_jR_jB_j$ and $A_i=B_i$ for $i\neq j$. *Theorem* 1. If each projection m_i of m on S_i is invariant under R_i and R_i is metrically transitive for m_i, then R is, in general, metrically transitive for m. (Certain weak regularity conditions for m are required.) *Corollary*. If m is the direct product of measures m_i invariant under R_i, then R is metrically transitive for m if and only if each R_i is metrically transitive for m_i. (Received November 26, 1954.)

289t. H. M. Schaerf: *Metric transitivity in product spaces*. II. Preliminary report.

The notations of the preceding report are kept. A set H of mappings of X into X is called *metrically transitive* for the measure m if so is the binary relation R where ARB means $A=hB$, $h\in H$. *Theorem* 2. For every i in I, let H_i be an at most countable set of 1-to-1 mappings of X_i into X_i which preserve both measurability and null sets of a measure m_i on S_i. Let H be the set of all sequences $\{h_j\}$ where $h_i\in H_i$ for some $i\in I$ while h_j is the identity mapping of X_j for $j\in I-(i)$. Then H is metrically transitive for the direct product of the measures m_i if each H_i is metrically transitive for m_i. All measures mentioned in these reports are assumed to be σ-finite. (Received November 26, 1954.)

290. Morris Schreiber: *Generalized spectral resolution for operators in Hilbert space.*

A "spectral" decomposition in terms of positive-operator-valued set functions is developed which holds for arbitrary bounded operators. Proceeding from a theorem of Nagy (Acta Szeged. vol. 15 (1953) pp. 87–92) stating that the powers of an operator A with $\|A\|\leqq 1$ are simultaneously compressions of the powers of a unitary operator on a larger space, it follows that the compression of the spectral measure of the unitary operator associated to A is a set function of the desired type ("operator measure") and is unique. It is proved that the support of the operator measure of A (always a subset of $|z|=1$) does not divide the plane if and only if A is unitary. A functional calculus of the usual type obtains for boundary values of functions analytic in $|z|<1$ and bounded on $|z|=1$. Those sequences of operators which can be expressed simultaneously in terms of a single operator measure are characterized (generalizing a result of Nagy, Acta Hungaricae vol. 3 (1952) pp. 289–292) by the condition that if $c_0+\sum c_n e^{in\theta}+\sum c_{-n}e^{-in\theta}$ is a positive trigonometric polynomial, then $c_0 A_0+\sum c_n A_n+\sum c_{-n}A_n^*$ is a positive operator, and related results on order and commutativity are obtained. (Received November 15, 1954.)

291. I. E. Segal: *Tensor algebras over Hilbert spaces.*

The algebra of all covariant tensors over a complex Hilbert space H is shown to contain no "completely characteristic" ideals that are maximal with respect to not containing all tensors from a certain rank onwards other than the symmetric and skew-symmetric ideals. The corresponding symmetric quotient algebra is shown to be unitarily equivalent to the space $L_2(H')$ of "square-integrable" functionals over a real Hilbert space H' of which H is the complexification, in a canonical fashion which clarifies the structure of various linear operators on the algebras. Among these is the Plancherel transform, which like some features of the theory of Lebesgue integration, is extended from the classical case when H' is finite-dimensional to the case of a space of arbitrary dimension. (Received November 3, 1954.)

292t. R. L. Shively: *Pseudo Laguerre polynomials.*

The classical Laguerre polynomials $L_n^{(\alpha)}(x)$ may be defined by the series $L_n^{(\alpha)}(x)=((\alpha+1)_n/n!)_1F_1(-n;\alpha+1;x)$. Most authors do not distinguish between the case in which α is independent of n, and the case in which α depends on n. For the purpose of this paper $L_n^{(\alpha)}(x)$ is defined to be a proper Laguerre polynomial if α is independent of n, and a pseudo Laguerre polynomial if α depends on n. The particular pseudo Laguerre polynomials $R_n^{(a)}(x)=((a+n)_n/n!)_1F_1(-n;\ a+n;\ x)$ are studied in order to learn something about the effect of changing a parameter $(\alpha+1)$ to $(a+n)$, a and α independent of n, in a hypergeometric polynomial. It is found that many of the important properties of the proper Laguerre polynomials are either ambiguous or totally meaningless when $(\alpha+1)$ is replaced by $(a+n)$, a and α independent of n. A generalization of $R_n^{(a)}(x)$, obtained by inserting an arbitrary number of numerator and denominator parameters all independent of n, is also studied in some detail. (Received November 12, 1954.)

293. C. J. Standish: *Inversion of a generalized Poisson transform.*

Let $f(x)=\pi^{-1}\int_{-\infty}^{\infty}[A(x-t)+B/(x-t)^2+C^2]g(t)dt$ where $g(t)$ is Lebesgue integrable on every finite interval. Convergence properties of the transform are investigated and real and complex inversion formulas are obtained. The real inversion formula in the special case of the Poisson transform ($A=0$, $B=C=1$) was discovered by Pollard (*The Poisson transform*, Trans. Amer. Math. Soc., to appear). (Received September 15, 1954.)

294B. Warren Stenberg: *On sequences with divergent total variation.*

Let $a_1, a_2, \cdots$ be a sequence of positive terms tending to zero with $\sum_{n=1}^{\infty} a_n=\infty$. It is proved that there is a univalent sequence $h_1, h_2, \cdots$ of positive integers such that the sequence $b_1, b_2, \cdots$, where $b_n=a_{h_n}$ has $\sum_{n=1}^{\infty} b_n=\infty$ and has the following property. If $c_1, c_2, \cdots$ is a subsequence of $b_1, b_2, \cdots$ with $\sum_{n=1}^{\infty} c_n=\infty$, then also $\sum_{n=1}^{\infty}|c_n-c_{n+1}|=\infty$. Even further, the sequence $b_1, b_2, \cdots$ may be so constructed that if $c_1, c_2, \cdots$ is a subsequence of $b_1, b_2, \cdots$ then for each positive integer m, $\sum_{n=1}^{m-1}|c_n-c_{n+1}|\geq 2\sum_{n=1}^{m}c_n-5b^*$, where b^* is the greatest of the terms of $b_1, b_2, \cdots$. Clearly the constant 2 cannot be increased; b^* cannot be replaced by c^*, the largest of the terms $c_1, c_2, \cdots$, even if 5 is replaced by an arbitrarily large positive number and 2 by an arbitrarily small positive number. (Received November 13, 1954.)

295. Wilhelm Stoll: *The singularities of meromorphic modifications.*

Let G and H be $2n$-dimensional complex manifolds, let τ be an analytic homeomorphism of an open subset $A\subseteq G$ onto $B\subseteq H$, and let $v=\tau^{-1}$. $\mathfrak{M}=\mathfrak{M}(G, A, M, \tau/H, B, N, v)$ is said to be a modification provided $M=G-A$, $N=H-B$ are subsets of analytic sets of dimension smaller than $2n$. The modification $\mathfrak{M}^{-1}=\mathfrak{M}(H, B, N, v/G, A, M, \tau)$ is the inverse of $\mathfrak{M}$. $\mathfrak{M}$ is open provided $\tau(U\cap A)\cup N$ is open when U is an open neighborhood of M. $\mathfrak{M}$ is said to be meromorphic provided that for each 2-dimensional complex manifold $L\subset G$ with $L\cap M=\{P_0\}$ the set consisting of all points $Q=\lim_{\mu\to\infty}\tau(P^\mu)$, with $\lim_{\mu\to\infty}P^\mu=P_0$ for a proper sequence $P^\mu\in L\cap A$, contains at most one point. For $n=2$ one has the theorem: If $\mathfrak{M}$ is meromorphic, if $\mathfrak{M}$ and $\mathfrak{M}^{-1}$ are open, then for each singular point P_0 of τ there exist a 2-dimensional analytic set $C(P_0)\subseteq N$ and a set $D(P_0)\subset C(P_0)$ without an accumulation point such that v is regular on $C(P_0)-D(P_0)$ and $v^*(Q)=P_0$ for $Q\in C(P_0)-D(P_0)$, where v^* is the continuation of v. Therefore $P_0\to C(P_0)$ is a one-to-one map. An easy consequence is: If H has a countable base of open sets, then the singularities of τ are isolated. (Received November 8, 1954.)

296B. George Swift: *n-valued irregular Borel measures.*

A Borel measure is n-valued on a topological space X if it assumes exactly n different values on X, where n is a positive integer. In this paper, it is shown that every n-valued, totally finite, irregular Borel measure on a σ-compact Hausdorff space is outer irregular at some point of the space. Furthermore, an n-valued, totally finite, irregular Borel measure μ on a σ-compact or locally compact Hausdorff space X is outer irregular at at most m points where m is the greatest integer less than or equal to $\mu(X)/\inf\{\mu(B): \mu(B)>0, B \text{ is a Borel set}\}$. Necessary and sufficient conditions on a σ-compact Hausdorff space X for the existence of a totally finite n-valued irregular Borel measure μ are that X be uncountable and that X possess a point x_0 and a class $\mathcal{A}$ of Borel sets such that (1) $\{x_0\}\notin\mathcal{A}$ and $x_0\in A^-$ for all $A\in\mathcal{A}$, (2) $\{A_i\}_{i=1}^\infty\subset\mathcal{A}$ implies that there exists an $A\in\mathcal{A}$ such that $A\subset\bigcap_{i=1}^\infty A_i$, and (3) $A\in\mathcal{A}$ and $A=B_1\cup B_2$, where B_1, B_2 are disjoint Borel sets, imply that there exists an $A_0\in\mathcal{A}$ such that either $A_0\in B_1$ or $A_0\in B_2$. (Received November 12, 1954.)

297t. C. T. Taam: *Schlicht functions and linear differential equations of second order.*

It is known that an analytic function $f(z)$ is schlicht in a region R if and only if no solution of (A): $W''+Q(z)W=0$ has more than one zero in R, where $Q(z)$ is one-half of the Schwarzian derivative of $f(z)$ with respect to z. In this paper several sufficient criteria for the schlichtness of $f(z)$ are obtained for different regions R by proving that no solution of (A) has more than one zero in R. Two basic theorems are used to establish the results: one is an extension of a nonoscillation criterion of Wintner to the complex plane (A. Wintner, *On the non-existence of conjugate points*, Amer. J. Math. vol. 73 (1951) pp. 368–380) and the other is a comparison theorem (C. T. Taam, *On the solutions of second order linear differential equations*, Proc. Amer. Math. Soc. vol. 4 (1953) pp. 876–879). The second theorem yields results which are either the same as those obtained by Z. Nehari (Lecture Notes, Conference on Functions of a Complex Variable at University of Michigan, 1953, pp. 148–151) or slight generalizations of his. (Received November 15, 1954.)

298. C. J. Titus: *Linear vector spaces of interior mappings.*

Let D be a domain in the xy-plane. Let F be the family of continuously differentiable mappings of D into the plane which are nowhere sense-reversing. Let F_0 be the family of mappings in F with the property that the Jacobian of any mapping in F_0 is zero only at points where it is rank zero. All maximal real linear vector spaces in F_0 are characterized. If V is such a vector space, then V is maximal even in F. A space V is completely determined by a pair of mappings which are independent in a certain sense. The relationship to quasi-conformal mappings and to pseudo-analytic functions of Bers is discussed. (Received November 15, 1954.)

299. Albert Wilansky (p) and Karl Zeller: *Controlled summability matrices.*

Classical treatments of summation methods have begun by showing that a matrix A satisfies (*) $\left|\sum_{k=0}^{m} a_{rk}x_k\right| \leqq M\sup_{0\leqq n\leqq r}\left|\sum_{k=0}^{n} a_{nk}x_k\right|$. (Cf. Hardy-Riesz, *Dirichlet's series*, 1915, p. 28, Lemma 7, and p. 29, lines 12–17.) We apply functional analysis to unify the methods and results, calling A *controlled* if (**) $\left\{\sum_{k=0}^{m} a_{rk}x_k\right\}$ is bounded for each $x\in c_A=\left\{x \mid \lim_n \sum a_{nk}x_k \text{ exists}\right\}$. This implies (*) for triangles ($a_{nn}\neq 0$, $a_{nk}=0$ for $k>n$). By p. 107 in Banach's treatise, (**) is equivalent to "with $\delta^n=(0, 0, \cdots, 0, 1, 0, \cdots)$, $\{\delta^i\}$ is a basis for its linear closure in c_A (one uses $\|x\|=\sup_n\left|\sum_k a_{nk}x_k\right|$), a maximal subspace of, or all of, c_A." Applications of the uniform boundedness principle, weak-star completeness, and other (F)-space techniques yield the classical results, often with strengthening and converses. Criteria occur in the general setting, e.g. (M. Riesz): $B=A^{-1}$, $b_{nk}\leqq 0$ for $k<n$, $\sum_k b_{nk}\geqq 0$ imply that A is controlled. Typical results: $x_n=o(a_{nn}^{-1})$ for all $x\in c_A$, hence, sometimes, A sums no divergent sequences, Corollary: Mercer's theorem. There are results on convergence factors, summability factors, comparison theorems, inversion theorems. Closely related concepts are *PMI*, *AK* used by the authors in earlier, separate, papers. (Received November 10, 1954.)

300. Y. K. Wong: *On projections in reflexive Banach spaces.*

E. R. Lorch proved that (Trans. Amer. Math. Soc. vol. 45, p. 202) if P, Q are permutable projections, then PQ and $P+Q-PQ$ are projections. In this paper, the converse is shown. If P, Q are projections, the preceding three properties are equivalent. Other equivalent conditions are stated in terms of the associated manifolds. Nagy gave an expression for $P_1 \dotplus \cdots \dotplus P_n$ in Hilbert space, which can be extended to reflexive Banach spaces. Lorch's result is a special case of that of Nagy, in which $n=2$. When $n\geqq 3$, the converse does not hold, in general, without further restriction. (Received November 12, 1954.)

301*t*. Y. K. Wong: *Some properties of the proper values of a matrix.*

In the paper on *Nonnegative-valued matrices* (Proc. Nat. Acad. Sci. U.S.A. (1954) pp. 121–123), the author showed that if (1) the column-sums $\sum_i a_{ij}$ are at most 1, then (2) the maximal proper value of $A=(a_{ij})$ is less than 1 if and only if (3) $\sum_{i=1}^{j} a_{ij}$ is less than 1 for all j. It is now shown that if property (1) is not assumed for nonnegative matrices, then (2) implies (3), but not conversely. However, if (1) is modified with absolute value for complex-valued matrices, then $\sum_{i=1}^{l} |a_{ij}| < 1$ is sufficient for the validity of (2), but not necessary. Without assuming condition (1), equivalent conditions for nonnegative matrices to have property (2) are given. The aim is to find some simple criteria for the validity of property (2) so as to facilitate certain numerical computations. (Received October 14, 1954.)

Applied Mathematics

302t. Y. W. Chen: *On a problem of minimal surface.* II.

The present paper extends previous results (see Bull. Amer. Math. Soc. vol. 60, p. 338) to the case when the boundary is not a simple polygon but a simple closed convex curve with a finite number of corners. As before, the admission of corners on the boundary is significant. The convergence of the approximating polygon solution is based upon the following property of the minimizing integral J (which is not positive definite): If one boundary curve C is contained in the other, C', then $J(C) > J(C')$. This property is proved by an application of the subordination principle of conformal mapping. Uniqueness is obtained for the symmetrical curves, using results of Lavrentieff and Gilbarg. The discontinuous behavior of the solution $z(x, y)$ at the corner is not the same as that of a harmonic function; indeed z remains continuous in two sectors formed by the tangents and normals at the corner. The formulation of the results in terms of conjugate minimal surface is of interest. It shows that when a surface is unable to meet the boundary requirement, it may smooth out the boundary condition by itself. This explains certain well-known features in the soap films experiments. (Received November 15, 1954.)

303t. Jim Douglas, Jr.: *On the numerical integration of quasi-linear parabolic differential equations.*

The numerical solution of the differential equation $u_{xx} = F(x, t, u)u_t + G(x, t, u)$, $0 \leqq x \leqq 1$, $0 \leqq t \leqq T$, is treated by means of the backwards difference equation $\Delta_x^2 w_{i,n+1} = F(x_i, t_n, w_{in})(w_{i,n+1} - w_{in})/\Delta t + G(x_i, t_n, w_{in})$, where $\Delta_x^2 w_{i,n+1} = (w_{i+1,n+1} - 2w_{i,n+1} + w_{i-1,n+1})/(\Delta x)^2$. Let $\beta = \min(1, 2a)$. Then, under reasonable assumptions on F, G, the initial condition, and the boundary conditions, if $\Delta x/(\Delta t)^a$, $a > 0$, is held fixed $|u_{in} - w_{in}| = O((\Delta t)^\beta)$ as $\Delta t \to 0$, uniformly in $0 \leqq x \leqq 1$, $0 \leqq t \leqq T$. It is also shown that, in the sense of least work, the best choice of a is one-half. (Received November 9, 1954.)

304. T. C. Doyle: *Higher order invariants of stress or deformation tensors and their syzygies.*

As a preliminary investigation motivated by the admission of higher order derivatives of displacements into the elastic strain energy density, a fundamental system of *irrational* joint invariants of order p of a euclidean metric tensor $\boldsymbol{g}$ and a generic two index tensor $\boldsymbol{a}$ is derived. A *rational* system of 21 invariants of order 1 is also determined. On identifying $\boldsymbol{a}$ with a stress tensor $\boldsymbol{t}$ in the absence of body force or with an isochoric deformation tensor $\boldsymbol{c}$, certain syzgyies occur which are exhibited in a canonical form, i.e., as the vanishing of properly chosen base invariants. A repetition of this specialization of $\boldsymbol{a}$ in the second order irrational invariants introduces 6 compatibility syzygies for $\boldsymbol{a} = \boldsymbol{c}$ in addition to the once differentiated first order syzygies. (Received November 10, 1954.)

305. R. J. Duffin: *A minimax theory for overdamped networks.*

Linear dynamical systems with n degrees of freedom are considered. Such a system is characterized by three positive-definite quadratic forms: the kinetic energy form a, the dissipation form b, and the potential energy form c. If $b \equiv 0$, the system is conservative and the frequencies of the normal modes of vibration are determined by the stationary values of the Rayleigh quotient c/a. In this paper a system is termed "overdamped" if $d^2 = b^2 - 4ac$ is positive for all virtual modes of vibration. It is shown that the $2n$ eigenvalues (decay constants) of an overdamped system are the stationary values of $(-b \pm d)/2a$. A minimax treatment of this functional is developed, in analogy to the treatment for the conservative case. It results that a constraint on the system tends to increase the eigenvalue of each normal mode. Among other things this property justifies the use of the Rayleigh-Ritz procedure for the approximate determination of the eigenvalues. (Received November 15, 1954.)

306. R. J. Duffin and Elsa Keitzer (p): *A trilinear formula relating network matrices.*

A positive real function $g(z)$ is a rational function with real coefficients whose real part is positive when the real part of z is positive. An important trilinear formula $T(f, g, z) = 0$, due to P. I. Richards [Duke Math. J. vol. 14 (1947) pp. 777–786] defines a new positive real function f in terms of g. Following a suggestion of R. Bott it is shown that this formula has a generalization $T(f, g, h) = 0$ where h is a positive real function whose poles are all on the imaginary axis. A positive real matrix is essentially a matrix whose quadratic form is a positive real function. The above trilinear formulae are generalized to matrices, and their properties are investigated. (Received November 15, 1954.)

307*t*. William H. Durfee: *Heat flow in a fluid with eddying flow.*

The differential equation for the flow of heat in a fluid flowing with eddying flow in a pipe as derived by Latzko (Zeitschrift für Angewandte Mathematik und Mechanik vol. 1 (1921) pp. 268–290) is solved by numerical methods with the aid of high speed computing machinery. The first nine eigenvalues and the tabulated values of the corresponding functions together with their first derivatives were computed. The results represent a considerable improvement both in completeness and accuracy over Latzko's computations. (Received November 12, 1954.)

308. David Gale: *The law of supply and demand.*

Let M be a market consisting of n *goods* G_j. A *price vector* $p = (\pi_1, \cdots, \pi_n)$ is a non-negative vector such that $\sum \pi_j = 1$, where π_j is the unit price of G_j. A commodity bundle $x = (\xi_1, \cdots, \xi_n)$ is a vector in R_n where ξ_j, the amount of G_j in the bundle x, is positive or negative according as G_j is supplied or consumed. Let there be m economic units (individuals or firms) U_i. For each price vector p the unit U_i chooses a set of commodity bundles $S_i(p)$ subject to the condition (1) if $x_i \in S_i(p)$, then $p \cdot x_i \geqq 0$ (this is the requirement that U_i be able to afford x_i at prices p). The function S_i is called the ith units *supply function.* A price vector p_0 is said to balance supply and demand if there exist bundles $x_1, \cdots, x_n$ where $x_i \in S_i(p_0)$ and $\sum x_i \geqq 0$, the last condition being the requirement that the net amount of each G_j supplied be at least equal to the amount consumed. Theorem: Let the functions S_i satisfy: (a) $S_i(p)$ is compact and convex for all p; (b) the graph of each S_i is closed. Then there exists a price vector p_0 which balances supply and demand. The proof depends on a well-known lemma of elementary combinatorial topology. (Received November 5, 1954.)

309. W. T. Guy, Jr.: *A generalized Laplace transform.* II.

Two convolution and composition type results were presented for a generalized Laplace transform. The kernel of the integral transform used was a Meijer G-function which had been previously defined. (Received November 15, 1954.)

310B. W. C. Hoffman: *Solution of the wave equation for a randomly inhomogeneous medium.*

The Helmholtz equation $\nabla^2\psi+k^2\psi=0$, where $k=k_0\kappa^{1/2}$ is a random function of position and $|\nabla\kappa|\lambda\ll 1$, possesses plane wave solutions of the form $\psi(x, y, z) =\psi_0 \exp \{i(\omega t-k_0S)\}$, where S and ψ_0 are functions of position, the latter being slowly-varying. The differential equation $(\nabla S)^2+(i/k_0)\nabla^2S=\kappa$ then results for S. Letting $S=\sum_0^\infty S_n\tau^n$ and $\kappa=\sum_0^\infty \kappa_n\tau^n$, $\tau\ll 1$, perturbation theory yields the following differential equations for S_0 and S_1: $(\nabla S_0)^2+(i/k_0)\nabla^2S_0=\kappa_0=\text{const.}$; $2\nabla S_0\cdot\nabla S_1 +(i/k_0)\nabla^2S_1=\kappa_1$. The former is solved by separation of variables, and certain properties of ψ and its mean value that are implied by the form of S_0 are studied. The first perturbations of the phase and the refractive index, S_1 and κ_1 respectively, are next supposed to be stationary random processes and their respective spectral representations in terms of the orthogonal processes $\int_{\Delta\omega}s_1(\omega_1, \omega_2, \omega_3)\prod_1^3 d\omega_j$ and $\int_{\Delta\omega}\zeta_1(\omega_1, \omega_2, \omega_3)\sum_1^3 d\omega_j$ are introduced. This leads to the equation $s_1(\omega_1, \omega_2, \omega_3) =ik_0[\boldsymbol{\omega}\cdot\boldsymbol{\omega}-2k_0\nabla S_0\cdot\boldsymbol{\omega}]^{-1}\zeta_1(\omega_1, \omega_2, \omega_3)$. The relation between the covariance functions S_1 and κ_1 is then determined by the *faltung* theorem, and it is shown that the covariance function of S is proportional to that of S_1. (Received November 10, 1954.)

311t. A. B. Lehman: *A note on modular lattices and series-parallel circuits.* Preliminary report.

Let $a, b, c, \cdots$ denote the elements of a series-parallel electric circuit configuration. Let $x'(a, b, c, \cdots)$ denote the polynomial representation of the two terminal circuit viewed from the nodes connecting element x. Theorem: In a modular lattice, $x'(a', b', c', \cdots)=x'(a, b, c, \cdots)$ as lattice polynomials. The proof is by substitution and induction. The necessity of the modular law may be shown by applying the theorem to a triangular circuit. More general lattice theorems on the substitution and shorting algebras of general circuits have been established. (Received December 14, 1954.)

312B. R. D. Luce: *k-stability of symmetric and of quota games.*

Let v be the 0, 1-reduced form of the characteristic function of an essential n-person game, $X=\|x_i\|$ an imputation of the game, and τ a proper partition of the players into coalitions. The pair (X, τ) is *k-stable*, where k is an integer, $0\leqq k\leqq n-2$, [Luce, R. D., Ann. of Math. vol. 59 (1954) pp. 357–366] if (i) for each subset S of players such that there exists a $T\in\tau$ with $|(S-T)\cup(T-S)|\leqq k$, then $v(S)\leqq\sum_{i\in S}x_i$ and if (ii) for each $i\in T$, where $T\in\tau$ and $|T|>1$, then $x_i>v(\{i\})=0$. The game is k-stable if it has a k-stable pair. Theorem 1. A symmetric game $[v(S)=v(|S|)]$ is k-stable if and only if $v(i)\leqq i/n$ for $0\leqq i\leqq k+1$. Theorem 2. A quota game [Shapley, L. S., *Contributions to the theory of games*, II, Annals of Mathematics Studies, no. 28, Princeton, 1953, pp. 343–359] is 1-stable if and only if there is no weak player. Theorem 3. If (X, τ)

is k-stable in a quota game with quota $Q=||q_i||$, then $X=Q$ if n is odd or if n is even and $k\geqq 2$. If n is even and $k=1$, then either $X=Q$ or $|T|$ is even and $v(T)=\sum_{i\in T}x_i=\sum_{i\in T}q_i$ for $T\in\tau$. Theorem 4. A quota game without weak player is k-stable if (*) $v(S)\leqq\sum_{i\in S}q_i$ for all S with $|S|\leqq k+1$. If it is k-stable (*) holds for all S with $|S|\leqq k$; if it is k-stable and (*) is violated for some S with $|S|=k+1$ and if (X,τ) is k-stable pair, then $|T\cap S|=1$ and $|T|$ is even for all $T\in\tau$ such that $T\cap S\neq 0$. (Received November 8, 1954.)

313. Y. L. Luke: *Simple formulas for the evaluation of some higher transcendental functions.*

A certain class of transcendents can be simply computed by using the trapezoidal or modified trapezoidal rule on definite integrals defining these functions. Computation is simplified. Technique requires evaluation of elementary functions usually available as a subroutine on automatic computers. Let $I=\int_0^b f(t)dt$. Apply first and second Eulerian summation formulas. Correction series is usually divergent and involves $E_r=[f^{(2r-1)}(b)-f^{(2r-1)}(0)]$. If $E=E_r=0$ for all r, series is null. Highly accurate representations for I using either form of trapezoidal rule are obtained. If b is finite, f an even periodic function of period b, $E=0$. If b is infinite, f an even integral function, $E=0$. Error is conveniently studied by expressing it as an infinite sum of Fourier coefficients of f. Important example is Bessel function $J_n(z)$. Three integral representations are considered: Bessel's integral, Poisson's integral, and representation for $J_n(x^2-y^2)^{1/2}$; simple estimates of error are found. Seven terms of modified trapezoidal rule gives $e^{-y}J_0(z)$, $z=x+iy$ accurate to at least $9d$ for all $|z|\leqq 10$. Three kinds of complete elliptic integrals are also studied. Another example is modified Bessel function $K_\nu(z)$, Re $(z)>0$ ($\nu=1/3$ yields Airy's integral, essentially). Other examples are error function and various rocket functions. (Received November 12, 1954.)

314t. T. S. Motzkin: *Evaluation of rational functions.*

For an algorithm starting from constants and v variables and consisting of an unbounded number of additions and n other binary operations, the number p of parameters of the family F obtained is $\leqq 2n+1$. Even for $v=1$, $p=2n+1$ is obtained by n divisions, e.g., in the algorithms of partial and (getting all real rational functions from real constants) recurrent fractions. In these algorithms a result obtained is used only once; under this assumption, for m multiplications and $d=n-m$ divisions, the exact bound for k is $2n-[(m-1)/(d+1)]$, where [ ] means largest integer. (Received November 12, 1959.)

315B. T. S. Motzkin: *Evaluation of polynomials.*

Let A_m denote an algorithm consisting of $m>0$ multiplications and arbitrarily many additions, starting from real constants and one variable, and F the family of polynomials obtained. A_m can be chosen so that F consists of almost all (for $m\leqq 3$: all) polynomials of degree $2m$ with highest coefficient 1 and contains a given polynomial. To obtain almost all polynomials of degree n with unrestricted highest coefficient, the lowest m is $[n/2]+1$ except for $n=3, 5, 7, 9$ where it is $[n/2]+2$. Ostrowski has shown that to obtain all polynomials of degree n, $n\leqq 4$, m must be $\geqq n$; this holds also for $n=5$, but not for $n>5$. As an application of these improvements on the Newton-Horner synthetic division procedure it is possible, using Tchebyshev approximation to $1/x$, to halve the time for division on divisionless automatic computers. (Received November 12, 1954.)

316. J. A. Nohel (p) and W. K. Ergen: *Stability of solutions of a particular delay-differential equation*. Preliminary report.

In a circulating fuel nuclear reactor the power $x(t)$ satisfies the equation (1) $x'(t) = (-c/\theta)\int_0^\theta(\theta-h)[f(x(t-h))-1]dh$, $'=d/dt$, where c and θ are positive constants, and t is real. The function $f(x)$ satisfies the conditions: (a) $f(x)>0$ for all x, (b) $f'(x)>0$ for all x, (c) $f(0)=1$. Assumption (c) implies that $x=0$ is a solution of (1). The stability and asymptotic stability in the sense of Liapounov of the zero solution of (1) are investigated after these concepts are redefined so as to apply to equations of the type (1). Asymptotic stability, and in one case only stability, is established for the zero solution of the equation of first variation associated with (1). The method used is similar to that of F. G. Prohammer, *Note on the linear kinetics of the circulating-fuel nuclear reactor*, Y-F10-99 (Declassified), where an entirely different result is obtained. The general stability problem of the zero solution of the nonlinear equation (1) is not completed. For the case of periodic solutions of (1) which are known to exist (F. H. Brownell and W. K. Ergen, A *theorem on rearrangements and its application to certain delay differential equations*, Journal of Rational Mechanics vol. 3 (1954)) the stability problem is formulated, but remains unsolved. (Received November 10, 1954.)

317. L. E. Payne: *Proof of a conjecture of A. Weinstein*.

The following conjecture is due to A. Weinstein. *The first eigenvalue in the buckling problem for a clamped plate is not less than the second eigenvalue of the membrane of the same shape which is fixed on the boundary*. The author proves this conjecture by using the first partial derivatives of the first eigenfunction for the clamped plate in the variational principle for the membrane eigenvalues. It is also shown that the two eigenvalues are equal if and only if the plate is circular. This last statement follows from some well known properties of nodal lines of a vibrating membrane. The research of this author was supported by the United States Air Force through the Office of Scientific Research. (Received October 7, 1954.)

318. George Seifert: *On solutions of pendulum-type equations*.

Let $\ddot{\theta}+f(\theta)\dot{\theta}=g(\theta)$, where f and g have continuous first derivatives everywhere, have period 2π in θ, and let $\int_0^{2\pi}f(\theta)d\theta>0$. If $z=\dot{\theta}$, then $\dot{z}=g(\theta)-f(\theta)z$, and this system may be described in terms of a cylindrical phase space, the θ and z being the usual cylindrical coordinates on, say, $x^2+y^2=1$. It is shown that for each solution $\theta(t)$, the corresponding phase trajectory (or characteristic) has a finite positive limiting set. If $g(\theta)\neq 0$ for each θ, then the limiting set of each trajectory is some closed curve surrounding the phase cylinder; i.e., a limit cycle of the second kind. If $g(\theta)$ has only simple zeros, the system will have elementary critical points and the limiting sets may also include these points, ordinary limit cycles, or closed separatrices. (Received November 12, 1954.)

319. Domina E. Spencer: *Bessel wave functions*.

Bessel functions have been thoroughly studied but the *Bessel wave functions*, which occur when the Helmholtz equation is separated in parabolic coordinates, have hardly been considered. The Bessel wave equation is the Bôcher equation designated as $\{24\}$, while its degenerate case, the Bessel equation, is classified as $\{23\}$. The solutions are called *Bessel wave functions* $J_p(k, qz)$ and $Y_p(k, qz)$. The paper develops series expansions, asymptotic expansions, recursion formulas and considers orthogonality and integral representation. (Received November 8, 1954.)

320t. R. L. Sternberg: *Successive approximation and expansion methods in the numerical design of microwave dielectric lenses.*

Two essentially complete formal solutions suitable for computation are given for the problem of designing either zoned or unzoned microwave dielectric lenses of revolution having two finite and two infinite focal points pairwise symmetrically displaced about the axis of symmetry and perfect with respect to meridional rays when the refractive index of the lens, the diameter of the lens, the normal distance to the finite focal points, the off axis angle to the infinite focal points, and the wave length of the incident radiation are given. The first of these formal solutions is a characteristic value and successive approximation method while the second is a series expansion technique similar to the Cauchy-Kovalevsky method in the solution of analytic differential equations. The results are particularly useful for designing extremely short focal length and wide aperture lenses for scanning antenna applications. (Received November 15, 1954.)

321. H. F. Weinberger: *An error estimate for "box normalization" in the one-dimensional Schroedinger equation.*

An error estimate is given for the approximation of the negative eigenvalues λ_k of (1) $u''+(\lambda-q)u=0$ on $0<x<\infty$ with (2) $ru'(0)+su(0)=0$, where $q(x)$ is piecewise continuous and bounded, with $\lim_{x\to\infty} q=0$. The eigenvalues are approximated by the eigenvalues $\lambda_k(c, a)$ of (1), (2), and (3) $u'(a)+cu(a)=0$ on the interval $0\leqq x\leqq a$. It is known that $\lim_{a\to\infty} \lambda_k(c, a)=\lambda_k$, but it is here shown how to find an a such that the error is less than a prescribed ϵ. It is further known that $\lambda_k(0, a)\leqq\lambda_k\leqq\lambda_k(\infty, a)$ and that $\lambda_k(c, a)$ increases with c. Thus, for $c>0$, it is sufficient to make $\lambda_k(\infty, a)-\lambda_k(0, a)<\epsilon$. It is shown that if (4) $q_1\leqq q(x)\leqq q_2$ for $x\geqq b$, where q_1 and q_2 are constants, then for all k such that $\lambda_k(\infty, a)\leqq q_1-\beta^2$ one has (6) $\lambda_k(\infty, a)-\lambda_k(0, a)\leqq q_2-q_1+(q_1-q_m)(\exp(2\pi K/(4-K^2)^{1/2})-1)$, where $q_m=\min q$ and (7) $K=2[\exp\{2(a-b)\beta\}-1-2(a-b)\beta]^{-1}$. Since $q(x)\to 0$, b and then a may be chosen to make the right-hand side of (6) arbitrarily small. The inequality (6) is first proved when q is constant for $x\geqq b$ by explicit computation. Use is then made of the fact that the eigenvalues decrease if q is replaced by q_1 and increase if q is replaced by q_2 for $x\geqq b$, and that the eigenvalues of the modified problems differ by less than q_2-q_1. This work was supported by the U.S.A.F. through the Office of Scientific Research. (Received November 15, 1954.)

322. R. A. Willoughby: *A stability consideration for matrix inversion.* Preliminary report.

The statement, C is an approximate inverse of A, has many possible interpretations. Usually it means that for some definition of the norm of a matrix $N(B)\leqq\epsilon$ (B being one of the matrices $CA-I$, $AC-I$, $(AC+CA)/2-I$). For example: $N(B)=\max|bij|$ is a norm which is easy to calculate. Two factors omitted in the preceding definition are: (1) Does C possess properties A^{-1} is known to have (e.g. symmetry)? Are the number of significant digits in the various cij adequate? These questions arose in connection with the inversion of a 41×41 complex symmetric matrix on a high speed digital computer. The process involved replacing each complex number $a+ib$ by its matrix equivalent $\binom{ab}{-ba}$ and inverting the resulting 82×82 real matrix partially by partitioning and partially by elimination. An approximate inverse of a 12×12 principal sub-matrix did adequately represent a 6×6 complex symmetric matrix, but such was not the case for a 28×28 even though the ϵ seemed as small as could reasonably be expected in such a problem considering the accuracy of the input data. (Received November 8, 1954.)

Geometry

323. Shreeram Abhyankar: *The theorem of local uniformization on algebraic surfaces over modular ground fields.*

Let K be an algebraic function field of two independent variables over an algebraically closed ground field k of characteristic p. Let v be a valuation of K/k. One says that v can be uniformized if a projective model S of K/k can be found on which the center of v is at a simple subvariety of S whose dimension is equal to the dimension of v. The theorem of local uniformization states that all the valuations of K/k can be uniformized. Zariski proved this theorem for $p=0$ (Ann. of Math. vol. 40 (1939) pp. 639–689). In this paper the theorem is proved for $p\neq 0$. Assuming only the validity of the local uniformization theorem, Zariski has established: (1) the existence of a nonsingular projective model of K/k, (2) the existence of the arithmetic genus of K/k, and (3) the Hilbert-Zariski theorem for K/k, which states that if R is a finite integrally closed integral domain over k whose quotient field contains K, then $R\cap K$ is finite over k (unpublished). These three results, hitherto completely proved only for $p=0$, may be therefore now regarded as completely demonstrated for $p\neq 0$ as well. Further, let T be a rational map of a normal V^{*r} onto a nonsingular V^r without fundamental points on either variety. Then the branch locus B of T on V is pure $r-1$ dimensional; if $r=2$ and if Q is a simple point or normal crossing of B, and if P is a corresponding point on V^*, the local Galois group is characterized. (Received November 15, 1954.)

324. E. F. Allen: *The Feuerbach configuration applied to central conics.*

Tangents at three distinct points t_1, t_2, t_3, of a central conic A, form a triangle T, whose vertices are x_1, x_2, x_3, in which the conic is either inscribed or escribed. The author proved that the conic A is tangent to the nine-point conic D of the triangle T (E. F. Allen, *On a triangle inscribed in a rectangular hyperbola*, Amer. Math. Monthly vol. 48, p. 675), and thus extends Feuerbach's theorem to central conics as well as circles. The conic D is the homothetic of the conic A, with center $K=S_3\overline{S}_1/S_1$, having the homothetic ratio $1/(S_1\overline{S}_1-1)$. S_1, S_2, S_3 are symmetric functions of t_1, t_2, and t_3. Certain conics of the Feuerbach configuration are inverse with respect to the conic A. The author defines the inverse C''' of a conic C' with respect to a conic C'', where C' and C'' are conics of the same kind, having parallel axes, and shows how to obtain the equation of C'''. Thus if B is the nine-point conic of the triangle t_1, t_2, t_3 and C the conic circumscribing triangle T, it is shown that B and C are inverse conics with respect to the conic A. The method of proof is analytic using the notation of the extended inversive geometry (E. F. Allen *An extended inversive geometry*, Amer. Math. Monthly vol. 60, p. 233). (Received October 14, 1954.)

325t. P. O. Bell: *A projective curvature tensor for hypersurfaces.*

Let $x(u^1, u^2, \cdots, u^{n-1})$, $x'(u^1, u^2, \cdots, u^{n-1})$ denote corresponding points of two hypersurfaces V, V' of n-dimensional projective space. In the present note a projective curvature tensor $R_{\alpha\beta}$ of V at x with respect to V' at x' is defined which is shown to be a generalization of the well known Ricci curvature tensor of a hypersurface V in

Euclidean space. Among the results obtained is the following geometric characterization of the invariant form $\Phi = R_{\alpha\beta}du^\alpha du^\beta$. Let V'_μ denote the hypersurface generated by a point $y_\mu = x' - \mu x$ collinear with x, x'. Let X, Y_μ denote the points of V, V'_μ which correspond to the parameter values $u^\alpha + du^\alpha$ $(\alpha = 1, 2, \cdots, n-1)$ and let p, P denote $(n-1)$-planes tangent to V at x, X, respectively. Let z_μ, Z_μ denote the points of intersection of the line (y_μ, Y_μ) and the $(n-1)$-planes p, P, respectively. The principal part of the cross-ratio $(y_\mu, z_\mu, Y_\mu, Z_\mu)$ is the quadratic differential form denoted by ϕ_μ. Let μ_α $(\alpha = 1, 2, \cdots, n-1)$ denote the functions which correspond to the foci of the line (x, x') (of the hypercongruence generated by (x, x')). The invariant Φ is given by $\Phi = (n-2)\phi_0 - \sum \phi_{\mu_\alpha}$, in which ϕ_0 is the function ϕ_μ for $\mu = 0$ and the sum is extended over the range $\alpha = 1, 2, \cdots, n-1$. If the general coordinates of the point x are homogeneous rectangular cartesian coordinates $(1, x^1, x^2, \cdots, x^n)$ and x' is the point "at infinity" on the normal to V at x, Φ becomes the quadratic form whose coefficients form the Ricci curvature tensor of V. (Received November 16, 1954.)

326. W. M. Boothby: *Bochner's lemma for Hermitian manifolds.*

In a recent paper (Amer. J. Math. vol. 76 (1954) pp. 509–534) the author derived a form of Bochner's lemma for Hermitian manifolds. However, the form of the lemma obtained in this instance was unnecessarily complicated by the presence of a term involving the torsion, and here a better proof shows that in the Hermitian case the statement is formally the same as in the Kähler case. Although results relating curvature and Betti numbers do not appear to be any more accessible in consequence, two theorems of Bochner can be generalized to Hermitian manifolds, namely: 1. If T_{ij^*} is positive definite on a compact, Hermitian manifold M, then there are no analytic differential forms on M, and 2. If T_{ij^*} is negative definite on a compact, Hermitian manifold M, then M does not admit a one parameter group of analytic homeomorphisms. In both cases $T_{ij^*} = R_{ij^*hh^*}$ is one of the three "Ricci tensors" of the Hermitian curvature. (Received November 12, 1954.)

327B. Leo Branovan: *Umbilics on hyperellipsoids and related closed surfaces in four dimensions.* Preliminary report.

The subject of umbilics on hypersurfaces in S_4 has rarely been considered in mathematical literature. The only previous treatment of this subject is a paper by Sommer (*Focaleigenschaften quadratischer Mannigfaltigkeiten im vierdimensionalen Raum*, Math. Ann. vol. 53 (1900) pp. 113–160) in which he considered a confocal manifold in S_4. He proved synthetically that the confocal manifold cannot have real umbilics. However, he did not obtain the coordinates of the umbilics under any conditions. In this paper, the author investigates the conditions under which umbilics on hyperellipsoids and related closed surfaces in S_4 exist; then he actually obtains the coordinates of the real umbilics on these surfaces. Furthermore, he compares umbilics on ordinary surfaces in S_3 with those on hypersurfaces in S_4 and draws interesting conclusions. A differential geometry approach is used to solve the problem. (Received November 4, 1954.)

328. H. S. M. Coxeter: *On Laves' graph of girth ten.*

Consider, in Euclidean 3-space, the infinite sets of points whose coordinates are $(0, 0, 0)$, $(1, 2, 3)$, $(2, 3, 1)$, $(3, 1, 2)$, $(2, 2, 2)$, $(3, 0, 1)$, $(0, 1, 3)$, $(1, 3, 0)$ $(\text{mod}\, 4)$. Certain pairs are distant $2^{1/2}$, and by joining all such pairs one obtains an infinite graph of degree 3 and girth 10: the smallest circuit is a skew decagon. Spheres of diameter $2^{1/2}$, with these points as centres, form a "thin packing" which was discussed by F. Laves (Z. Kristallogr. vol. 82 (1932) p. 10). When the infinite space is reduced to a three-dimensional torus by identifying points whose coordinates differ by multiples of $4n$, the resulting finite graph has $8n^3$ nodes and $12n^3$ branches. The girth is still 10 when $n \geqq 3$, but only 8 when $n=2$. (Received October 25, 1954.)

329. S. I. Goldberg: *Note on metric manifolds with torsion.*

Let V_n be a compact n-dimensional manifold on which there is given a positive definite metric $ds^2 = g_{jk}dx^j dx^k$ and a metric connection Γ^i_{jk}, so that the covariant derivative $g_{jk;1}$ of g_{jk}, taken with respect to Γ^i_{jk}, vanishes. Γ^i_{jk} is the most general nonsymmetric connection that complies with the condition that $g_{jk;1}=0$. Denote the torsion tensor by $S_{jk}{}^i = (\Gamma^i_{jk}) - \Gamma^i_{kj})/2$ and assume that $S_{ji}{}^i = 0$. Consider a pseudo-harmonic vector $\xi^i(\xi_{i;j} = \xi_{j;i}$ and $\xi^i{}_{;i} = 0)$ and form the vector $G_{ij}\xi^{i;k}\xi^j$ where G_{ij} is an arbitrary symmetric tensor. Using Green's theorem and then putting $G_{ij} = \rho g_{ij}$ where ρ is a nonzero scalar one obtains $\int_{V_n} \rho[(E_{js} + E_{sj} - (\Delta\rho/\rho)g_{js})\xi^j\xi^s - 2(S_{krs} + S_{ksr})\xi^k\xi^{r;s} + (g_{rt}g_{su} + g_{ru}g_{st})\xi^{r;s}\xi^{t;u}]dv = 0$ where E_{jk} is the contracted curvature tensor referred to Γ^i_{jk} and $\Delta\rho$ is the Laplacean of ρ. This gives a generalization of Theorem 7.9 (cf. *Curvature and Betti numbers*, K. Yano and S. Bochner) in case V_n is orientable. In a similar manner one can generalize Theorem 7.14 (loc. cit.) by considering pseudo-Killing vector. These results can easily be extended to cover pseudo-harmonic and pseudo-Killing tensors. Furthermore, ρ can be adjusted to yield some interesting formulae. (Received November 15, 1954.)

330. Frank Harary: *On the notion of balance in signed graphs.*

The concepts and results of this note were motivated by some problems in social psychology. A signed graph (s-graph) results when one takes an ordinary linear graph and regards some of its lines as positive and the remaining lines as negative. A cycle of an s-graph is positive if it contains an even number of negative lines. An s-graph is balanced if all its cycles are positive. It is shown that a necessary and sufficient condition for an s-graph to be balanced is that its point set can be divided into two disjoint subsets such that each positive line joins two points of the same subset and each negative line joins two points of different subsets. (Sponsored by a grant from the Rockefeller Foundation to the Research Center for Group Dynamics.) (Received November 12, 1954.)

331. C. C. Hsiung: *An integral formula for a hypersurface in a Riemannian space.*

Let V^n be an orientable hypersurface twice differentiably imbedded in a Riemannian space R^{n+1} of $n+1 \geqq 3$ dimensions; $\kappa_1, \cdots, \kappa_n$ the n principal curvatures at a point P of V^n; and M_1 the first mean curvature of V^n at P defined by $M_1 = \kappa_1 + \cdots + \kappa_n$. Let dA be the area element of V^n at P, and p the scalar product of the unit normal vector of V^n at P and the position vector of P with respect to any orthogonal frame in R^{n+1}. The purpose of this paper is to extend a recent result of the author [Bull. Amer. Math. Soc. Abstract 60-4-441] by showing that for an orientable hypersurface V^n with a closed boundary V^{n-1} of dimension $n-1$ the integral $\int_{V^n} (M_1 p + n)dA$ can be expressed as an integral over the boundary V^{n-1} and hence that the integral vanishes for a closed orientable V^n. (Received November 9, 1954.)

332. Wilhelm Klingenberg: *Configurations in euclidean planes.*

Removing a line L_∞ and the points on it from a projective plane π one gets a euclidean plane π^*. It is well known (M. Hall, Trans. Amer. Math. Soc. vol. 54) that by choosing a fundamental quadrilateral, which has two vertices on L_∞, π^* may be described by means of the elements of a ternary ring T. T is a field (or a commutative field) if and only if the Desargues Theorem (D) holds in π^* (or the Pappos Theorem (P)). The author denotes by (R) ("Reidemeisterfigur") or by (T) ("Thomsenfigur") the configuration which corresponds to the associative law or the commutative law of multiplication in a ternary ring T. Then the following theorem holds: (R) is equivalent to (D) and (T) is equivalent to (P). This means: If for every ternary ring T, defined by π^*, the associative law of multiplication (or the commutative law of multiplication) holds, then T is a field (or a commutative field) and vice versa. It is well known that (R) is a consequence of (T). Thus this theorem includes another proof of the theorem of Hessenberg, i.e. that (D) in a consequence of (P). (Received November 5, 1954.)

333. L. F. Markus: *The universe and Mr. Euler.*

It is known that a (C^∞) differentiable n-manifold M admits a continuous line element field if and only if M admits a differentiable, covariant, symmetric, second order tensor field with signature of one at each point, that is, a Lorentz metric on M. It is shown here that a compact differentiable n-manifold M admits a continuous line element field, and thus a Lorentz metric, if and only if the Euler characteristic is zero. A cosmological interpretation of this result for a bounded space of general relativity is that the characteristic of the universe is zero. In particular, the universe is not the product of two pretzels. (Received November 15, 1954.)

334t. Albert Nijenhuis: *On local and infinitesimal holonomy groups.*

The methods and results of the author's paper, which dealt with the connections in linear bundles only [K. Nederl. Akad. Wetensch. A vol. 56 (1953) pp. 233–249; A vol. 57 (1954) pp. 17–25] can be extended to the general case of bundles with Lie groups. Some results along that line were obtained independently by Ambrose and Singer [Trans. Amer. Math. Soc. vol. 75 (1953) pp. 428–443]. The local holonomy group $h^*(p)$ at $p \in M$ is the intersection of all restricted holonomy groups $h^0(U, p)$ at the base point p, where U ranges over the open neighborhoods of p; the infinitesimal holonomy group $h'(p)$ is determined by explicit expressions in terms of the curvature and its derivatives at p. Some of the results are: (1) $h'(p) \subset h^*(p)$; (2) If dim $h^*(x)$ is constant for all $x \in M$, then $h^0(M, p) = h^*(p)$; (3) If $h'(x)$ and $h^*(x)$ are equal for all $x \in M$, then $h^0(M, x)$ is equal to these; (4) If the bundle and the connection are analytic, then $h^0(M, p) = h^*(p) = h'(p)$; (5) If Ω^* is the set of $x \in M$ where dim $h^*(x)$ is discontinuous, and if Ω' is the same for $h'(x)$, then $\Omega^* \subset \Omega'$; both are closed in M and have no interior points. (Received November 16, 1954.)

335. T. K. Pan: *Angular spread and indicatric torsion.*

Different forms of angular spread, analogous to those of geodesic curvature due to Beltrami, Bonnet and others, are derived; and a modified expression of Gauss-Bonnet theorem is obtained with respect to a vector field in a surface. The arc-rate of turning along a space curve of the plane, which is determined by a vector of a field in space and its derived vector along the curve, is defined as the torsion of the field along the curve. This leads to a generalization of the Frenet formulas in the theory of space curves. When a vector field is in a surface, a torsion, called the indicatric torsion of the field along the curve, is defined to include the geodesic torsion of a curve as a special case. It is found that the indicatric torsion of the asymptotic field of a surface vector field along the curve of the field is numerically equal to the principal curvature of the field and that the indicatric torsion of a vector field along a curve is zero if and only if the curve is the line of curvature of the field, the first part being a generalization of the theorem of Enneper. (Received November 12, 1954.)

336. Valdemars Punga: *The derivation of O. Varga's affine connection by H. Friesecke's method.*

O. Varga in *Über affinzusammenhängende Mannigfaltigkeiten von Linienelementen inbesondere deren Äquivalenz*, Publicationes Mathematicae, Tomusl, Debrecen, 1949, introduces the parallel displacement of a vector ξ^i in the space of line elements (x^i, x'^i) by $D\xi^i = d\xi^i + C^i_{kj}\xi^k\pi^j(d) + \Gamma^{*i}_{kj}\xi^k dx^j$. In the present paper the author shows that applying to the space of line elements the same arguments as H. Friesecke in his *Vektorübertragung, Richtungsübertragung, Metrik*, Math. Ann. vol. 94 (1925) one will naturally arrive at O. Varga's affine connection defined by C^i_{kj} and Γ^{*i}_{kj}. (Received November 5, 1954.)

337t. P. V. Reichelderfer: *On the product of two essentially absolutely continuous transformations.*

Given bounded domains D' and D'' in euclidean n-space let T' be a continuous transformation from D' into D'' and let T'' be a continuous transformation from D'' into a bounded portion of n-space. Assume that both T' and T'' are essentially absolutely continuous. (T. Radó and P. V. Reichelderfer, *On n-dimensional concepts of bounded variation, absolute continuity and generalized jacobian*, Proc. Nat. Acad. Sci. U.S.A. vol. 35 (1949) pp. 678–681). Then a necessary and sufficient condition in order that $T''T'$ be essentially absolutely continuous is that $T''T'$ be essentially of bounded variation. If $j_e(u', T')$ and $j_e(u'', T'')$ are the essential jacobians for T' and T'' respectively, then a necessary and sufficient condition in order that $T''T'$ be essentially absolutely continuous is that $j_e(T'u', T'')\, j_e(u', T')$ be summable in D'. If $T''T'$ is essentially absolutely continuous then its essential jacobian $j_e(u', T''T')$ is equal to $j_e(T'u', T'')\, j_e(u', T')$ almost everywhere in D'. (Received November 8, 1954.)

338. H. W. E. Schwerdtfeger: *An approach to non-Euclidean geometry.*

The plane D of operation is the whole or a part of the complete plane of the complex numbers which serve as points in the geometries. The basic ideas are prescribed by F. Klein's Erlanger Program. Euclidean geometry is thus generalized by: 1. Assuming a group G of transformations of D into itself as motions; 2. Defining congruence with respect to G; 3. Defining straight lines, circles and cycles by its one-parameter subgroups; 4. Introducing distance as a "two-point invariant." The group G will be restricted by the following assumptions: A. G is simply, not twofold transitive. B. For every element $g \in G$ ($g \neq$ unit element) the powers g^s (s real) form a continuous one-parametric subgroup of G. C. Every stability subgroup is cyclic. D. One of the stability subgroups is similar to a rotation group. *Theorem*: There is a unique two-point invariant $f(z_1, z_2)$ of the group G. E. A transformation $h_z \in G$ for which $h_z(z) = 0$ has the property $|h_z(0)| = |z|$. F. The transformations of G are conformal. Under these restrictions on the motion group G the domain D is either the inner of a circle about 0 (hyperbolic geometry), or the plane without infinity, or the completed plane (spherical geometry). (Received November 12, 1954.)

339. Saly Ruth Struik: *Affine geometry*. II.

Enriques (Encyklopädie d. Math. Wiss. III, 1, 1) gives an axiomatics of area based on Euclid's axioms of congruence utilizing equality of decomposition for two equal triangles and utilizing the square for measure. For three dimensions Enriques only mentions that according to Dehn's proof the analogous definition of equality of volume for tetrahedrons fails. No alternative is proposed. Hilbert adds to this fact Dehn's assertion that a foundation for three-space will have to utilize another principle, e.g. the Cavalieri principle. In *Affine geometry*, I this has been done, by giving an axiomatics of affine geometry and basing equality of triangle and tetrahedron on the notion of affine two-space reflection and affine three-space reflection respectively. In the present paper two theorems of Euclidean geometry are added based on purely affine findings of the first paper. (1) Two triangles are of equal area if and only if they can be placed into a position where they are each other's two-space affine reflection. (2) Two tetrahedrons are of equal volume if and only if they can be brought into a position where they are both the affine three-space reflection of a third one. Proof is by construction. (Received November 8, 1954.)

340. N. Z. Wolfsohn: *On differentiable maps of Euclidean n-space into Euclidean m-space*. Preliminary report.

Let R be a subset of E^n and let $f: R \rightarrow E^m$ where $f \in C^1$. Then with $(x_1, \cdots, x_n)$ and $(y_1, \cdots, y_m)$ coordinate systems in E^n and E^m respectively, $\partial f(P)/\partial x_i$ is regarded as a vector in E^m. Using an extension of methods of Whitney (*The general type of singularity of a set of* $2n-1$ *smooth functions of n variables*, Duke Math. J. vol. 10 (1943) pp. 161–172) it can be shown that given f, there exists f^*, approximating f as closely as desired and such that the rank of f^* at every point P is at least min $(m, n)-1$ if $2(m+2)/3 < n \leqq m$ or if $m < n$ and $m < (n+4)/2$. If $n \geqq m$, if the rank of f is $m-1$ for all P, and if for some P, $\partial f(P)/\partial x_{m+i}=0$ $(i=0, \cdots, n-m)$ while $\partial f(P)/\partial x_1, \cdots, \partial f(P)/\partial x_{m-1}$ span an $m-1$ dimensional space parallel to that spanned by $(y_1, \cdots, y_{m-1})$ the quadratic rank and signature of f at P is defined to be equal to the rank and signature of the quadratic form $\sum_{i,j=0}^{n-m} (\partial^2 y_m(P)/\partial x_{m+i}\partial x_{m+j})w_i w_j$. F may be approximated by f^* so that the quadratic rank of f^* at any singular point is $\geqq n-m+1-p$ where p is the greatest integer such that $m \geqq (p^2+p+2)/2$. If $n \leqq m$, if the rank of f at every singular point is $n-1$, and if $\partial f(P)/\partial x_n=0$ at a singular point P, then f^* may be found, approximating f such that a certain matrix has rank m. In certain of these cases, local coordinate systems may be found which give the map a particularly simple form. (Received November 10, 1954.)

Logic and Foundations

341. J. W. Addison: *Analogies in the Borel, Lusin, and Kleene hierarchies*. II. Preliminary report.

The effective Lusin hierarchy (ω_1 levels) on N (natural numbers) and on N^N is defined (using images of recursive transforms from N^N, complements, and recursively enumerable unions). Recursive sieves and recursive operations (A) are applied in the theory. On N and N^N the finite effective Lusin (effective projective) hierarchy is equivalent to the Kleene function-quantifier $(\mathcal{J})$ hierarchy. Parallel theorems about the projective hierarchy on N^N and $\mathcal{J}$ hierarchies on N and N^N are demonstrated by essentially identical proofs. A set of recursive predicates which are 'notations' for 'effective ordinals' is defined; it is an $\forall^1$- but not an $\exists^1$-set of N^N, i.e. it is expressible in $\hat{\phi}(\psi)(Ey)R(\phi, \psi, y)$ but not in $\hat{\phi}(E\psi)(y)R(\phi, \psi, y)$ form, where $\phi, \psi \in N^N$, $y \in N$, and R is recursive. Using this, any two disjoint $\exists^1$-sets of N are separable by an effective Borel set (and hence by an $\exists^1 \cap \forall^1$-set). Kleene's theorem (any $\exists^1 \cap \forall^1$-set is an effective Borel set) follows as a special case. There exist two disjoint $\forall^1$-sets of N not separable by any $\exists^1 \cap \forall^1$-set. The analogy of the projective with the $\mathcal{J}$ hierarchy thus lacks the imperfections of the earlier analogy with the Kleene (number-quantifier) hierarchy (cf. Kleene, Neder. Akad. Wetensch. vol. 53 (1950) pp. 800–802). (Received November 15, 1954.)

342t. Solomon Feferman: *Sum operations on relational systems.*

The arithmetic theory of a given relational system $\mathfrak{A} = \langle A, R_0, \cdots, R_{n-1} \rangle$ is here denoted by $Th(\mathfrak{A})$; the fact that a sentence ϕ of $Th(\mathfrak{A})$ is true in $\mathfrak{A}$ is expressed by $\vdash_{\mathfrak{A}} \phi$ (cf. Mostowski, Journal of Symbolic Logic vol. 17 (1952) pp. 2–3). The relation $\equiv$ of arithmetical equivalence between systems is defined by Tarski (Proc. Int. Cong. Math. 1950, vol. 1, p. 712). For any two systems $\mathfrak{A}$ and $\mathfrak{B}$ with $A \cap B = \Lambda$ it is possible to define a new system $\mathfrak{A} + (\phi)\mathfrak{B}$ (where ϕ is any finite sequence of formulas out of a certain theory $Th([\mathfrak{A}, \mathfrak{B}])$ comprehending $Th(\mathfrak{A})$, $Th(\mathfrak{B})$) having the following properties: I^+. $+(\phi)$ preserves $\cong$. II^+. If θ is any sentence of $Th(\mathfrak{A} + (\phi)\mathfrak{B})$, there exist effectively constructible sentences $\psi_0, \cdots, \psi_k$ of $Th(\mathfrak{A})$ and $\chi_0, \cdots, \chi_k$ of $Th(\mathfrak{B})$ such that $\vdash_{\mathfrak{A}+(\phi)\mathfrak{B}} \theta$ iff for some $j \leq k$, $\vdash_{\mathfrak{A}} \psi_j$ and $\vdash_{\mathfrak{B}} \chi_j$. III^+. $+(\phi)$ preserves $\equiv$. IV^+. If $Th(\mathfrak{A})$, $Th(\mathfrak{B})$ are both decidable, so also is $Th(\mathfrak{A} + (\phi)\mathfrak{B})$. By suitable specifications of ϕ, $\mathfrak{A}$ and $\mathfrak{B}$, the operation $+(\phi)$ may be made to agree with the cardinal sum or the ordinal sum. Thus IV^+ generalizes a result of Beth (Collection de Logiques Mathématique Série A (Paris-Louvain 1954) fasc. V pp. 29–35) concerning the ordinal sum of simply ordered systems. (Received November 12, 1954.)

343t. Solomon Feferman: *Product operations on relational systems.*

For notation see preceeding abstract. By selecting a certain class P out of the class of formulas of $Th([\mathfrak{A}, \mathfrak{B}])$, it is possible to define a new system $\mathfrak{A} \times (\phi)\mathfrak{B}$ for any systems $\mathfrak{A}$ and $\mathfrak{B}$ and any finite sequence ϕ of formulas of P. The properties I^+-IV^+ of the operation $+(\phi)$ hold as well for the operation $\times(\phi)$ (under the restrictions to P given above), giving new theorems $\mathrm{I}^\times$–$\mathrm{IV}^\times$. By suitable specification of ϕ, $\mathfrak{A}$ and $\mathfrak{B}$, the operation $\times(\phi)$ may be made to agree with the cardinal product or the ordinal product or with a mixture of these operations. Using this last fact leads to an interesting application of $\mathrm{IV}^\times$: For each integer n, let W_n be the set of ordinals less than ω^n, $<_n$ the well-ordering relation in W_n, and S_n and N_n the ternary relations in W_n corresponding respectively to ordinary and natural addition of ordinals. $\langle W_n, N_n, <_n \rangle$ may be expressed as a product $\times(\phi)$ of n identical factors $\langle W_1, S_1, <_1 \rangle$ in which S_1 is cardinally multiplied, $<_1$ ordinally multiplied. Since $Th(\langle W_1, S_1, <_1 \rangle)$ is decidable by a result of Presburger (Comptes-rendus du I Congres des Mathematiciens des Pays Slaves (Warszawa 1929) pp. 92–101 and 395), and since S_n is elementarily definable in terms of N_n and $<_n$, it follows that $Th(\langle W_n, S_n \rangle)$ is decidable for each integer n. The general problem (originating with Tarski), whether the theory of addition of arbitrary ordinals is decidable, remains open. (Received November 12, 1954.)

344B. Richard Montague: *Non-finitizable and essentially non-finitizable theories.*

For terminology consult Tarski, Mostowski, Robinson, *Undecidable theories*. A theory is ***non-finitizable*** if it is not finitely axiomatizable, ***essentially non-finitizable*** if no consistent extension with the same constants is finitely axiomatizable. *Theorem* 1. *Zermelo-Fraenkel set-theory is essentially non-finitizable.* A theory T is *strongly arithmetical* if, intuitively speaking, Peano's arithmetic (with $+$, $\cdot$, and 0) can be interpreted in T in such a way that induction and definition by recursion (involving, respectively, arbitrary formulas of T and arbitrary terms definable in T) are permissible. *Theorem* 2. *If T is strongly arithmetical and Φ either is a universal sentence interpretable in T or follows logically from such a sentence, then the consistency of Φ can be proved in T.* With the aid of Theorem 2, Theorems 3 and 4 are proved. *Theorem* 3. *The following theories are essentially non-finitizable*: (i) *Peano's arithmetic,* (ii) *Zermelo's set-theory without the axiom of infinity but with a form of the axiom of constructibility. Theorem* 4. *If Zermelo's set-theory without the axiom of infinity is consistent, then it is non-finitizable.* Theorems 2 and 3(i) are obtained by generalizing a method of Mostowski (Fund. Math. vol. 39 (1952) pp. 133–158). (Received November 13, 1954.)

345B. R. M. Robinson: *Arithmetical representation of recursively enumerable sets.*

A set S of natural numbers is called recursively enumerable if there is a general recursive function $F(x, y)$ such that $y \in S \leftrightarrow (\bigvee x)[F(x, y) = 0]$. Martin Davis has shown [J. Symbolic Logic vol. 18 (1953) pp. 33–41] that every such set S can be represented in the form $y \in S \leftrightarrow (\bigvee b)(\bigwedge w)[w \leqq b \rightarrow (\bigvee x_1, \cdots, x_\lambda)\ P(y, b, w, x_1, \cdots, x_\lambda) = 0]$, where P is a polynomial with integer coefficients, but no estimate for λ was given. In the present paper, it is shown that it is possible to take $\lambda = 4$. (On the other hand, it is easily seen that one cannot always take $\lambda = 0$, so that the smallest possible λ is 1, 2, 3, or 4.) The proof is based on the fact that the set S, if non-empty, is the range of a primitive recursive function of one variable. Now all such functions can be obtained from $x+1$ and Kx by repeated use of the formulas $Fx = J(Ax, Bx)$, $Fx = BAx$, and $Fx = B^x0$ [Bull. Amer. Math. Soc. Abstract 61-1-162]. Applying these three formulas in a systematic way, we obtain a sequence F_nx which enumerates all primitive recursive functions. The condition $(\bigvee x)[y = F_nx]$ is then expressed arithmetically. Various devices are used to reduce the number of quantifiers needed. (Received October 25, 1954.)

346. J. R. Shoenfield: *The independence of the Axiom of Choice.*

The Axiom of Choice is not provable from axioms A–C of Godel's *The consistency of the continuum hypothesis.* The proof uses an extension of techniques developed by Fraenkel and Mostowski. (Received November 12, 1954.)

347B. R. L. Vaught: *On the arithmetical equivalence of free algebras.*

Let $\mathcal{K}$ be a class of algebras $\mathfrak{A}$ formed by a set and a binary operation over the set, and suppose that $\mathcal{K}$ is closed under taking of isomorphic images, subalgebras, and arbitrary direct products. Then the free algebra $\mathfrak{C}$ over $\mathcal{K}$ with any given cardinal number of generators (as defined in Birkhoff, *Lattice theory,* p. viii, with the added condition that $\mathfrak{C} \in \mathcal{K}$) exists and is unique up to isomorphism. *Theorem*: Any two free algebras $\mathfrak{A}$ and $\mathfrak{B}$ over $\mathcal{K}$, each with infinitely many generators, are arithmetically equivalent (i.e., the same sentences of a language L with the equality symbol, a binary operation symbol, sentential connectives, and quantifiers, hold in $\mathfrak{A}$ and $\mathfrak{B}$). Indeed, if, moreover, $\mathfrak{A}$ is a subalgebra of $\mathfrak{B}$, then any formula of L which is satisfied in $\mathfrak{A}$ by some elements is satisfied in $\mathfrak{B}$ by the same elements. (Thus $\mathfrak{B}$ is an arithmetical extension of $\mathfrak{A}$, an unpublished notion due to Tarski.) The theorem may be extended to algebras formed by a set and arbitrarily many finitary operations. These investigations arose from a still unresolved conjecture of Tarski's that any two free groups with at least *two* generators are arithmetically equivalent. (Received October 25, 1954.)

Statistics and Probability

348. D. G. Austin: *Some differentiation properties of Markoff transition probability functions.*

Let $p_{ij}(t)$ denote the transition probability functions of a denumerable Markoff chain satisfying $\lim_{t\to 0} p_{ij}(t)=\delta_{ij}$. It is known that $p_{ii}(t)$ has a derivative, finite or infinite, at $t=0$ which will be denoted by $-q_i$. It is shown that if either q_i or q_j is finite, then $p_{ij}(t)$ has an ordinary derivative almost everywhere and has a difference quotient bounded above by q_i and below by max $(-q_i, -q_j)$. If q_i is finite each of the series $\sum_j D^*p_{ij}(t)$ and $\sum_j D_*p_{ij}(t)$ converge absolutely to numbers between q_i and $-q_i$. In the special case where the q_i are bounded, the $p_{ij}(t)$ have a Taylor series expansion and the higher order derivatives satisfy a generalized Kolmogoroff differential equation $D^{(m+n)}p_{ij}(t_1+t_2)=\sum_k D^{(m)}p_{ik}(t_1)D^{(n)}p_{kj}(t_2)$. (Received November 10, 1954.)

349t. H. W. Becker: *Lattice interpolation between rhyme schemes and factorial words.*

If $@_n$, W_n, and $n!=F_n$ enumerate *RS*, 2-limit *FW* (Bull. Amer. Math. Soc. vol. 57 (1951) p. 297), and *FW*, then $e^{t@}=\exp\ (e^t-1)$, $e^{tW}=\sec^2 t+\sec t \tan t$, and $e^{tF}=1/(1-t)$, with $@_n<W_n<F_n$, $n\gtreqqless 4$. But $@_n\not\subset W_n$, $n>2$, *RS* with letters rhymed more than once not being members of *W*. $@_n\subset I_n\subset F_n$ is satisfied by I_n, the n-letter factorial words in which r is both the variety of letters and the highest of them. The subset of F_{n+1} with s different letters, r highest, is $F_{n+1,r;s}=sF_{n,r;s}+(r-s+1)F_{n,r;s-1}+F_{n,r-1);s-1}$, the) denoting summation from 1 through $r-1$. A valuable check is the table of $F_{n+1,r}=rF_{n,r}+F_{n,r-1)}$. In particular, $F_{n,r;2}=2^{n-r+1}-1$, $F_{n,n;s}=F_{n-1;s-1}$ (a crosscheck, congruences due to Carlitz and Riordan, Duke Math. J. vol. 20 (1953) p. 341), and $F_{n,r;r}=I_{n,r}=I_{n,n-r+1}$. Independent formulation of I_n seems impracticable, so its recurrence and generating functions are unknown, beyond $I_{n,1}=1$, $I_{n,2}=2^{n-1}-1$, $I_{n,3}=1+3(I_{n-1,3}+I_{n-2,2})$. Breakdowns of I_n by terminal letter, number of singletons, and a's, and position of last singleton, and a, are tabulated. The latter is curious, in that the table is symmetrical but montone *de*creasing to midtable. $0\leqq n\leqq 10$, $I_n=1$, 1, 2, 5, 16, 63, 294, 1585, 9692, 66275, 501106. $W_n=I_n=S_n$, $n<5$; $W_n<I_n<S_n$, $n\gtreqqless 5$: where $S_{n+1}=$superfactorial $n=(F+1)^n=1+nS_n\sim n!e$. By Erdélyi et al., *Higher transcendental functions*, 1953, 1.18(2), 1.20(9), 1.21(8, 14), $W_n\sim 2^{n+7/2}n^{n+1/2}(n+1)/e^n\cdot\pi^{n+3/2}$, so $W_n/W_{n-1}\sim(n+1)2/\pi$. Empirically $I_n\sim 1.01(W_n^2\cdot S_n)^{1/3}$, a cubic mean, so $I_n/I_{n-1}\sim n(2/\pi)^{2/3}=0.74n$. (Received November 12, 1954.)

350t. Volodymyr Bohun-Chudyniv: *On a new method of constructing canonical sets of Latin $2^k\times 2^k$ squares $(k\geqq 2)$.*

In the works of W. L. Stevens and R. S. Bose and K. R. Nair is given a method of construction, by using some properties of Galois fields, for each value of power k, of only one canonical set of Latin squares that can be orthogonized. In the author's paper (see Proceedings of the International Congress of Mathematicians, 1954, Amsterdam, pp. 281–282) was given with the help of author's schemes of K-nions, a general method of determining for each value of K not only one canonical set of Latin

$2^k \times 2^k$ squares, but all possible Latin squares that can be orthogonized, and it was shown that canonical sets of this type, determined by Bose and Stevens, can be obtained from the author's schemes as their separate cases. In the present paper the author determines another method of constructing canonical sets of Latin $2^k \times 2^k$ squares ($k \geqq 2$), simpler than the aforementioned methods, by using pairs of mutually conjugate solutions in positive integers from the following expressions: (I) $1, X, Y$; $1, 1+2^k-Y, X+2^k-Y$; $1, Y-X+1, 1+2^k-X$ which satisfy the following conditions: (1) $X < Y$, and all values of elements (I) in the second and third triples of both solutions must be different; (2) in the Latin square pertaining to each of the solutions, for each pair $A_{\beta,\alpha}; A_{\gamma,\alpha}$ belonging to any given column, exists a reciprocal pair of these elements in another column of the same row. Illustrative examples for $K=3, 4, 5, 6$, and 7 have been worked out by the author. (Received November 15, 1954.)

351*t*. Volodymyr Bohun-Chudyniv: *On determining all possible orthogonalizable sets of Latin $2^3 \times 2^3$ squares and pertaining to them completely orthogonal Latin squares.*

Using the known Bose and Nair's method, pointed out in paper *On complex sets of Latin squares* (Sankhya vol. 5, p. 34), one can obtain for every value of K only one set with two mutually conjugate Latin $2^k \times 2^k$ squares, called a *canonical set*, which can be orthogonalized. In Ronald Fisher's *The design of experiments* (Oliver and Boyd, Edinburgh, 4th ed., 1947, p. 81) is pointed out an example of orthogonalizable Latin $2^3 \times 2^3$ square, which cannot be obtained by the method mentioned above. By the author's methods pointed out in the paper, *On a general method for constructing completely orthogonal $2^k \times 2^k$ square by using closed orthogonal systems of K-nions* (see Proceedings of the International Congress of Mathematicians, 1954, pp. 281–282), one can determine for every k all possible orthogonalizable sets of Latin $2^k \times 2^k$ squares and pertaining to them completely orthogonal Latin $2^k \times 2^k$ squares. Bose and Stevens' *canonical sets* and Fisher's example mentioned above can be obtained from the author's schemes as separate cases. The aims of this paper are: (1) To determine the number of all possible sets with two mutually conjugate Latin $2^3 \times 2^3$ squares that can be orthogonalized, and to construct all of them; (2) To prove the existence of not only one completely orthogonal Latin $2^3 \times 2^3$ square for each orthogonalizable Latin $2^3 \times 2^3$ square, but 8. (3) To give a method for constructing all completely orthogonal Latin $2^3 \times 2^3$ squares for each type of orthogonalizable Latin $2^3 \times 2^3$ squares and to construct them. (Received November 17, 1954.)

352. Maria Castellani: *Fantappiè's analytical indicators and Levy's characteristic functions.*

This is a preliminary investigation on relations among Fantappiè's "analytica functionals" and Levy's characteristic functions. Given on R_n the distribution function $F(x_1, x_2, \cdots, x_n)$ of n random variables, the linear operators to be considered are: $E[g(z_1, z_2, \cdots, z_n; x_1\ x_2 \cdots x_n] = \int_R g dF = f(z_1, z_2, \cdots, z_n)$. F.A. indicators are given as follows: $u(z_1, z_2, \cdots, z_n) = E(\prod 1/(z_i - x_i))$ (antisymmetric indicators); $v(z_1, z_2, \cdots, z_n) = E(\prod 1/(1-z_i x_i))$ (symmetric indicators); $w(z_1, z_2, \cdots, z_n) = E(1/(1+\sum z_i x_i))$ (projective indicators). Relations among indicators are given by using convenient operators. Relations among F.A. indicators and Levy's characteristic functions are easely obtained when the n random variables are independent. The operator to be used is a product of nR_1 transforms, viz: $\phi(it) = E(e^{itx}) = e^{t\dot{x}} w(i\dot{x}) = e^{it\dot{x}} w(\dot{x})$. For random variables not independent the projective indicator is advisable. When F.A. indicators are regular at the origin the problem of moment could be solved by known procedures. (Received November 10, 1954.)

353. K. L. Chung: *Continuous parameter Markov chains.* Preliminary report.

In part one only the case of stable states is considered, namely all $q_i = -p'_{ii}(0+) < \infty$. A complete description of the sample functions of such a chain is given. From this we can define the successive intervals of sojourn and the various random variables connected therewith. Recurrence and ergodicity properties follow, in the same pattern as in the discrete parameter case. Many known results in the latter case, including a number of them in my *Contributions to the theory of Markov chains* (Journal of Research, Bureau of Standards vol. 50 (1953) pp. 203–208; Trans. Amer. Math. Soc. vol. 76 (1954) pp. 397–419) are extended to the continuous parameter case. Comparisons with discrete skeletons are given. The main limit theorem is an extension of Doeblin's ratio theorem, from which the average limit theorem follows. An ergodic theorem in the ratio form is proved in any recurrent class. This is preceded by a discussion of certain stochastic integrals. Some of these results have been extended to instantaneous states (with $q_i = \infty$) and will be given in part two. (Received September 20, 1954.)

354. Joseph Gillis: *Random walk with correlation.*

Random walk on a d-dimensional lattice is investigated such that, at any stage, the probabilities of the step being in the various possible directions depend upon the direction of the previous step. The motion may be characterized by a generating function which is here derived. The generating function is then used to obtain some general properties of the walk. Certain special cases are considered in greater detail. The existence of recurrent points is investigated in particular, and it is shown that Pólya's result (Math. Ann. vol. 84 (1921) pp. 149–160) (that every point is a recurrent point for $d=1$, 2 but that there are no such points for $d \geqq 3$) is unaffected by the correlation between adjacent steps. (Received November 1, 1954.)

355t. H. S. Konijn: *Properties of two classes of bivariate distribution functions.*

Let F denote the joint distribution of two independent random variables Y and Z. The paper investigates properties of (1) the joint distribution F_λ of the linearly transformed random variables Y_λ and Z_λ, of (2) the distributions (previously examined by Fréchet) F_+ [and F_-] of random variables which have the same marginal distributions as Y and Z but which are nondecreasing [nonincreasing] functions of each other, and (3) mixtures F_κ^+ [F_κ^-] of F and F_+ [F_-] with coefficient κ. (Received November 12, 1954.)

356t. H. S. Konijn: *On the power of certain tests for independence in bivariate populations.*

Let $\mathfrak{T}_0$ be the Spearman rank correlation test, $\mathfrak{T}_1$ the difference sign correlation test, $\mathfrak{T}_2$ the unbiased grade correlation test (which is asymptotically equivalent with $\mathfrak{T}_0$), $\mathfrak{T}_3$ the medial correlation test, and $\mathfrak{R}$ the ordinary (parametric) correlation test. (Whenever discussing $\mathfrak{R}$, existence of fourth moments is assumed.) Let the joint distribution of two independent random variables be a continuous function F. Properties of the power of the above-mentioned tests are found for the alternatives F_λ, F_κ^+ and F_κ (defined in the preceding abstract), particularly for alternatives "close" to the hypothesis of independence and for large samples. Against alternatives F_λ the efficiency of $\mathfrak{T}_3$ is found to depend strongly on local properties of the densities, which should invite caution, and the efficiency of $\mathfrak{T}_1$ with respect to $\mathfrak{T}_0$ is often unity. Against alternatives F_κ^+ and F_κ^- these tests are shown to be unbiased and equally efficient with efficiency independent of F, and the population parameters corresponding to the nonparametric test statistics are simply related to κ. Incidentally, Pitman's result on efficiency is extended in several directions. (Received November 12, 1954.)

357B. R. B. Leipnik: *Cycling a stationary process.*

If X is a stionary (wide sense) stochastic process, a the autocorrelation function of X, $a(s)=E[X_tX_{t+s}]$, and $E[X_t]=0$, and $\sum_{n=0}^{\infty} a(ns)$ converges for each $s>0$, then for each $T>0$, the processes $Y_{t,n}^{(T)}=(1/n)^{1/2}(X_t+X_{t+T}+\cdots+X_{t+(n-1)T})$ converge in distribution to a stationary process $Y^{(T)}$ on $[0, T)$ with an autocorrelation function $a^{(T)}$ such that $a^{(T)}(s)=a^{(T)}(T-s)$ and $\lim_{T\to\infty} a^{(T)}(s)=a(s)$. The autocorrelation matrix $(a^{(T)}(t_i, t_j))$ is simpler in several respects than $(a(t_i, t_j))$ which is important for the problem of estimation in stationary processes. (Received November 12, 1954.)

358B. Emanuel Parzen: *On the convergence of distribution functions at points of discontinuity.* I.

Let $F_n(\mathbf{x})$ for $n=1, 2, \cdots$ and $F(\mathbf{x})$ be distribution functions on Euclidean N-space E_N, with lower limit 0 and upper limit 1. For simplicity, results are stated only for $N=2$, although they hold for general N. F_n is defined to converge to F at the point $\mathbf{x}$ if $\Delta_n(\mathbf{x})=|F_n(x_1+0, x_2+0)-F(x_1+0, x_2+0)|+|F_n(x_1+0, x_2-0)-F(x_1+0, x_2-0)|+|F_n(x_1-0, x_2+0)-F(x_1-0, x_2+0)|+|F_n(x_1-0, x_2-0)-F(x_1-0, x_2-0)|\to 0$ as $n\to\infty$. Usually considered in the literature is "complete" convergence: $F_n\to^c F$ if $\Delta_n(\mathbf{x})\to 0$ at every continuity point of F. "Total" convergence is defined: $F_n\to^t F$ if $\Delta_n(\mathbf{x})\to 0$ at every point $\mathbf{x}$. A condition involving convergence of jumps is also introduced, denoted $F_n\to^J F$, which has the basic property that: $F_n\to^t F$ if, and only if, $F_n\to^c F$ and $F_n\to^J F$. Necessary and sufficient conditions are given for these modes of convergence to hold in terms of the conditions stated in the following abstract (which lead to sufficient conditions in terms of characteristic functions). In the notation of the following abstract, total convergence is equivalent to (IV), complete convergence to (V), and jump convergence to (III). The results for jump convergence lead to conditions for convergence of conditional distribution functions in certain cases. (Received October 26, 1954.)

359*t*. Emanuel Parzen: *On the convergence of distribution functions at points of discontinuity.* II.

Given a distribution function F on Euclidean 2-space E_2, define $D(F; \mathbf{a}, \mathbf{h})$ for $\mathbf{a}$ in E_2 and $\mathbf{h}=(h_1, h_2)>0$ by $h_1h_2D(F; \mathbf{a}, \mathbf{h})=\int_0^{h_1}\int_0^{h_2} dz_1dz_2\{F(a_1+z_1, a_2+z_2)-F(a_1+z_1, a_2-z_2)-F(a_1-z_1, a_2+z_2)+F(a_1-z_1, a_2-z_2)\}$. Given a sequence of distribution functions F_n, similarly define $D(F_n; \mathbf{a}, \mathbf{h})$. Probabilistic significance is given to the following possible hypotheses. Suppose, for every $\mathbf{a}$ in E_2, $D(F_n; \mathbf{a}, \mathbf{h})\to D(F; \mathbf{a}, \mathbf{h})$ as (I) $n\to\infty$, $h_1\to 0$, $h_2\to 0$; (II) $n\to\infty$, $h_1\to 0$, h_2 fixed; (III) $n\to\infty$, $h_1\to 0$, uniformly in $h_2>0$; (IV) $n\to\infty$, uniformly in h_1, $h_2>0$; (V) $n\to\infty$, h_1, h_2 fixed. The foregoing conditions lead to conditions in terms of the characteristic functions f and f_n of the distribution functions. In particular, (IV) holds if $|f_n(\mathbf{t})-f(\mathbf{t})|\to 0$ as $n\to\infty$ uniformly in

$t \in E_2$, and (IV) holds if $|f_n(t)-f(t)| \to \infty$ as $n \to \infty$ and $\|t\| \to \infty$. (Received October 26, 1954.)

360. J. M. Shapiro: *Error estimates for certain probability limit theorems.*

Consider a system of random variables (x_{nk}) with distribution functions F_{nk}, $k=1, 2, \cdots, k_n$; $n=1, 2, \cdots$, such that for each n, $x_{n1}, \cdots, x_{nk_n}$ are independent. Let $S_n = x_{n1} + \cdots + x_{nk_n}$ and let $F_n(x)$ be the distribution function of S_n. Under the assumption that the x_{nk} have two moments, estimates are obtained on $M_n = \sup_{-\infty < x < \infty} |F_n(x) - F(x)|$ where $F(x)$ is any infinitely divisible distribution with mean μ and finite second moment such that $dF(x)/dx$ is bounded. Let $G(u)$ be the nondecreasing function given by Kolmogorov's formula for the representation of the characteristic function of $F(x)$. The estimates involve $|\sum_{k=1}^{k_n} \int_{-\infty}^{x_i} \mu^2 dF_{nk}(\mu + E[x_{nk}]) - G(x_i)|$ evaluated at a finite number of points x_i, the variances of the x_{nk}, and $|E[S_n] - \mu|$. It is shown that under necessary and sufficient conditions for $F_n(x)$ to approach $F(x)$, the bounds on M_n obtained approach zero as n becomes infinite. In the special case of the classical Poisson theorem (i.e. where the x_{nk} are determined by $P\{x_{nk}=0\} = 1-\lambda/n$, $P\{x_{nk}=1\} = \lambda/n$, $k=1, \cdots, n$) it is shown that $M_n \leqq c[4\lambda^2/n - 3\lambda^3/n^2 + \lambda^4/n^3]$ where $F(x)$ is the Poisson distribution with parameter λ and c is a constant. (Received October 28, 1954.)

361*t*. F. L. Spitzer: *On interval recurrent sums of independent random variables.*

The partial sums S_n of independent identically distributed random variables X_i, $i=1, \cdots, n$, $n=1, 2, \cdots$, are known to be interval recurrent if the X_i have a density $f(x)$ in $L^p(-\infty, \infty)$ for some $p>1$, and if the distribution of $n^{-1/\alpha} S_n$ converges to the symmetric stable distribution of index α for some $1 \leqq \alpha \leqq 2$. The following limit theorems show how the value of α determines the recurrence behavior of S_n satisfying the above conditions. For every a and every $x>0$, $\lim_{n\to\infty} \Pr [\min_{1\leqq k \leqq n} |S_k - a| > xC(n, \alpha)] = E_{1-1/\alpha}(-x)$, where $C(n, \alpha) = (\alpha/2) n^{1/\alpha - 1} \sin (\pi/\alpha)$ if $1 < \alpha \leqq 2$, and $(1/2)\pi(\log n)^{-1}$ if $\alpha = 1$, and where $E_p(z)$ is the Mittag-Leffler function. Further, if $\{a_n\}$ is any monotone positive sequence, then the probability that $|S_n - a| \leqq n^{1/\alpha} a_n$ for infinitely many values of n is zero or one according as the series $\sum_{n=1}^{\infty} a_n$ converges or diverges. (Received November 17, 1954.)

Topology

362. R. D. Anderson: *Zero-dimensional infinite compact groups of homeomorphisms.*

The purpose of this paper is to contribute to the knowledge of the ways in which zero-dimensional infinite compact groups of homeomorphisms can operate effectively on locally connected metric continua. A group G of homeomorphisms is said to operate on a closed point set X in a strongly effective way if for any element g of G and any point x of X, $g(x) = x$ only if g is the identity. The principal result is to show that the dyadic group (all of whose elements except the identity are of infinite order) can operate in a strongly effective way on the (one-dimensional) universal curve. It is also noted that, rather trivially, the compact infinite-dimensional torus also admits the dyadic group operating in a strongly effective way. Such an example apparently is not generally known. Standard known examples of p-adic groups operating effectively on locally connected continua all involve a type of "branching" clearly not present if the group operates in a strongly effective way. The examples cited may also be used to exhibit regular almost periodic homeomorphisms of locally connected spaces with no periodic points. (Received November 15, 1954.)

363. R. D. Anderson and Mary-Elizabeth Hamstrom (p): *On spaces filled up by continuous collections of arcs.*

The principal result of this paper is the theorem that if G is a nondegenerate continuous collection of arcs filling up the compact metric continuum M, then no connected open subset of M is separated by any closed and totally disconnected point set. It follows immediately that M must be at least 2-dimensional at each of its points. A slight extension of the methods establishes that if in addition to the hypotheses above G is a Cantor manifold of dimension greater than 1, then M is not separated by any rational curve. (Received November 15, 1954.)

364*t*. R. M. Baer: *A characterization theorem for lattices with Hausdorff interval topology.*

A characterization theorem for Boolean algebras with Hausdorff interval topology has been given by Katetov [Colloq. Math. vol. 4, pp. 229–235] and Northam [Proc. Amer. Math. Soc. vol. 4 (1953) pp. 824–827]. The general lattice is the subject of the following theorem. First, write (with respect to some underlying partially ordered set) $\hat{a}$ for the set $[x: x \leqq a]$, and $\breve{a}$ for the set $[x: a \leqq x]$, and define, for nonempty *subsets* X and Y in the space, $X < Y$ to mean: if x in X, y in Y, then either $x<y$ or x and y incomparable (or $X \leqq Y$, in which case $x \leqq y$ or x and y incomparable). Theorem: a necessary and sufficient condition that the interval topology of a lattice L be Hausdorff is that, for every pair of elements a, b in L with $a<b$, there exist finite, nonempty subsets A and B in L such that both the following (i) and (ii) are satisfied. (i) $a<A\leqq b$, $a\leqq B<b$; (ii) $(\breve{x})$, $x\in A$, $(\hat{y})$, $y\in B$ is a covering of L. (Received September 23, 1954.)

365. Lida K. Barrett: *Regular curves and regular points of finite order.*

J. R. Kline has raised the question whether for any positive integer n greater than two, there exists a continuous curve such that between each pair of its points there are exactly n simple arcs mutually exclusive except for end points. J. H. Kusner has settled this question for n equal to three or four (*On continuous curves with cyclic connection of higher order*, Comptes Rendus de Varsovie vol. 25 (1932) Classe III, pp. 71–92). In this paper it is shown that for any integer n greater than two no such curve exists. The final part of this paper answers two questions raised by W. L. Ayres (*On the regular points of a continuum*, Trans. Amer. Math. Soc. (1931)) concerning continuous curves containing points of only two orders. Ayres conjectured that if $m>n>2$ then (1) the points of order m must be countable and (2) for some integer k, $m=k(n-1)$. Examples are given to show that neither of these conjectures is true. (Received November 10, 1954.)

366. Raoul Bott: *On the Sturm intersection theory.*

The complete set $X_\lambda(t)$ of fundamental solutions of a regular self-adjoint system of second order differential equations $Ly=\lambda y$ gives rise to a map X^L: $(t, \lambda)\rightarrow X_\lambda(t)$ of the t, λ plane into a certain Lie group H. It is shown that every self-adjoint boundary condition B defines a cycle $\phi(B)$ in H, such that the intersections of the curve $\lambda \to X_\lambda(a)$ with $\phi(B)$ (in the sense of algebraic topology) correspond precisely to the spectrum of the problem $Ly=\lambda y$ subject to B at $t=0$ and $t=a$. This point of view permits the generalisation of the classical oscillation and comparison theorems of Sturm (for the case $n=2$) to higher dimensional systems (as was done by Morse in *The Calculus of variation in the large,* Amer. Math. Soc. Colloquium Publications, vol. 18) and to general boundary conditions. They are all consequences of the fact that as X^L is defined on all of the (t, x) plane, the image of an oriented curve F in that plane under X^L will be homologous to zero and hence have total algebraic intersection zero with $\phi(B)$. These methods are applied to refine comparison theorems announced by M. Morse and E. Pitcher (*On certain invariants of closed extremals,* Nat. Acad. Sci. vol. 20 (1934) pp. 282–287) concerning periodic operators. (Received November 12, 1954.)

367. Eldon Dyer: *Certain transformations which reduce dimension. II.*

In this paper it is shown that if G is a continuous collection of mutually exclusive dendrons filling up a compact metric n-dimensional space, then G with respect to its elements as points is an $(n-1)$-dimensional space. The above statement is true if "dimension" means either Brouwer-Menger-Urysohn dimension or homology dimension over the integers modulo k, $k \geqq 2$. A corollary of this theorem is that if $\mathfrak{A}$ is either the additive group of real numbers modulo 1 or that of the integers modulo k, $k \geqq 2$, and L is a compact metric n-dimensional space (homology dimension with coefficient group $\mathfrak{A}$) having a nontrivial nth-homology group over $\mathfrak{A}$, then L is not filled up by a continuous collection of mutually exclusive dendrons. (Received November 12, 1954.)

368. W. M. Faucett: *Topological semi-groups and trees.*

A mob S is a Hausdorff space that admits a continuous associative multiplication. A compact connected Hausdorff space S is a tree if for every pair of points in S there exists a third point separating them. A preliminary result states that if there exists $k \in K$, the minimal two-sided ideal of the compact connected mob S, such that $K-k$ is the union of two separated sets, then either (i) every element in K is a left zero for S, or (ii) every element in K is a right zero for S. Let S be a metric tree and N the set of end points of S. It is shown that if S is a mob and if N is a group, then S has a zero. Further, if N is an abelian group, then S is abelian. Another result states that if S has a unit, then S is abelian if and only if N is contained in the center of S. All of these results have been verified for the compact Hausdorff case. (Received November 15, 1954.)

369B. H. A. Forrester: *Acyclic models and de Rham's theorem.*

Let $\mathcal{A}$ be a category, $\mathcal{G}_\Lambda$ the category of left Λ-modules, Λ a ring with unit. $F(\mathcal{A}, \mathcal{G}_\Lambda)$ denotes the category of covariant functors $T:\mathcal{A}\to\mathcal{G}$. A direct (inverse) model theory (R, Γ) is formed by a covariant functor $R: F(\mathcal{A}, \mathcal{G}_\Lambda) \to F(\mathcal{A}, \mathcal{G}_\Lambda)$ and a transformation $\Gamma: R\to I$ (resp. $\Gamma: I \to R$), where I is the identity functor. This is a natural version of the Eilenberg-MacLane concept of models (S. Eilenberg and S. MacLane, *Acyclic models,* Amer. J. Math. vol. 75 (1953) pp. 189–199). A representative Ψ of a functor $T:\mathcal{A}\to\mathcal{G}_\Lambda$ is a transformation $\Psi: T\to R(T)$ such that $\Gamma(T)\Psi$ is the identity transformation (for inverse model theories the order must be reversed);

if $\Psi(A): T(A) \to R(T)(A)$ for each $A \in \mathcal{A}$, but no naturality conditions are required, Ψ is called a semi-representation. The fundamental theorems of Eilenberg-MacLane can be established. Further, *if* (R, Γ) *is an inverse model theory,* K, L *are cochain functors,* $R(K)$ *and* $R(L)$ *are acyclic, and* K, L *are representable, and if* $f: K \to L$ *is a cochain map which is a homotopy equivalence in low dimensions, then* $f_*: H(K) \to H(L)$ *is an isomorphism.* If ϕ is the de Rham functor of differential forms, C is the real singular cochain functor, and $\int: \phi \to C$ is the Stokes' map, it follows that $\int_*: H(\phi) \approx H(C)$. It is easy to see then that $\int_*$ preserves products. (Received October 25, 1954.)

370t. F. B. Jones: *A note on homogeneous metric continua.*

Every compact, decomposable, metric continuum has a nondegenerate, aposyndetic decomposition such that (1) the decomposition (with respect to its elements as points) is a compact, homogeneous, metric continuum and (2) each element of the decomposition is a homogeneous continuum. Roughly speaking, the elements of the decomposition are atomic. (Received November 15, 1954.)

371B. V. L. Klee, Jr.: *Separation properties of convex cones.*

Except in rather special cases, the known separation theorems for convex sets require that the sets involved have no common point, and thus fail to cover the interesting case of two convex cones whose intersection and common vertex is the origin ϕ. This case is discussed in the present note, the principal results extending theorems of Aronszajn and of Tagamlitzki. A *ϕ-cone* in a topological linear space is a closed convex cone with vertex ϕ. For a ϕ-cone A, $A' = A \cap -A$. Theorem. *Suppose* E *is a locally convex topological linear space,* A *and* B *are* ϕ*-cones in* E *such that* $A \cap B = \{\phi\}$, *and* A *is locally compact. Then* E *admits a continuous linear functional* f *such that* $f < 0$ *on* $A \backslash A'$, $f = 0$ *on* $A' \cup B'$, *and* $f \geqq 0$ *on* $B \backslash B'$. *If, in addition, either* B *is locally compact or* E *is normable and* B *separable, then* E *admits* f *such that* $f < 0$ *on* $A \backslash A'$, $f = 0$ *on* $A' \cup B'$, *and* $f > 0$ *on* $B \backslash B'$. Examples show that these results cannot be substantially improved. In particular, Hilbert space contains a disjoint pair of linearly bounded closed convex sets which cannot be separated by any hyperplanes and hence contains a pair of ϕ-cones A and B which cannot be separated even though $A' = B' = A \cap B = \{\phi\}$. (Received October 18, 1954.)

372. Harold W. Kuhn: *A combinatorial lemma equivalent to the Brouwer fixed-point theorem.*

The following combinatorial lemma is equivalent to the Brouwer fixed-point theorem: Denote by I_n the cube in E_n formed by all points $x = (x_k)$ with $|x_k| \leqq 1$ for $k = 1, \cdots, n$ and let K be any subdivision of I_n by hyperplanes orthogonal to the coordinate directions. Suppose further that each vertex x of the subdivision is labeled with $\lambda(x) = (\lambda_k(x))$, where $\lambda_k(x) = \pm 1$ subject only to $\lambda_k(x) = x_k$ if $x_k = \pm 1$ for $k = 1, \cdots, n$. Then there is a rectangular n-cell of the subdivision with both labels in each coordinate direction appearing among its vertices. (Received November 12, 1954.)

373. E. B. Leach: *Cohomology theory in the space of n complex variables.*

Let R be an exact (schlicht) existence domain for an analytic function. Let Ω^p be the faisceau of pure differential forms of degree p, with complex analytic coefficients, defined near points of R. Dolbeault [C. R. Acad. Sci. Paris vol. 236 (1953) pp. 175–

177] has given an analytic characterization of $H^q(\Omega^p)$, the group of cohomology with complex-analytic p-form coefficients. In the case $q=1$, it has been shown that $H^1(\Omega^p)$ is trivial, i.e., it contains only the identity element. A proof of this is given by H. Cartan [Bull. Soc. Math. France vol. 78 (1950) pp. 29–64]. In this paper, it is shown that the groups $H^q(\Omega^p)$ are trivial for all $q \geqq 1$. By Dolbeault's theorem, this is accomplished by showing that certain systems of partial differential equations which possess local solution possess solution throughout R. This is done by a method similar to that of Cartan. (Received October 13, 1954.)

374. L. F. McAuley: *A note on generalized upper semi-continuous collections.*

With the knowledge that an aposyndetic connected topological space is a Hausdorff topological space, the following generalization of upper semi-continuous collections is useful in a study of aposyndeteic decompositions of continua. Suppose that G is a collection of point sets. A subcollection R of G is said to be a region in G provided that there exists no point set P in G^*-R^* such that (1) for g in G, $g\cdot P$ has no limit point in R^* and (2) R^* contains a limit point of P. An element g of G is said to be a limit element of a subcollection H of G provided that every region R in G which contains g also contains an element of $(H+g)-g$. A collection G of point sets is said to be upper semi-continuous provided that if (1) H is a subcollection of G and (2) g and h are two limit elements of H, then there exists a subcollection K of H such that (a) either g or h is a limit element of K and (b) the limiting set of K either (i) contains no point of $h-h\cdot g$ or (ii) contains no point of $g-g\cdot h$. (Received November 15, 1954.)

375*t*. P. J. McCarthy: *Non-archimedean uniform structures.*

A uniform structure on a set E is called non-archimedean if it has a fundamental system of entourages consisting entirely of entourages V satisfying $V\circ V=V$. A uniform space is called non-archimedean if its uniform structure is non-archimedean. These uniform spaces were defined and studied by A. F. Monna (Proc. Kon. Nederl. Akad. Wetensch. vol. 53, pp. 470–481, 625–637). It is shown that if E is a 0-dimensional space (Hausdorff or not), then there is a non-archimedean uniform structure on E compatible with the topology of E. Furthermore, if E is Hausdorff, it is precompact with respect to this uniform structure. Using this result it is proved that every 0-dimensional Hausdorff space is homeomorphic to a dense subset of a compact 0-dimensional space. (Received November 9, 1954.)

376. F. P. Palermo: *The cohomology rings of product spaces.*

Let K_1, K_2 be finite simplicial complexes. It may be shown by examples that $H(K_1\times K_2)$, the integral cohomology ring of $K_1\times K_2$, is not determined by $H(K_i)$, the integral cohomology rings of K_i, $i=1, 2$. In this paper it is shown that $H(K_1\times K_2)$ is completely determined by the following: $H(K_i)$ and $H(K_i, n)$, the cohomology ring of K_i with coefficients in the ring of integers mod n, for all integers n, together with the group homomorphisms $\Delta_n: H(K_i, n)\rightarrow H(K_i)$ and $h_{m,n}: H(K_i, n)\rightarrow H(K_i, m)$ for all integers m, n $(i=1, 2)$. Here Δ_n is the Bockstein coboundary homomorphism and $h_{m,n}$ is induced by the homomorphism which takes a cocycle x mod n into $(m/(m, n))\cdot x$ which is a cocycle mod m (J. H. C. Whitehead, Comment. Math. Helv. vol. 22 (1949) pp. 48–92). A ring $R(K_1, K_2)$ which is naturally isomorphic to $H(K_1\times K_2)$ is constructed from the rings $H(K_i)$ and $H(K_i, n)$ and the homomorphisms Δ_n and $h_{m,n}$ for all integers m, n. This construction gives a functor in the sense of Eilenberg and MacLane (Trans. Amer. Math. Soc. vol. 58 (1945) pp. 231–294). The construction can be extended to infinite simplicial complexes. This completes and extends the earlier work of M. Bockstein (C. R. (Doklady) vol. 40 (1943) pp. 339–342). (Received October 25, 1954.)

377t. J. P. Roth: *An application of algebraic topology of numerical analysis: Kron's method.* I.

It is shown that the process of solving an electrical network in terms of itself is formulable in terms of a "twisted" isomorphism of the homology and cohomology sequence of the 1-dimensional complex defined by the network. Specifically, the space of mesh currents coincides with the first homology group $H^1(K)$ (complex coefficient field), the space of branch (coil) currents with the group $C^1(K)$ of 1-chains; the equations (Kron, *Tensor analysis of networks*, New York, Wiley, 1939); $i = Ci'$ an expression of the natural homomorphism of $H'(K)$ into $C'(K)$. The space of node currents corresponds to the group of bounding zero chains, the map $I' = A_t I$ being an expression of the boundary operator (also $A_t i = 0$). The space of node potentials is the group of 0-cochains, $C_1(K)$ the space of coil voltages, $E = AE'$ the coboundary operator, $H_1(K)$ the space of mesh emf's and $e' = C_t e$ the natural homomorphism of $C_1(K)$ on $H_1(K)$. The mapping Z and its inverse Y are isomorphisms defined by the "hardware" of $C^1(K)$ and $C_1(K)$. It is shown that $Z' = C_t Z C$ has an inverse if Z is "power-definite —a generalization of Hermann Weyl's result (Revista Math. Hispano-Americana vol. 5 (1923))—as well as $A_t Y A = Y'$. (Received November 15, 1954.)

378. J. P. Roth: *An application of algebraic topology to numerical analysis: Kron's method.* II.

By suitable interpretation of Kron's method of solving a network by means of "tearing" in terms of homomorphisms of homology and cohomology sequences induced by simplicial mappings of 1 dimensional complexes, the author obtains a proof of the validity of this method. Specifically let K, $\tilde{K}$ be 1 complexes with the same number of branches, ϕ a simplicial map of $\tilde{K}$ on K. Let N be a matrix defining the isomorphism of $C_1(K)$ on $C_1(\tilde{K})$, bases for each space being chosen. Then if $\tilde{Y}$ is the inverse of $\tilde{Z}$, the inverse of Z is $N_t \tilde{Y} N$, computed exclusively in terms of the inverse $\tilde{Y}$ of $\tilde{Z}$ and the matrix N of the map ϕ having entries of 0, 1, -1. The application to numerical analysis arises from the fact that finite difference equations can be represented by "electrical" networks (Kron, Journal of Applied Physics vol. 24 (1953)). (Received November 15, 1954.)

379. Mary E. Rudin: *A property of normal spaces related to the existence of a Souslin space.*

A set S is called a Souslin space if (a) S is linearly ordered, (b) S is not separable, (c) there are at most a countable number of mutually exclusive segments in S. The existence of Souslin spaces is an open problem. It is proved that if a Souslin space exists, then there exists a normal Hausdorff space X with the following property: there is a sequence $M_1, M_2, M_3, \cdots$ of closed sets in X such that $M_1 \cdot M_2 \cdot M_3 \cdot \cdots = 0$ but $N_1 \cdot N_2 \cdot N_3 \cdot \cdots \neq 0$ for every sequence $N_1, N_2, N_3, \cdots$ of open sets such that $M_i \subset N_i$. (Received November 8, 1954.)

380. Annette Sinclair: *Pseudo locally compact spaces.*

The author defines a topological space $(\mathcal{R}, \mathfrak{N})$ to be $\mathfrak{N}'$-*pseudo locally compact* if any neighborhood by the $\mathfrak{N}$-topology of an arbitrary point p of $\mathcal{R}$ contains some neighborhood $N_p(\mathcal{R}, \mathfrak{N})$ by the $\mathfrak{N}$-topology such that the point set $\overline{N}_p(\mathcal{R}, \mathfrak{N})$ is compact by the $\mathfrak{N}'$-topology. Examples are given of a set of functions analytic in a region with a topology such that the function space so defined is not locally compact but has a property close to local compactness. If $\mathcal{R}$ is the set of functions analytic in an open region R and $\mathfrak{N}$ is the topology induced by the metric $d(f, g) = \sup_{z \in R} |f(z) - g(z)|$, then $(\mathcal{R}, \mathfrak{N})$ is not locally compact. Let $\mathfrak{N}'$ be the topology induced by the metric $\rho(f, g) = \sum_{n=1}^{\infty} (1/2^n) d_n(f, g)/(1+d_n(f, g))$, where $d_n(f, g) = \max_{z \in R_n} |f(z) - g(z)|$ for $\{R_n\}$ a certain monotone increasing sequence of R-covering sets. Then $(\mathcal{R}, \mathfrak{N}')$ is also not locally compact. However, the space $(\mathcal{R}, \mathfrak{N})$ is $\mathfrak{N}'$-pseudo locally compact. (Received November 8, 1954.)

381. Fred Supnick: *On the Hauptvermutung for polyhedral 3-manifolds with boundary.*

E. E. Moise proved the Hauptvermutung for 3-manifolds (Ann. of Math. (1952) pp. 96–114); R. H. Bing extended this to 3-manifolds with boundary (Ann. of Math. (1954) p. 150). In their work the 3-simplexes of the triangulations and their subdivisions are not necessarily *rectilinear* (i.e. straight-edged, flat-faced (open) tetrahedrons in E^3). In the present work this *rectilinear* restriction upon the simplexes of the triangulations and their subdivisions is made. (Two n-complexes are *isomorphic* if there is a one to one correspondence between the i-simplexes of one, and those of the other, such that incidence relations are preserved.) The following theorem is proved: *Let K_1 and K_2 be locally finite rectilinear simplicial 3-complexes in E^3, which are homeomorphic 3-manifolds with boundaries B_1 and B_2 respectively, B_1 and B_2 being locally finite rectilinear simplicial 2-complexes. Then K_1 and K_2 possess isomorphic rectilinear simplicial subdivisions. In particular, if K_1 and K_2 are finite (in number of simplexes), then K_1 and K_2 possess isomorphic rectilinear simplicial finite subdivisions.* Theorem 3 of Bing (ibid. pp. 148–149) plays an essential role in the proof. (Received November 12, 1954.)

382. Hidehiko Yamabe: *Compact transformation groups on spheres.*

Let G be a compact group of differential homeomorphisms on d dimensional sphere S^d, admitting one fixed point p_0. Assume there is a Riemannian structure invariant under G. Then eigenspaces of the Laplace Bertrami operator Δ give representation spaces of G, which are decomposed into irreducible spaces under G. On the other hand tangent space at p_0 turns out to be another representation space. Hence by applying approximation theorem and Schur's Lemma, one has d eigenfunctions of Δ which form a coordinate system around p_0. By applying a fixed point theorem, another point p_0' is obtained where all these d functions take the value 0. If the Riemannian structure is analytic, and if G is connected, this p_0' can be proved to be another fixed point of G by considering the intersection of zero points of these d functions. (Received October 4, 1954.)

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