voted to the discussion of upper and lower bounds of quadratic functionals and to Friedrichs' analysis of Trefftz's method. In the new addition, the author emphasizes a geometrical approach given by Prager and Synge (Quarterly of Applied Mathematics vol. 5 (1947) pp. 241–269). Similar questions were widely discussed by various methods in recent publications, and a part of the literature is mentioned by Courant in one of the longest footnotes of the book. In view of the great and deserved influence of the book, such a footnote cannot be passed over without comment. The footnote states that the theories indicated in §9 have been recently rediscovered and advanced by several authors. Let us emphasize that the word rediscovered is incorrect, since all papers refer either to Trefftz's method or to its analysis by Friedrichs. The purpose of the recent investigations was to simplify the method of Trefftz and to obtain stronger results. Incidentally, the footnote omits a reference to the work of Diaz and Weinstein (see e.g. *Schwarz' inequality and the methods of Rayleigh-Ritz and Trefftz*, Journal of Mathematics and Physics vol. 26 (1947) pp. 133–136) which uses an analytic approach. The various methods were recently analyzed by Diaz, Collectanea Mathematica vol. 4 (1951) pp. 1–47, specially pp. 41–46, who pointed out the advantages of the analytic method, which yields simpler formulas and more correct results than the geometric procedure.

The tendency of understatement of the work done outside the author's circle reappears in the footnote on page 175, where Lord Rayleigh's contributions to variational methods are evaluated as follows: "Even before Ritz, such ideas were successfully employed by Lord Rayleigh."

The book preserves not only all the high points of the original, but also some of its misprints, which, being by now classical, only add to the pleasure of the reader. It is with great expectations that the reviewer is looking forward to the translation and additions to the second volume.

A. WEINSTEIN

Forced oscillations in non-linear systems. By C. Hayashi. Osaka, Nippon Printing and Publishing Co., 1953. 14+164 pp. $4.50.

This book was written "to provide engineers and physicists with practical knowledge concerning the important subject of non-linear oscillations," in particular forced oscillations governed by non-linear equations of the second order.

The text is divided into two parts; the first concerns the stability of steady state oscillations, whereas the second is devoted to a dis-

cussion of the geometry of solutions of a slightly perturbed system of two first order equations. The first chapter considers the equation $v'' + f(v)v' + g(v) = e(t)$ $(' = d/dt)$, where e is periodic. The existence of a periodic solution of the same (or integral multiple of the) period of e is assumed, and the stability of this solution is defined in terms of the characteristic exponents of the corresponding equation of the first variation. A discussion of the Mathieu equation and Hill's equation follows in which "approximate" characteristic exponents are obtained, and corresponding stability regions are plotted. The question of the degree of approximation is not considered. In the next three chapters special cases are worked out in detail, a typical equation being $v'' + 2\delta v' + (c_1 v + c_3 v^3) = B \cos \nu t$, where δ, c_1, c_3, B, ν, are constants, with a subharmonic of the form $v = k_1 \sin t + k_2 \cos t + k_3 \cos \nu t$ assumed. Various experiments with electrical oscillatory circuits are described and the results compared with the approximate stability regions determined. In the last two chapters (Part II) the integral curves of some special cases of the system $x' = X(x, y)$, $y' = Y(x, y)$ are depicted in the vicinity of the equilibrium points. The solutions of the van der Pol equation are also sketched.

The book is replete with excellent illustrations.

Although from the mathematical viewpoint the equations and solutions considered are rather special, this work should serve its purpose well, and in addition provide mathematicians with a supply of nicely illustrated examples of forced oscillations.

E. A. CODDINGTON

Hypergeometric and Legendre functions with applications to integral equations of potential theory. By Chester Snow. (National Bureau of Standards, Applied Mathematics Series, no. 19.) Washington, Government Printing Office, 1952. 11+427 pp. $3.25.

This is the second edition of a monograph of which the first edition appeared in 1942 (as MT15 in the Mathematical Tables Series of the National Bureau of Standards) and was exhausted within a year, a second printing being sold out soon afterwards. Continuing, and indeed increasing, demand for the monograph and the impending retirement of the author from active service with the National Bureau of Standards were the motivations for the present second edition. Known misprints have been corrected, some chapters have been rewritten and expanded, and a new chapter (on confluent hypergeometric functions) has been added.

A brief indication of the contents, chapter by chapter, follows.

I. Definitions and preliminary formulas relating to the gamma function and Gauss' hypergeometric series.