BOOK REVIEWS

Tensor analysis. Theory and applications. By I. S. Sokolnikoff. New York, Wiley, 1951. 10+335 pp. $6.00.

This volume is an outgrowth of a course of lectures given by the author over a period of years at the University of Wisconsin, Brown University, and the University of California. It consists of a general introduction to vector analysis and tensor calculus with special emphasis on the various applications. A short bibliography of some of the texts of the subject is given on p. 327 and an index is on pp. 329–335.

The book consists of six chapters. The first two deal with linear vector spaces, matrices, and the calculus of tensors. The last four chapters consist of applications to geometry, analytical mechanics, relativistic mechanics, and mechanics of continuous media. Throughout the book there are exercises to illustrate the already established concepts and formulas.

In the first chapter, the student is introduced to the concept of vector by intuitive and geometrical means. Thereafter, linear vector spaces over the fields of real and complex numbers, Euclidean spaces of n dimensions, linear transformations and matrices, reductions of quadratic forms, orthogonal and Hermitean matrices are discussed. In the second chapter, the usual concepts of the tensor calculus are introduced with a short section on relative tensors. After a short treatment of the Riemannian metric tensor and the Christoffel symbols, covariant differentiation is defined by writing the well known formulas. By means of Ricci's commutation rule for the covariant differentiation of tensors, the Riemann-Christoffel tensor is obtained. After treating the Ricci tensor and the Riemannian (including the Euclidean) spaces, this second chapter closes with a discussion of the generalized epsilons and Kronecker deltas together with applications to determinants.

The third chapter, on geometry, begins with a short exposition of the development of non-Euclidean geometries. The author defines the metric of a general Finsler space of which Riemannian geometry is a special case. The remainder of the chapter is devoted to the study of classical differential geometry of curves and surfaces in Euclidean three-dimensional space, as is done in such books as *An introduction to differential geometry*, by L. P. Eisenhart, Princeton University Press, and *Applications of the absolute differential calculus*, by A. J.

McConnell, Blackie. After showing how the sphere and pseudosphere are physical representations of Riemannian elliptic geometry and Lobachevskean hyperbolic geometry, the author obtains the Serret-Frenet formulas, curvature, and torsion of a space curve. The differential equations of the geodesics of a Riemannian manifold are obtained by means of the Euler-Lagrange differential equations in the calculus of variations. In the theory of surfaces, it is shown how the Gaussian curvature is related to the Riemann-Christoffel tensor. Among the topics discussed on surface theory may be mentioned the first and second fundamental forms, the equations of Gauss, Mainardi-Codazzi, and Weingarten, Meusnier's theorem, principal curvatures, and the lines of curvature. This third chapter concludes with a short excursion into n-dimensional space.

In the fourth chapter, on analytical mechanics, the three general laws of motion of Newton are stated on p. 199. The motion of a particle in three-dimensional Euclidean space is studied by means of general curvilinear coordinates. Among the subjects covered are such topics as work, energy, the Lagrangian equations of motion, the energy equation in a conservative field of force, Hamilton's Principle of Least Action, the Hamiltonian equations, holonomic and non-holonomic systems, and virtual work. After showing how this applies to a constrained motion on a surface, the author extends the preceding concepts to general holonomic dynamical systems with n degrees of freedom. In the concluding part of this fourth chapter, there is a discussion of potential theory including the theorem of Gauss and the equation of Poisson. Elsewhere the notions concerning dynamical systems have been extended so as to include not only dynamical trajectories, but also brachistochrones, catenaries, velocity systems, and other families of curves in a given field of force of a Riemannian space of n dimensions. Also there is a development of potential theory in a Euclidean universe of n dimensions in other works.

The fifth chapter, on relativistic mechanics, is mainly concerned with the special theory of relativity. The two Einstein postulates of 1905 are stated on p. 265. From these, the Lorentz-Einstein transformation is deduced. After a discussion of the Minkowski velocity and acceleration, the energy equation and the gravitational equations of Einstein are obtained. Next it is shown how to solve Einstein's equations for the gravitational field produced by a spherically symmetric mass particle, which situation corresponds to the gravitational field of the sun. This is followed by a discussion of the planetary orbits in such a field. This chapter concludes with some remarks concerning the advance of the perihelion of Mercury.

The sixth and final chapter, on mechanics of continuous media, contains a general formulation of the basic concepts of mechanics of continua and a derivation of the fundamental equations governing the behavior of continuous media. The treatment of this chapter follows the ideas of F. D. Murnaghan. (See the paper, *Finite deformations of an elastic solid*, Amer. J. Math. vol. 59 (1937) pp. 235–260.) For the deformation of a continuum of identifiable material particles from its undeformed state into its deformed state, the Lagrangian and Eulerian strain tensors are obtained. After discussing the physical significance of the components of these tensors, the author finds the compatibility relations on the Eulerian strain tensor, which for the linearized case reduce to the conditions first obtained by B. Saint Venant in 1860. The deformation of a medium at any point is characterized by a certain quadratic form, which is represented geometrically as a quadric surface, called the strain quadric. From this is deduced the fact that the principal directions are those orthogonal directions in the undeformed state which remain orthogonal after deformation. Next it is shown that the state of stress at a point of a body, in equilibrium under prescribed surface and body forces, is characterized by a certain symmetric tensor, called the stress tensor. For this state of stress, the stress quadric of Cauchy is derived. The differential equations of a body in a state of equilibrium under the action of prescribed body and surface forces are obtained. It is proved that if the virtual displacement consists of translations and rotations, then Killing's equations hold, that is, the virtual work in an arbitrary rigid virtual displacement is zero. In the study of stress-strain relations, the author makes use of the first law of thermodynamics and deduces a law that includes the stress-strain relations of the classical theory of elasticity as a special case. This final chapter is concluded with the derivation of the equations of elasticity and also of fluid motion.

The material of this book is given in a straightforward and elementary fashion. The ideas and concepts are presented in such a manner that it is possible for a student with only a course in advanced calculus to have no difficulty in comprehending the subject matter. The last few chapters on the various applications would perhaps influence the more advanced student of tensor analysis to desire its possession. This book is a good introduction to the subject of tensor analysis, its usefulness, and its various applications.

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