folds and manifolds of constant curvature are considered and results on the existence of harmonic and Killing tensors obtained. Several measures of deviation from constancy of curvature are defined and it is shown that if this deviation remains within certain limits the Betti numbers are unaltered. To the reviewer these relations between curvature and Betti numbers are among the most interesting consequences of the theory.

A chapter is devoted to the special case of semi-simple group manifolds, and here the deviation from flatness is explicitly calculated. Following this is a chapter on Riemannian manifolds carrying additional structure in the form of an affine connection with torsion (as in the case of the group space). A chapter on Kähler manifolds completes the main body of the book. This latter chapter includes important applications of the theory; for example, it is shown that if the deviation of the curvature of a Kähler manifold from constant positive holomorphic curvature remains within prescribed limits, then there are no effective harmonic tensors, and hence the Betti numbers are those of the complex projective space.

A final chapter by S. Bochner entitled "Supplements" is perhaps the most significant since it contains indications of new directions in which the theory is proceeding. Important as it is, however, the topics are so diverse as to make a brief summary impossible. The titles of the eight sections are: (1) Symmetric Manifolds, (2) Convexity, (3) Minimal Varieties, (4) Complex Imbedding, (5) Sufficiently Many Vector or Tensor Fields, (6) Euler-Poincaré Characteristic, (7) Noncompact Manifolds and Boundary Values Zero, (8) Almost Automorphic Vector and Tensor Fields.

The book contains in addition to the above a chapter outlining the relevant differential geometry and tensor analysis and a brief introduction to complex-analytic manifolds. It should be remarked that in addition to interesting theoretical contributions, Yano should be commended for the careful, readable exposition he has given here of this topic in global differential geometry.

WILLIAM M. BOOTHBY

Higher transcendental functions. By A. Erdélyi, W. Magnus, F. Oberhettinger, and F. G. Tricomi. Based, in part, on notes left by Harry Bateman and compiled by the Staff of the Bateman Manuscript Project. New York, McGraw-Hill, 1953. Vol. I, 26+302 pp., $6.50. Vol. II, 17+396 pp., $7.50.

These two volumes compiled by the "Bateman Manuscript Project" represent a stupendous accomplishment. Under the able directorship of A. Erdélyi, the thirteen chapters of the two volumes were written by W. Magnus, F. Oberhettinger, and F. G. Tricomi, with the aid of a number of research assistants. The project was inspired by the idea of the late H. Bateman, from the California Institute of Technology, to prepare a compilation on a grand scale of the "special functions" (the definition of this term is somewhat arbitrary) occurring in mathematical analysis. When Bateman died in 1946, he left a considerable material of notes, system-cards, and drafts pertaining to such a plan. In the present undertaking they were used only to a limited extent, but in a way they served as an inspiration and the basis of the whole plan.

The work on the "Bateman Manuscript Project," that is, on the writing of the present volumes, extended with varying intensity from 1948 to 1952. It was the result of a joint effort on the part of the Office of Naval Research and the California Institute of Technology. Its history is told in two Prefaces to the first volume, written by Mina Rees and E. C. Watson, representing the institutions mentioned. The difficulties of such a compilation as was planned by Bateman and is carried out in the present work are enormous. They are due not so much to the vastness of the pertinent material but rather to the intrinsic difficulty of formulating and following clear and consistent principles in organizing it. (The term "Higher Transcendental Functions" appears just as arbitary as the term "Special Functions.") In whatever way one designs the underlying viewpoints, there will be ample interconnections and overlappings resulting in the occurrence of the same subject matter, or part of it, under different headings. (For instance, in the present system, Legendre polynomials appear in Chapter III as a special section and in Chapter X likewise.)

The single chapters have been compiled and written by various authors; see below, where we shall indicate the authors by M, O, and T, respectively. The Foreword is modestly silent about the share of Professor Erdélyi which, judging from diverse signs, must have been considerable—not only in terms of organization and coordination but also in actual contributions. By and large it seems to be a good policy that unity of style, form, and procedure were not too rigidly enforced. In some chapters more details (proofs) are presented than in other chapters which are more concise with respect to derivations.

References are given in each chapter, following the principle of usefulness rather than completeness. The symbols employed were chosen with particular care. Other notations occurring frequently in the literature are listed. An important feature is the listing of exhaustive tables and systematic collection of formulas which will greatly increase the efficiency of the work.

The books produced by this imposing effort make delightful reading with much color and usefulness at the same time. As is pointed out in the Foreword to the first volume, they remind one somewhat of a new-fashioned version of the *Modern Analysis* of Whittaker and Watson, except for the absence of interesting problems and exercises (which were of course outside the scope of the Project).

We now list briefly the contents of the single chapters and their outstanding features.

Chapter I (O), The Gamma functions. We find here sections dealing with the generalized zeta function, with the Riemann zeta function, and with the Bernoulli and related polynomials.

Chapter II (M), The hypergeometric function. We find here a rich collection of well known formulas and relations. The 15 relations of Gauss between contiguous functions and Kummer's 24 solutions of the hypergeometric equation are given, moreover 15 integral representations. In addition, this chapter lists the less generally known quadratic and higher transformations (47 items) and the degenerate cases (29 items) which are hardly available except in original papers.

Chapter III (O), Legendre functions. The representations of $P_\nu^\mu(z)$ and $Q_\nu^\mu(z)$ (first and second solution of the Legendre differential equation) are given in terms of hypergeometric functions of various arguments; this list contains 18 items for each function. Further useful sections deal with toroidal and conical functions.

Chapter IV (M), The generalized hypergeometric series.

Chapter V, Further generalizations of the hypergeometric function, dealing with MacRobert's E-function, Meijer's G-function (with a list of 75 special formulas) and with hypergeometric functions of several variables. Horn's list is given, containing 34 items; moreover, an extensive list of systems of partial differential equations, transformations, etc.

Chapter VI (T), The confluent hypergeometric function, involving many special functions under this heading which receive systematic and more extensive treatment in other parts of the work. The sections dealing with the asymptotic behavior (if the argument or some of the parameters become large), and those relating to the position of the zeros, deserve particular mention.

Chapter VII (O), Bessel functions. Asymptotic expansions (for large values of the argument and the order) are treated with particular exhaustiveness listing numerous results which do not appear in the Treatise of Watson (1922); they are due to Watson, van der Corput, Langer, and others. The same is true for the section on the zeros. The second part contains a huge system of formulas.

Chapter VIII (T), Functions of the parabolic cylinder and of the paraboloid of revolution.

Chapter IX (M, T), The incomplete gamma functions and related functions.

Chapter X (T), Orthogonal polynomials. Various parts of this chapter go considerably beyond the material dealt with in the reviewer's book on the same topic: for instance, the work of Tricomi on the asymptotic behavior and the zeros of Laguerre polynomials and the properties of the important polynomials of Pollaczek.

Chapter XI (M), Spherical and hyperspherical harmonic polynomials, a very elegant and important chapter much of which was taken from unpublished notes of a course given by G. Herglotz.

Chapter XII, Orthogonal polynomials in several variables.

Chapter XIII (T), Elliptic functions and integrals.

Both volumes have a subject index and index of notations which will greatly increase the usefulness of the work.

The mathematical public will be indebted to the collaborators and to the editor of this project for their accomplishment.

G. SZEGÖ

The algebraic theory of spinors. By C. Chevalley. New York, Columbia University Press, 1954. 8+131 pp.

Most of the results of the theory of spinors are due to its founder E. Cartan; and, until this year, the only place where they could be found in book form was E. Cartan's own *Leçons sur la théorie des spineurs*, published in 1938. Strangely enough, the deep and unerring geometric insight which guided Cartan's researches, and places him among the greatest mathematicians of all time, is too often smothered in his books under complicated and seemingly gratuitous computations: witness, for instance, his fantastic definition of spinors (at the beginning of the second volume of the work quoted above) by means of the coefficients of a system of (non-independent) linear equations defining a maximal isotropic subspace! The reason for this is most probably to be found in the fact that E. Cartan's generation did not have at its disposal the geometric language which modern linear algebra has given us, and which now makes it possible to express in a clear and concise way concepts and results which otherwise would remain hopelessly buried under forbidding swarms of matrices.

The remarkably skillful way in which this language is used is cer-