identified with perfectly good mathematical operators (unbounded, but closed and densely defined). This development was independently arrived at by J. M. Cook, whose approach was perhaps less concrete but more invariant, using tensors over a Hilbert space rather than square-integrable functions of several variables. The large number of applications and comments that are made in the book may be judged from the fact that the mathematical part of Cook's paper is 10 pages long while the corresponding material in the book is scattered through some fifty odd pages, without including proofs of such results as the self-adjointness of the canonical P's and Q's.

There is no doubt that the author has succeeded in his aim "to present basic sections of field quantum theory in a consistent mathematical language without carrying out all mathematical deductions with complete rigor." It is regrettable that the standard of rigor and the mode of presentation are not such as to make the mathematical consistency of many of the sections wholly manifest and explicit, but one must be thankful that the ice has been broken with this first large-scale attempt to deal mathematically with quantum fields. While the book is not easy reading in detail and is not closely integrated in the large (the relation between statistics and dynamics seemed blurred to the reviewer), a mathematician interested in a more rigorous and searching approach to the specific problems of quantum field theory may well find this a stimulating and useful book.

I. E. SEGAL

Collected mathematical works. By H. Bohr. Ed. by E. Følner and B. Jessen. Copenhagen, Dansk Matematisk Forening, 1952. Vol. I, 34+771 pp., 1 plate; vol. II, 9+852 pp., 1 plate; vol. III, 10+985 pp., 1 plate. 110 kr.

These volumes are an impressive reminder that Harald Bohr was one of the great mathematicians of the first half of this century. Most of his work is now generally regarded as "classical" and much of it has been superseded or subsumed in more general results, but this fact is in itself a tribute to the importance and fundamental character of his work and to the stimulation which it gave to other mathematicians. His papers are sure to be admired by generations of future mathematicians and the publication of his collected works is accordingly most welcome.

Aside from the intrinsic interest of Bohr's papers, they are models of mathematical exposition. Their eminent readability makes them excellent reading for students of analysis at practically all levels. While this readability is largely due just to good mathematical writ-

ing, it stems to some extent from the use of a more leisurely style and the inclusion of more expository material than seems fashionable or permissible nowadays. Evidently Bohr was not a believer in the dictum that "authors must present their results in as compact a form as possible and should avoid space-consuming exposition and discussion" —with the happy result that almost any one of his papers can be read as an entity in itself.

This collection contains all of Bohr's mathematical writings with the exception of obituaries, elementary articles in Danish, textbooks in Danish, and the address he delivered at the 1950 International Congress in connection with the presentation of the Fields medals. In particular it includes his joint article with Cramér *Die neuere Entwicklung der analytische Zahlentheorie* in the Enzyklopädie der mathematischen Wissenschaften (1923) and his monograph *Fastperiodische Funktionen* in the series Ergebnisse der Mathematik und ihrer Grenzgebiete (1932). His doctoral dissertation *Bidrag til de Dirichlet'ske Rœkkers Theori* (1910) is given both in the original Danish and in English translation, and English summaries are provided for the other research papers in Danish. An English translation of an interesting lecture *Looking Backward*, given by Bohr on his 60th birthday, appears as a preface. Aside from the translations, the papers have been reproduced by a photographic method. The last volume contains a fifteen-page general index and a dozen pages of notes giving cross-references among the papers and the most important references to later work by other authors.

Bohr's work is described rather aptly by the following quotation from an introductory note by the editors of these volumes, Erling Følner and Børge Jessen: "Bohr's contribution to mathematics is one of great unity. His investigations in his youth on Dirichlet series and the Riemann zeta-function established his reputation as one of the leading analysts of his time. The problem as to which functions may be represented by Dirichlet series led to his main achievement, the theory of almost periodic functions, on which the greater part of his later work is concentrated. With the exception of a small number of papers, dealing mostly with problems in the general theory of analytic functions, his papers on other subjects are all closely connected with, or are even auxiliaries of, his investigations on these main subjects." A fuller discussion of Bohr's life and the significance of his work may be found in Jessen's memorial address, which is given (in English) as a fitting postlude to this collection.

Mathematicians everywhere will be grateful to Bohr's countrymen for publishing these collected works, especially in view of their low

price and handsome physical appearance. They are a "must" on the analyst's bookshelf.

PAUL T. BATEMAN

Methods of mathematical physics. By R. Courant and D. Hilbert. Vol. I. 1st English ed. New York, Interscience, 1953. 16+561 pp. $9.50.

This book is mainly a translation of the second German edition of the celebrated textbook written by Courant and his collaborators at the University of Göttingen. While Hilbert did not take any active part in its preparation his name was put as a co-author to indicate the tremendous influence which he exerted on the mathematical thinking of his surroundings and indeed of the whole mathematical world. Since the content of the book is well known to every worker in the field, let us recall only that the text covers the following subjects: linear transformations and quadratic forms, development of arbitrary functions in series of orthogonal functions, linear integral equations, calculus of variations, eigenvalue and vibration problems, application of variational calculus to eigenvalue problems, and special functions.

This work presents a cross-section of the subject matter as it appeared to Courant's school in Göttingen in 1931. It is, of course, not a valid criticism that the translation contains no newer developments, inasmuch as the author states in his preface that the pressure for publication of an English "Courant-Hilbert" became irresistible. However, regardless of its contents as seen today, one may reasonably ask what will be the reaction of some students who will miss the possibility of using the work as a reference book in which each theorem is stated and numbered in a precise way. Instead of a catalogue of theorems, the reader will find an artistic exposition of the profound meaning of mathematical thinking. The author is greatly aided in his exposition by his natural inclination to somewhat fluid statements which greatly stimulate the imagination of the reader. The reviewer gratefully acknowledges being one of the large community of scientists outside of Göttingen who were influenced by Courant's book.

Very few additions and alterations have been included in the English edition; of these, only the interesting appendix by W. Magnus has been mentioned in the preface. This appendix deals with the question of how a set of linearly independent spherical harmonics in three variables is transformed if the coordinate system is rotated.

The main addition not mentioned in the introduction is a paragraph entitled: Reciprocal quadratic variational problems (chapter 4, §11, pp. 252–257), which complements a preceding section (§9) de-